A FORMULATION OF INELASTIC STRUCTURAL DESIGN UNDER UNCERTAINTY FOR COST-EFFECTIVENESS AS GAME AND DECISION THEORY PROBLEMS

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CERTIFICATE

This is to certify that the thesis entitled 'A Formulation of Inelastic Structural Design Under Uncertainty for Cost-effectiveness as Game and Decision Theory Problems' by N. Gopalakrishnan Nair, for the award of the Degree of Doctor of Philosophy, of the Indian Institute of Technology, Kanpur is record of bonafide research work carried out by him under my supervision and guidance. The thesis work, in my opinion, reached the standard fulfilling the requirements for the Doctor of Philosophy Degree. The results embodied in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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SYNOPSIS

The thesis is an attempt to make use of the modern concepts of engineering system design and theories of decision-making to evolve a methodology for design of " well-balanced" structures. The application of several decision techniques extensively used in optimal control theory, management science, operations research etc. are investigated for this purpose with special emphasis on the feasibility of applying game theory for inelastic structural design.

The nature of structural design decisions is analyzed and a methodology for structural design that pays attention not only to the structural safety aspects, but also to the economic, social, functional and other involvements of design is proposed. The approach to the design is one of dividing the problem into decisions at comparatively smaller levels at which the decisions can be tackled rationally and more effectively, and of synthesizing the decisions so made to global decisions by means of appropriate methods. Structural design is divided into five major

decisions namely the selection of site, selection of functional configuration, choice of structural concept, arrangement of members and selection of connections, and finally the choice of materials and proportioning of members. The decisions are to be sequentially carried out in the framework of a learning mechanism or heuristic procedure, and may be iterative in nature. Decision models are proposed for all the first four steps which include gaming, checklist method, and other subjective decision models. Of these decisions, the choice of structural concept is further studied and illustrated through examples. The last decision namely the selection of materials and proportioning of members is studied in detail as follows.

The proposed formulation of design for the choice of structural members treats structural design as a decision making process under certainty, risk or uncertainty depending upon the nature of information or design data available. Also, the method uses a direct design concept in which the force-deformation relations of cross sections of structural members are obtained as the output of the solution. Knowing the force-deformation relations, the cross sections are subsequently chosen subject to certain optimization criterion(cost, cost-effectiveness etc.). The method is essentially developed for inelastic materials with or without creep effects although it is equally applicable to elastic cases

as well. A cost-effectiveness criterion that trades off the cost of the structure with the expected cost of failure considering several modes of failure for the same structure is developed and used as the criterion of design. This method also determines the optimum safety level consistent with economy, and also considers the consequences of failure and ductility requirements. Serviceability requirements are satisfied for an arbitrarily specified normal load condition.

Inelastic structural design for given load conditions is formulated as an optimal control problem which satisfies the serviceability requirements under normal load conditions. A design method for risk situations (random state of loading) making use of cost-effectiveness criterion is also proposed. The two methods together give the force-deformation relations satisfying the values in design; safety, serviceability, ductility and economy. The concepts in these two formulations are subsequently made use of to formulate a general stage by stage decision process for proportioning the members under conditions of uncertainty.

The decisions involved in the proportioning of members under uncertainty are divided into four stages. In the first two stages, a game theory model in which structural action is simulated as a game played by structure against nature is considered to construct the force-deformation

relations required for serviceability and optimum cost-effectiveness under uncertain load conditions. Two consecutive plays of the game are considered. The first play called normal play is formulated as a differential game problem for satisfying serviceability requirements. The uncertainties in the timing, sequence, position and direction of loads are considered as the strategies of nature. The second play called survival play is formulated as a statistical game of minimax type in which the unknown parameters of probability density functions of the loads are taken as the strategies of nature. Survival play makes use of cost-effectiveness criterion as the payoff. In both plays, the force-deformation relations of structural cross sections are the strategies of structure. The optimal strategy of structure called task curve in the proposed game is the design output.

In the third stage of decision, the task curves are corrected for nonmeasurable uncertainties mainly arising due to 'human errors' in design, construction and operation by introducing proper marginal factors evaluated by a decision theory approach. In the fourth stage of decision, the material and dimensions of the members are chosen matching their force-deformation relations with the corrected task curves. This choice of cross section is for minimum cost or cost-effectiveness and is to be considered as choices under certainty, risk or uncertainty. The

random behaviour of structural members is idealized as a Markov process. The uncertainty in the randomness is taken into consideration by a Bayesian Markov process. The concepts involved in the proposed method are compared with those in the existing design methods.

The emphasis in the present investigation is on the analysis and formulation of the generalized structural design problem rather than on the computational aspects of any specific problem. The method is illustrated at all stages by means of 13 suitable illustrative examples of both elastic and inelastic structures with or without creep effects. The method attempts to eliminate the arbitrariness in the design process and takes into account many of the problems that may arise in a design activity. The proposed method has a deterministic format in spite of the probabilistic nature of data.

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NOTATIONS

$\mathcal{A}$ = admissible set of control variables;

$\underline{\bar{A}}$ = admissible region of the Euclidean n-space;

$A(t, \underline{x})$ = upper bound to the control variable α in the separable type of constraints;

A_1 = boundary surface of the region R on which load $\underline{w}$ is specified;

A_2 = boundary surface of the region R on which displacements are specified;

A, B, C = component structural elements of a total system;

A_i, B_i, C_i, D_i = alternative actions in each of A, B, C, D respectively;

Λ^{-1} = covariance matrix in a multivariate distribution;

a = material constants defining the creep characteristics;

a_{ij} = coefficient matrix in equilibrium equations;

$a_1^i \ldots a_k^i$ = the k attributes of the ith alternative action;

$\underline{\bar{B}}$ = boundary surface to the state variable space that limits deformation;

$B(t, x)$ = lower bound to the control variable $\underline{\alpha}$ in the separable type of constraints;

B = alternative events;

$\underline{\bar{C}}$ = hypersurface in Euclidean space defined by compatibility conditions;

C_T = total cost derived by cost-effectiveness (generalized cost);

C_{Tm} = cost-effectiveness of member m;

C_s = cost of the structure;

C_{si} = cost of ith alternative structure;

C_{sm} = cost of member m in a structure;

C_i = cost of ith mode of failure;

$C(\underline{w})$ = cost of failure when load vector of magnitude $\underline{w}$ is acting on the structure;

C_{sc} = cost of unserviceability of structure;

C_d = cost of damage of structure;

C_p = cost of partial collapse of structure;

C_w = cost of collapse of structure with warning;

C_c = cost of collapse of structure without adequate warning

C_{IN} = loss in the input cost of the system;

C_F = total expected cost of failure;

C_L = cost of consequence of failure or loss in the output or benefit of the system

C_R = cost of structure salvaged after failure or operation terminated;

C_f = cost of failure (single mode);

C_O = cost of one sequence of design;

$C_1, \ldots C_n$ = cost of the structure obtained in successive trials of design;

c_j = the change in energy in the jth cross section in an interval ds;

d_i = the allowable permanent deformation in the loading process;

d_i' = the allowable permanent deformation in the unloading process;

d_i^* = the constant in Ramberg-Osgood function;

E^n = Euclidean n-space;

$E(\cdot)$ = represents the expected value of an event;

e_i = change in the external work + complementary work done by the ith load in an interval ds;

$\bar{\underline{F}}$ = force space;

$\underline{F}=\{F_1, \ldots F_n\}$ = force vector representing the forces in n force-deformation relations at N cross sections, state variable in force space $\bar{\underline{F}}$;

F_i^c = maximum force F_i attained by the cross section in the normal load condition;

F_i^* = maximum force attained by the ith force in the structure (in the cost-effectiveness analysis);

$\underline{F}^*(s)$ = optimal path of state vector $\underline{F}$;

$F_p(\cdot)$ = cumulative distribution function, also marginal distribution function;

F_i' = force at which an unloading or reloading starts;

F_M = arbitrarily specified force that enters as a constant in Ramberg-Osgood function;

$\underline{F}(s)$ = the path of state vector $\underline{F}$;

$\underline{F}(0)$ = initial value of state vector at s or t = 0;

$F^1, \ldots F^N$ = states that may be occupied by the force at y = y in the Markov idealization of stochastic behaviour of force-deformation relations;

F_R, Y_R, Y_R^* = force, deformation and maximum deformation respectively in the task curve;

F_L, Y_L, Y_L^* = force, deformation and maximum deformation respectively of shifted task curve;

$\underline{f} = (f_1, \ldots f_n)$ = the functions in state equation;

G_0 = change in the performance index or pay-off in an interval ds;

G_{01} = change in the internal energy for the entire volume of structure in ds;

G_{02} = the sum of the change in external work and complementary work done by all the loads $\underline{w}$ in an interval ds;

g_i = $\frac{dzi}{dt}$ = functions in state equation representing creep;

g_0 = the value of G_0 in a time-dependent deformation with jumps in state variable;

g_A, g_B, g_C = gradation factors used in the calculation of utilities;

H = Hamiltonian

h = terminal pay-off in differential game;

i, j, k, l, m, n, p, = numbers used as subscripts and superscripts;

K_T = cost-effectiveness factor;

K_i = the ratio of the cost of ith mode of failure to the cost of structure;

$K(\underline{w})$ = ratio of the cost of failure at load $\underline{w}$ to the cost of structure;

$K(\underline{x})$ = functions in compatibility condition;

k = number of force deformation relations in a section;

L = length of the member;

$\underline{\bar{M}}$ = manifold;

$\underline{\bar{M}}^F$ = manifold in force space;

$\bar{\underline{M}}^w$ = manifold in load space;

M(s) = minimum value of performance index in optimal control formulation and the value of the game in differential game at s;

m_f = interaction factors in the choice of structural concept;

m = number of loads acting on the structure;

$\underline{m} = \{m_1, \ldots m_n\}$ = means of n forces derived from those of loads through equilibrium equations;

$N(\cdot)$ = represents normal distribution;

N = number of cross sections;

n = number of forces and deformations;

P = performance index;

$P(\cdot)$ = probability of an event;

P(y) = stochastic matrix at y;

$P(\underline{w})$ = probability of occurrence of load $\underline{w}$;

P^* = value of differential game

p_f = probability of failure;

p_1, p_2, p_3 = probability of the states of serviceability; unserviceability and collapse of the structure;

p = number of equilibrium equations;

$Q(\underline{x})$ = function representing the boundary $\bar{\underline{B}}$

q = number of compatibility equations;

$R(\underline{x})$ = equilibrium equations;

R = region of space or body;

r = number of jumps, also as power it appears in Ramberg-Osgood function;

r_s, r_c, r_d = marginal uncertainty factors;

S = final value of stage variable

s = stage variable

$s = \upsilon + j$ = modified stage variable to take jumps into account;

T = total life of the structure;

t = time as stage variable, also as state variable;

$U(\cdot)$ = utility of an alternative;

U_0 = effectiveness of a structure at zero reliability;

U_1 = effectiveness of a structure at reliability one;

u_{ij} = change in the energy in the ith force-deformation relation at the jth section;

$\underline{u} = \{u_1 \ldots u_m\}$ = strategies of nature in the normal play of the structural action game;

$u(a_1{}^i)\ldots$ = utility assigned to the alternative action i corresponding to the attribute a_1;

$\underline{u}^L$ = = lower bound to the strategies $\underline{u}$;

$\underline{u}^u$ = upper bound to the strategies $\underline{u}$;

V_n = amount to be spent by a designer if he carries out n trials of design of the structure;

$\underline{v} = \{v_1, \ldots v_n\}$ = control variables in optimal control problem, n strategies of player 2 in differential game;

v_0 = value of G_0 during a jump in state variable $\underline{x}$;

$\bar{\underline{W}}$ = load space;

$W^u = \underline{W}$ = upper limit of load in normal load condition;

W^L = lower limit of load in normal load condition;

$\underline{w} = \{w_1, \ldots w_m\}$ = magnitude of m loads acting on the structure; state vector in load space;

w_j = weightage factor of jth attribute;

$\underline{w}(s)$ = load as a function of stage variable s;

$\underline{w}(0)$ = initial value of $\underline{w}$ at s=0;

w_0 = 0 or 1 depending upon whether a jump occurs or not in optimal control problem. Strategy of player I who can choose 0 or 1 in differential game;

$X=\{\underline{x}, t\}$ = augmented state variable in $\bar{\underline{Y}}$ space;

$\underline{x}=\{x_1, \ldots x_n\}$ = state vector in deformation space;

$(\underline{x})^0$ = initial value of x at s=0;

$(x)^f$ = final value of $\underline{x}$ at t = T;

$\underline{x}^*(s)$ = optimal path of $\underline{x}$;

$\underline{x}'$, $\underline{x}''$ = points on $\bar{\underline{B}}$ when $\underline{x}(s)$ intersects with it;

$\underline{x}^p$ = permanent deformation;

$\bar{\underline{Y}}$ = deformation space;

$\underline{y}=(y_1, \ldots y_n)$ = time-independent component of state variable $\underline{x}$;

$\underline{y}^p$ = plastic component of y;

$\underline{y}^e$ = elastic component of $\underline{y}$;

$\underline{y}_i^c$ = maximum value of deformation undergone by a section during normal load condition;

$\underline{z}$ = time-dependent component of $\underline{x}$;

$\dot{\underline{z}}$ = rate of change of $\underline{z}$ with t;

$\underline{\alpha}, \beta$ = interaction factor;

$\underline{\alpha}=(\alpha_1, \ldots \alpha_n)$ = stiffnesses, the control variable in control problem and the strategy vector in game;

β = maintenance factor;

γ = constant in the creep equation;

$\underline{\Delta}$ = deformation in the direction of load $\underline{w}$;

ΔL = length between the sections in a member;

ΔF = interval of state F^i in Markov process;

π = energy potential in variational theorem;

$\pi(y)$ = state probability in Markov process;

$\underline{\mu}=\{\mu_1,\ldots\mu_m\}$ = means of load $\underline{w}$;

λ, μ, ν = Lagrangian multipliers in optimal control and game problems;

μ^u = upper bound to mean vector $\underline{\mu}$;

μ^L = lower bound to mean vector $\underline{\mu}$;

$\underline{\lambda}=\{\lambda_1,\ldots\lambda_n\}$ = costate variables;

$\underline{\theta}(\underline{x},\underline{\alpha})$ = control variable constraints;

$\underline{\theta}=\{\theta_1,\ldots,\theta_m\}$ = standard deviation of the normal distributions of loads $\underline{w}$;

$\underline{\theta}^L$ = lower bound to $\underline{\theta}$;

$\underline{\theta}^u$ = upper bound to $\underline{\theta}$;

$\nu_i(F)$ = factor that represents nonlinearity of ith force deformation relation;

ν_q = an arbitrary constant in the jump condition;

$\phi(s,\underline{\alpha},\underline{F},\underline{x})$ = function obtained by differentiating $K(x)$;

ρ = correlation co-efficient;

$\rho(A,B)$ = outcome matrix in decision problem;

$\Omega(A,B)$ = pay-off matrix in the decision theory problem based on preferences;

$\omega(A,B)$ = an element of $\Omega(A,B)$

$(.)^-$ = indicate value of a variable just ahead of a jump;

$(\cdot)^+$	=	indicates value of a variable just after the jump;
$(\cdot)^*$	=	indicates the optimal values of variables in optimal control formulation and differential game formulation;
$\cdot(s)$	=	indicates the path of a variable with respect to stage variable s;
$\succ$	=	is preferred to;
$\sim$	=	is indifferent to;
$\succsim$	=	is preferred or indifferent to;

CHAPTER ONE

INTRODUCTION

1.1 GENERAL

Design of structures is perhaps one of the oldest design activities. Wasiutynski and Brandt (1)* trace back the origin of structural design to the first work of Galileo. However, the progress in the earlier period was slow and a comprehensive approach to design was absent for a long time. Design was performed in the 19th century and the earlier period of 20th century through a process of analysis within the framework of continuum mechanics, strength of materials etc. The mid twentieth century has introduced a new enthusiasm in the field of structural design. The probabilistic concept of design was introduced as an alternative to the conventional deterministic approach. The two other achievements during this period are the introduction of electronic computer as a design aid, and the increased use of the theories of decision-making and modern decision techniques of operations research, and statistical decision theory. Decision theory is defined as the formalisation of decision problems and the application of the mathematical and statistical techniques to their solution. The modern concepts and tools given above help the structural designer to arrive at logical

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* Numbers within paranthesis indicate references listed at the end.

decisions as contrasted to intuitive decisions and to consider to an increased measure the economic, social, and other involvements of design, which were considered at one time to be intractable. The modern systems engineering methods together with the theories of decision-making have opened up new hopes in making the design more logical systematic, and have found wide applications in industrial engineering, electrical engineering, aerospace engineering etc. Many decision techniques that are used in management science, war problems, and optimal control problems need to be investigated for suitability to structural design problems.

Structural design need not be dealt with in the narrowest sense of a process of mathematical computation to find a set of cross sections for the structural members, such that a 'factor of safety' of say 2.0, or a probability of failure of 10^{-6} is obtained. It has got deeper and wider implications ranging from satisfaction of human needs, both physical and psychological, working with uncertain data, the systematic analysis of the economics of the system, and working out of a harmonious combination of geometry with material consistent with the functional and aesthetic needs. Pugsley(2) in his Wright memorial lecture characterizes the essentials of a well balanced structure. He emphasizes that the design of a structure calls for a skilful balancing of many factors. Thus, for an aeroplane, not only the matching of loads and strengths is to be done, but also the matching of the

structure to the aerodynamic and performance needs as well as to the economics of aircraft operation and to public views on safety needs to be incorporated in the design. An economic and efficient system requires the simultaneous consideration of three aspects; material, structural form and design conditions (3), as shown in Fig. 1.1. The designer has to operate at the central region marked A of the figure. In this process numerous other related problems, like, the choice of a structural configuration, choice of idealization, and method of structural analysis, the selection of a criterion, jointing and fabrication methods, collection of data with dependable precision about loads and materials, the problem of safety, failure mode, etc. make the design a complex affair. The object of this chapter is to briefly present current state of knowledge of structural design processes and related areas, so as to pave the way for the development of the theme of the present investigation. Through the review of current research, the need for the present investigation to develop a rational design methodology for structural design is identified. The object and scope of this investigation is described in Sections 1.7 and 1.8. Specifically, a philosophical format for making logical structural design decisions and investigating the applicability of some mathematical decision techniques like game theory, statistical decision theory etc. to a class of optimum structural design problems with nonlinear material behaviour

is presented and explained through examples. In Chapter 2, an attempt to analyze in detail the nature and problems of structural design decisions is made with the intention of relating structural design to the available methods in systems engineering and decision-making.

1.2 ENGINEERING DESIGN AND DESIGN OF STRUCTURES

Design is a creative decision process leading to concrete activity like a finished product (4). Design methods have been in existence in various fields from the time of Greeks and in modern times may readily be traced back to Leonard da Vinci (5). In recent years, particularly in the last two decades, attention has been devoted progressively to the general methods of design of engineered products. This is termed by some as design engineering (6). Engineering design can also be considered as the creation of a product in an optimal manner subject to problem solving constraints (such as level of knowledge of designer, time and facilities available for design, computation etc., and facilities for verification by model or prototype tests) and solution constraints (cost including time, availability of materials, construction equipment, construction methods, manufacturing skills etc.). Rosenstein (7) divides design into twelve stages starting from the identification of needs to the final stage of implementation of proposed design. Asimow (8) has dealt with the design aspects, and has suggested three

distinct phases of design namely feasibility study, preliminary design, and detailed design to be followed by three phases, which include planning for production, consumption and retirement. Each phase of design is characterized by a fine structure which is a repeated fundamental sequence of activities which consists of: problem statement and formulation, information collection, modelling, value statement, synthesis of alternatives, evaluation, optimization, communication and implementation. Mesarovic (9) discusses design activity as the dynamic relationship between the technological environment and the designer. In 1948, Zwicky (10) introduced the morphological concept which was later adopted by Norris (11). According to the approach, engineering design involves four major activities of problem definition, analysis, synthesis and presentation. Krick (12) describes a design process to consist of five steps namely problem formulation, problem analysis, search for alternatives, decisions and specifications. Eder (13) classifies the design methodologies into six as follows: a. Experience, b. Modification and running design, c. Check lists, d. Design trees, e. The fully-systematic method, and f. System search method.

Though design itself is not a science, scientific methods are used for the prediction and estimation associated with a design activity. They involve close observation of physical phenomena, creation of a theory or model, and prediction of observables from the theory or model by

mathematical, or logical deductions or through experiments. However, when an analytical treatment or experimental investigation is impossible, these scientific methods are not applicable. Simulation techniques like analogue or digital simulation, Monte Carlo Method, participative models like gaming, Scenario writing, Delphimethod (a form of gaming) etc. (14) have been suggested to deal with such situations. Planning activity, which has a high political, and social content, demands such simulation procedures. Civil Engineering planning is one such activity with decisions of social and psychological significance, and as such there is great potentiality for these techniques. These simulation techniques are in extensive use in other fields of planning and design like war, management, urban planning etc.

In spite of the developments of general design and simulation techniques, only very few applications of them have entered the field of structural design. Tung Au (15) suggests a heuristic game approach to planning and design of structures. Structural design activity is simulated as a game. The team of players will be designers engaged in the design activity, and the plays take place in a computer simulated environment to produce the most satisfactory design as envisioned by the players. Spillers (16) emphasizes the need of introducing the concept of artificial intelligence or learning process in design so as to minimize the human interference and judgement for automated design.

Khachaturian (17) has discussed the operations in the structural design process, and critically evaluated the role of structural optimization. This is shown in Fig. 1.2. The role of feasibility study, preliminary design and detailed design for routine as well as innovative situations is critically studied. The significance of value system, concept, information and their nature is critically discussed for preliminary design of structures.

1.3 APPROACHES TO STRUCTURAL DESIGN: A REVIEW

One important aspect of structural design is the effective utilization of available data to predict the behaviour of structure in a rational manner. The design data and technological information that we make use of regarding load and structural behaviour may be at various levels of sophistication. If the information is perfect, i.e. deterministic, the designer can make the decisions with certainty. However, it is very rarely possible to estimate with certainty the loads and strengths of materials. On the other hand, it is recognized that these are random phenomena, which as shown in this thesis, lead to a risk situation (i.e. the possibilities of the states and their probabilities known). The information about the statistical nature of the data must be perfect for such a situation, which may be too costly to obtain, though not impossible. Hence probabilities are very often unknown leading to uncertain situations. In addition, uncertainties of non-measurable type may arise

due to human errors in design (assumptions, idealizations, inaccuracies in computation etc.), construction(degree of quality control, workmanship etc.) and operation (operating under conditions not specifically designed for) of the system. A classification of the uncertainties in a structural design process is made in Chapter 4.

Several approaches to structural design have been suggested to deal with the above-discussed information problem with a view to satisfy strength (under all design conditions), serviceability, ductility and other design requirements. Broadly speaking, structural design can be divided into the following three approaches.

(1) Deterministic approach

(2) Probabilistic approach

(3) Some combinations of both deterministic and probabilistic approaches (Engineering Approaches)

In the deterministic approach, all the loads, strengths of materials and other design variables are considered to be deterministic quantities. The design methods based on working stress or allowable stresses were the most generally used during the first half of the present century. In the last 20 years there is a trend towards using overload factors for loads so that they can be designed using strength theories (e.g. plastic limit analysis for ductile members under quasi-static loading). Deterministic approach is infact an empirical approach in which all the relevant quantities are codified.

The method of codification itself does not contain any rules for defining the quantities. All the decisions regarding the quantities are made by code writers, and the designer is left with mostly the mathematical computations using the codified design specifications. Any uncertainties associated with the design informations are assumed to be compensated by certain 'magic factors' called '<u>safety factors</u>', either as overload factors or ratio of yield strength to maximum permissible stress. The greatest asset of the deterministic approach is the long-standing knowledge and experience accumulated in the past two hundred years, in terms of working stress design and plastic limit design. The current design procedures therefore involve the use of load factors to account for uncertainties that may exist in the applied loads and in the resisting strength of structural members.

The probabilistic approach on the other hand recognizes the random nature of loads, strengths etc. and resorts to mathematical procedures employing the calculus of probability. According to Moe (18), Max Mayer was the first to introduce the statistical concept in structural safety in 1926. The earlier proponents of this approach are Freudenthal (19), Prot (20) and Pugsley (21). Considerable literature exists today on the probabilistic approach to design. The probability density functions of the random quantities must be known to apply this method of design. This design under known probabili
is treated in this thesis as a decision under risk. Contrary

to our belief, a probability of failure does not cover all uncertainties which the safety factor is purported to do, in a deterministic approach. Classicial probability theories have serious limitations in structural design because of the imprecise nature of information available and difficulties in applying to practical cases as was pointed out by Sridhar Rao (23). Of late, attempts have been made to treat the probability distributions as either unknown or only approximately krown and to use statistical decision models. Turkstra (22) assumes design as a statistical decision problem of the Bayesian type. Benjamin (23) makes use of the extensive form of Bayesian decision due to Raiffa and Schlaifer (24) for decision under unknown probabilities. Sexsmith (25) also proposes a Bayesian decision process for design.

Turkstra's work (22) is a pioneer effort in the search for rationality and consistency in the structural design decisions. He emphasizes that a design cannot exclude engineering judgement, and be carried out on a purely factual basis. With this intention of incorporating judgement, he has formulated structural design as a decision under uncertainty. He assumes that the designer is playing a game against an imaginary opponent nature, the latter player being introduced only for the ease of formulation. Nature chooses a load and resistance and the designer chooses a design (material, cross-section, structural system). If the load is greater than the resistance, the designer suffers a loss. A probability distribution is

assigned over all moves except the designers. This distribution is assigned by the designer which represents the relative likelihood to the possible acts of Nature. Numerical utilities are assigned to describe the preferences. The complex game problem is to be solved with the help. of the assigned utilities. Knowing the utilities and the probabilities assigned to the states of nature, expected values are calculated and a choice is to be made based on the optimum expected pay off(returns). When new data become available the subjective probabilities may be improved by means of a Bayesian decision approach.

The third approach is essentially an engineering approach aimed at achieving the advantages of both the probabilistic concept and the safety factors in the deterministic approach. Three types of approaches have been suggested. The CEB approach (26) recognizes the statistical nature of loads and strengths by defining characteristic loads, and strengths on a probabilistic basis, and resorts to a deterministic format for design. Partial safety factors are introduced to take care of the uncertainties associated with the design. The characteristic loads and strengths are modified suitably using the partial safety factors into the design loads and design strengths for which an ultimate limit state design is proposed. The extended reliability method of Ang and Amin (27), on the other hand, introduces a factor of uncertainty (also called judgement factor) to take care of the non-measurable uncertainties. The limitations of classical reliability method

which is very sensitive to the type of distribution functions assumed for low probability of failure is partially overcome in this method. Ang (28) classifies uncertainties into objective uncertainties (considered as risk situation in this thesis) that arises due to the statistical variabilities and that can be treated in explicit probability terms and subjective uncertainties that arises due to the imperfect knowledge, information etc. This latter type of uncertainty is incorporated in design through the judgement factors which are derived by the designer for the situation using his judgement and discretion. The method uses CEB recommendations to evaluate the factor ϑ and is an improvement over classical probability methods to make use of available statistical information on loads and materials. The method given by CEB also considers the subjective uncertainties through partial safety factors, but at the same time makes use of reserve inelastic capacity of the structure for combating the additional burden due to these uncertainties. The nominal requirement for safety is given by

$$p\ (R \leq \vartheta S) \leq \alpha \tag{1.1}$$

where p stands for the probability of the event, R is the resistance, S is the load or load effect measured in the same unit as that of R, α an accepted probability of unsafety which takes care of the uncertainty arising from objective information and ϑ the factor that takes care of the

subjective uncertainties.

In a recent work, Ang (29) proposes to evaluate the judgement factor on the basis of subjective probabilities. In this case, the risk of failure is a product of the objective probability and the judgemental probability associated with the uncertainty factor. This approach to design is intended to retain the rationality of the reliability concept, and to possess the practical flexibility of the limit-state approach of CEB. However, the evaluation of the factor ν through judgement in an overall sense not considering the utilities associated with the consequences of the uncertainties may not be a rational procedure. A decision theory approach based on preferences is proposed in Chapter 7 of this thesis to evaluate a set of factors similar to the judgement factor.

The third type of approach is due to Lind (30) and Cornell (31). Lind suggests a deterministic codified design in which optimization is achieved not through design, but through the optimization of codes themselves. It is further proposed to derive partial safety factors using the probabilistic data of loads strengths and other variables (32). A schematic diagram of design approaches is shown in Fig. 1.3.

An associated problem in structural design, which attracted the attention of several research workers, is the problem of rationalizing the level of safety which is at present taken in an arbitrary manner. For example, one defines

the safety factor to be 2.0, or a probability of failure 10^{-6}. Turkstra (33) proposes a rational approach in which failure level is linked to the total cost of failure. Tabular method of the evaluation of safety factors (34), and the probabilistic approach to the evaluation of partial safety factors proposed by Ravindra and Lind (32) and Knol (35) are worth mentioning. In the tabulation method, the factors that affect the choice of overload factors are listed and the designer is to rate the relative influence of each of them on the design. The overload factor for the rating can be obtained from a table of factors. Knol (35) studies the problem of structural safety in both its theoretical and practical aspects, and discusses especially the influence of uncertainty arising from gross 'human error'. A system of safety factors for reinforced concrete structures similar to the one proposed by the British Institute of Structural Engineers is proposed.

1.4 REVIEW OF STRUCTURAL DESIGN PROCESSES

Several reviews of structural design processes have appeared in the literature (1, 4, 36-41). Therefore, it is not attempted to review all aspects of structural design here. Instead, a broad classification of structural design processes is made here so as to give a broad idea of the nature of structural design processes. In Chapter 2, they are further classified based on the structural design decisions.

Design methods are very often classified into direct

and indirect methods (42). In direct method, the best dimensions of the structure and the type of materials are directly determined so as not to exceed the limit values of stresses, displacements, deformations, cracking etc. In the indirect method, the shape, dimensions, and materials of the structure are chosen as a first approximation. Possible loads are also determined, and a verification whether the estimated behaviour under these loads is satisfactory is made. If the behaviour is not adequate, the structure is reanalysed with new data. This is an inverse method, trial and error or iterative analysis-design cycle.

From the view-point of decision making, structural design processes can be classified into two categories.

(i) Classical Nonutilitarian Process: The classical design procedures intended to assure a preassigned safety level (defined either deterministically or probabilistically) belong to this class. No utility function is explicitly made use of to prefer one design to other designs. So long as the design' obeys all the laws of mechanics and remains within the safety level, it is said to be acceptable. Though economy may be accepted as a basic requirement, it does not enter into the design computations specifically.

(ii) Modern Utilitarian Processes: A utility function representing some design criteria enters the design computations directly or indirectly, and a design having an optimum utility

defined by some acceptance rule is chosen as the optimum design. Uniform strength design is perhaps the oldest form of optimum design and was studied even in 17th century. Minimum weight as an objective was used by Maxwell (43), in 1890, and Michell (44), in 1904. The present state of minimum weight design is essentially due to the rapid growth of aerospace structures, and the introduction of limit analysis. Considerable literature exists in this area (36, 38, 1). In 1920, Forsell (45) suggested cost as the utility and postulated that 'a structure should be proportioned such that the total cost (including the initial cost, maintenance cost etc. and the expected value of the cost of failure) is a minimum'. Many other utilities like, energy, deflection, amplitude etc. are also used by some authors. The structural design processes for preliminary structural design, irrespective of the type of utility are classified into three based on the problem formulation as in Fig. 1.4.

1. Conventional methods of optimum design
2. Optimization based on theorems in structural mechanics
3. Mathematical programming formulation of optimum design.

Conventional methods of optimum design include uniform strength designs (fully-stressed designs, simultaneous failure mode approach etc.) iterative methods etc. Optimization based on theorems in structural mechanics form a group, in which

the utility function does not appear directly in the computations. Instead structures are automatically optimized through the use of structural theorems for minimum volume of material. Maxwell (43) and Michell (44) theorems give minimum weight design of trusses, while Heyman's (46) theorem gives the minimum weight design of beams and frames. Theorems for plastic minimum weight are suggested by Foulkes (47), Prager and Shield (48), Shield (49) and others (50, 51).

Taylor (52) gives a variational method based on energy principles to find the shape of the strongest column out of a given volume of material or to find a stiffest column for a given volume of material to carry a specified load. Prager (53) derives optimality criteria for minimum weight design of single purpose and multipurpose structures from the classical extremum principles of continuum mechanics. The minimum weight design reduces to the minimization of functionals representing energy potential. This functional is then minimized subject to the constraints, which automatically gives the minimum weight.

Among the utilitarian design methods, the problems formulated as mathematical programming problems (54) of optimization are the most popular. In general, optimization is done for two types of problems. In one type, the layouts of the members are not altered and optimum design is obtained by optimizing the geometry of the members only. In another type, the layout of members is also changed and the geometry

of member cross section is found out such that the weight is minimum. Optimization of truss geometry formulated as dynamic programming problems (55, 56) and nonlinear programming problems (57, 58) are examples of this class of problems.

In the former type of problems, namely the optimization of geometry of members keeping only the layout unchanged, elastic and limit design formulations are available. Methods based on limit design are generally linear programming problems irrespective of static or kinematic formulation (59). Infact a static formulation is a dual to the kinematic formulation. Schmit (60), Fox (61), Rozvany (62) and others have developed elastic optimum designs formulated as nonlinear programming problems. Optimum design for nonlinear materials (elastic or elastic-plastic) has not been attempted so far. Along with the problem formulation, as programming problems, several optimization techniques have also been developed.

The mathematical programming problems are suitable for deterministic approach to design. However, when probability of failure (reliability) is taken as the constraints or the objective function is replaced by an expected value function, these techniques can very well be used for optimum design. Several methods have been suggested in this manner (17, 63-66).

Design as discussed earlier makes use of a variety of data regarding loads, material behaviour, technological information etc. These informations are infact estimates of future events with the help of the knowledge of the past.

Very often, one can say in almost all cases, the data obtained may be uncertain. For example, the highway bridge loading specifications for the past thirty years or so have increased loadings because of the advances in the truck industry. Hence the design is performed under conditions of uncertainty. A variety of uncertainties may exist; some of these are discussed in Chapter 4. In spite of the fact that design is known as a decision under uncertainty, decision processes for uncertainty conditions are not specifically made use of in the usual procedure. Game theory and statistical decision theory can contribute significantly to this problem. The first formulation of structural design as a Bayesian game (subjective probability approach) was due to Turkstra (22). Later Benjamin (23) and Sexsmith (25) attempted to apply similar Bayesian approaches. This thesis is an attempt in this direction to make use of game and decision theoretic methods, and to assess their applicability to certain classes of problems in structural design.

Among other works, the expected value approach suggested by Haider (67), the cost effectiveness model of Blake (68) and the optimal control theory approach of Haug and Kirmser (69) are worth mentioning.

Haider (67) has developed an expected worth model that incorporates the safety, serviceability and economy of the structure. The expected worth R_i of any action i is

given by

$$R_i = \sum_{j=1}^{n_H} p_{j,i}\ Q_{j,i} \tag{1.2}$$

$p_{j,i}$ = the probability of the ith action jth state combination;

$Q_{j,i}$ = Consequence of ith action and jth state;

n_H = Total number of mutually exclusive and exhaustive states.

A design decision using the worth function given above is illustrated for a statically determinate truss. A worth function is developed in which the cost of structure and cost of failure are related to weight. Also, the minimum probabilities of failure and probabilities of other states are functions of the total weight of the structure. Thus the expected worth is given as a function of the weight of the structure. The optimum design problem thus reduces to finding that value of the structural weight which maximizes the worth function. This problem is solved by a sequential search process. In this process, a method is also developed by which the minimum probability of failure can be found out for a given total weight of the structure.

1.5 DESIGN OF MATERIALS

Design of materials consists of the production of some arrangement of constituents of one or more materials to yield

desired properties. This is infact a recent development essentially due to the development of polymeric materials and composite materials. Fibre composites are examples of such materials designed for specific use. Conventionally, only standard materials tested by long applications were taken for structural purposes. The concept of design of materials has brought a new thinking in design process to develop new 'need oriented' materials. The motivation comes from the special characteristics required, just as in the case of aerospace structures or from the attempt to bring down the product cost. The mix design of plain concrete and the design of reinforced concrete are the earliest formsof need oriented designs in Civil Engineering. Recently Krokosky (70), and Chamis (71) have tried to spell out the characteristics of optimal materials for various environmental conditions and functional requirements. Smolenski and Krokosky (72) have given an optimal multifactor design for sandwich panels.

1.6 CURRENT INTERESTS IN STRUCTURAL DESIGN RESEARCH

The review of the current literature on structural design processes presented in the preceding sections is selective and presents selected aspects of the design problem, that have a direct link with the theme of the thesis. Many related problems, that have attracted the attention of research workers in structural design, are knowingly left out. In general terms, the research work that is actively

pursued about structural design can be stated as follows:

1. A concerted effort to know more about the nature of loads and other environmental effects, strength and behaviour of structural materials and other data required for design.

2. Attempts to understand more about the structural behaviour, failure modes, long term effects of loads etc.

3. To improve the technological skill in design, construction, maintenance, etc.

4. To rationalize the design procedure and computational techniques with the help of systems engineering, theory of decision making, operations research and improved experimental procedures.

5. Effective utilization of computer, not only for numerical computation, but also for complete automation of routine designs reducing the dependence on human interference, judgements etc. by heuristic game approach or by introducing artificial intelligence.

6. To have a rational approach to structural safety and system performance of structures.

7. To bring in the design procedure, the social, psychological and economical involvements so as to have an acceptable economic structure consistent with optimum functional conformity and structural performance

Studies, in all these cases, have improved our understanding of the engineered system and its design. Any design procedure could make use of the knowledge derived from all these sources. However, the attempts made to have a practical procedure or methodology of design by which the problems like uncertainty of information, randomness of the design parameters, optimization, rational safety measures etc. can be systematically dealt with appear to be unsatisfactory. For effective research aimed at a better method of design, all the research activities must be made systematic within the framework of a common design process. In such a case, the research can be channelized to yield relevant information for refining the design method. Hence <u>it is worth while to attempt to have a design methodology consistent with modern concepts of system engineering design and decision making</u>. Perfect information of data, flawless technology, and complete independence of design from human interference, and absolute optimality are ambitions which one can never achieve or if possible, may not be commercially practicable. The design method must take into account these aspects. So also, an ideal design methodology that is comprehensive and applicable to all possible cases in a rational manner may be very difficult to obtain.

1.7 OBJECT AND SCOPE OF THE THESIS

In spite of the above-mentioned limitations, it may be possible to rationalise the design concepts and methodology

within the framework of the constraints mentioned above. The object of the thesis is to develop a rational design methodology (rational in the limited sense) that can take note of many of the problems, if not all, of the structural design process. With this intention, the nature of structural design decisions is studied in the light of system engineering design and the theories of decision making. A design methodology for the various decisions starting from the selection of site to the final selection of cross sections is proposed. Decision techniques of operation research and statistical decision theory like operational gaming, check-list method, subjective programming, game theory and decision theory are proposed for making the various decisions. The central theme of this methodology is the following.

Structure is a component of an overall engineered system which interacts with other components. The purpose of the structural component is to maintain the stable functional form with adequate safety to men and materials. The overall optimization of the engineered system with respect to certain objective (merit) function requires the optimization of the structural subsystem considering its interaction with other subsystems. Therefore, structural design is done considering this interaction, together with the performance of the structure in the future under the action of environment when it is built. This performance depends also on the precisi with which it is designed, prediction of conditions in

service, and the skill with which it is constructed. The errors, and omissions resulting from the human interference in both design and construction must be properly compensated for. Further, all uncertainties and randomness of the design data must be taken into account so that the decisions arrived at are dependable. The decision techniques chosen must be consistent with the design-designer relationship, and the methodology must have least dependence on human skill and judgement for possible automation of routine design or standardization in codes of practice by systematizing availabl experience. However, the creative aspects of structural design (choice of innovative structural types, development of new materials etc.) are important.

Of the various decisions, one decision namely the selection of the geometry and material of members for a given member arrangement, is studied in detail, and design methods are developed for the same. The selection consists of a set of decisions made by the structure in the face of uncertainty due to both load and its own behaviour. Game theory, decision theory and Bayesian Markov chain models are proposed for the various stages of decision. The game theory model assumes that structural action is a game played by structure against nature, and differential game and statistical game are made use of. The proposed method has got the following special features.

1. The force-deformation relations of the cross sections are the basic parameters made use of to divide the decisions into stages. The force-deformation relations are synthesized and obtained as an outcome of the game problem. In this way the method is a direct design method in contrast with the existing analysis oriented designs.

2. The method is developed for non-linear force-deformation relations and is applicable to elastic, or elastic-plastic materials with or without creep effects, and for deterministic and probabilistic information.

3. The design is considered to be decision under uncertainty, and uncertainty decision models are used for decisions in contrast to the present practice of bypassing uncertainty through safety factors, or judgemental probabilities.

4. A cost-effectiveness criterion in which structural efficiency expressed in terms of the expected cost of failure is traded-off with the initial cost of structure. This will eliminate the arbitrariness of safety levels, and will give an economic design consistent with safety, serviceability and ductility. In this sense, safety margines are obtained not by the conventional methods where an arbitrary safety level is assumed.

5. Serviceability for an arbitrarily specified normal load condition is guranteed. Hence the structure is serviceable (with regard to some criterion) so long as the loading is within the limit of normal load conditions.

6. Failure is not defined in terms of any particular mode. All possible modes of failure are considered with their probabilities of occurrence and costs of failure (economic as well as human losses).

7. In spite of the fact, that loads and structural behaviour are probabilistically defined, the design format is basically deterministic. This would be appealing in practical designs.

8. The concept of direct design gives a design for a class of material behaviour not necessarily specifying the material. Designer can choose the material from a class of materials. This would promote the modern concept of material design in which materials are designed for specific use rather than design the system for specific material.

9. The design as formulated here is applicable for quasi-static loading, but can be extended for repeated and dynamic loads and other environmental conditions.

10. The formulation, though general, is to be studied further in detail for two and three dimensional

structural systems. After further studies, and obtaining further meaningful data, this method may give a valid basis for either a rational design method and/or a rational methodology for improvement of codes of practice.

1.8 BRIEF OUTLINE OF CHAPTERS

The thesis is divided into 11 chapters. In Chapter 2, the nature of structural design decisions is discussed in the light of engineering design and decision making theories. A design methodology consisting of 5 decisions is proposed in Chapter 3. Chapters 4 to 10 are devoted fully to describe the selection of the material and geometry of members, for a given arrangement of the members of the structural systems.

Chapters 4, 5 and 6 prepare a basic framework for the systematic development of inelastic design for cost-effectiveness under uncertainty presented in Chapters 7, 8 and 9. The method of selection of the material and dimensions of cross sections is described in Chapter 10. Chapter 11 contains the conclusions and recommendations for further research.

Chapter 2: This chapter consists of an introductory discussion on the nature of structural design decisions, and the major problems involved in this process of decision. The importance of the design phase, the nature of the decisions involved, and the various techniques in operations research, and statistical decision theory available for making the

decisions are discussed. It is emphasized that structural design is a decision under uncertainty. The decisions are divided into five steps as selection of site or position, selection of the functional configuration, choice of a structural concept, arrangement of the members and connections, and finally the selection of the material and proportions and geometry of the members.

Chapter 3: A methodology of design for a stage-wise decision of the five steps of decision is proposed in this chapter. The proposed methodology is consistent with the nature of the decisions and the design-designer relationship. Also, it is attempted to take care of some of the problems discussed in Chapter 2. The importance of the introduction of a learning mechanism in design for systematic incorporation of human experience and judgement, and for automation of routine design is also presented. Decision models like gaming, checklist method, etc. have been discussed

Chapter 4: The uncertainties, failure modes and a cost-effectiveness criterion are presented in this chapter. Uncertainties are classified and mathematically stated. The various failure modes that may occur during the life time of the structure are also presented. A mathematical expression for the cost-effectiveness criterion is derived from an analogy with a mechanical system. A simplification of the criterion is also suggested.

Chapter 5: The inelastic design of structures for serviceability under normal load conditions (for a deterministic loading condition) is formulated as an optimal control problem. By this method, the relevant force-deformation relations of the cross sections of the members are obtained as the optimal controls required for the serviceability requirements for a given path of loading. Knowing the force-deformation relations that the structure needs, the cross sections can be chosen such that they possess the same set of relations. Structural design problems for serviceability controlled design using nonlinear materials are solved in which a simulated condition of certainty exists.

Chapter 6: This chapter is a supplimentary to the Chapter 5, and it consists of a method of design when the loading is a random phenomenon with known probabilities. The method is therefore designated as inelastic design under risk. As in Chapter 5, no uncertainties are considered. The proposed method extends and completes the force-deformation relations (called task curves) so as to achieve optimum cost-effectiveness. When members having cross sections with these force-deformation relations are chosen, it is expected that the structure is safe, serviceable and economic.

Chapter 7: Inelastic design is treated as a decision under uncertainty, and the outline of a method for optimum design under uncertainty for cost-effectiveness is presented. A design is to be completed in four stages. A structural action

game, with two consecutive plays, a decision based on personal preferences, and a Markov decision process are proposed as the models for the decisions. Of these, the evaluation of the marginal factors for non-measurable uncertainties by a decision based on preferences is presented in detail, while other stages of decision are presented in the subsequent chapters.

Chapter 8: The method proposed in Chapter 5 is extended to cover the uncertainties of the strategic type in the loading process. Structural action is assumed to be a game played by structure against nature. A differential game model which is a natural extension of the optimal control problem introducing the concept of a game is chosen. This particular play is called the 'normal play' of the game. The solution or optimal strategy of the structure is the force-deformation relations corresponding to the worst course of loading chosen by nature such that serviceability requirements are met with.

Chapter 9: The method proposed in Chapter 6 is extended to cover the uncertainties of the statistical type in the loading process. Structural action game is assumed, and the play of the game is called 'survival play'. A statistical game, in which the uncertain probability density functions form the strategy of the nature is made use of. The solution of the game extends the force-deformation relations obtained in Chapter 8 such that the pay-off namely the cost-effectiveness

attains a minimax optimum.

<u>Chapter 10</u>: This chapter presents the method of selection of the material and geometry of the cross sections once the task curves are obtained as explained in Chapters 5 to 9. Three cases are considered. They are choice under certainty when the material behaves deterministically, a choice under risk when material behaviour is probabilistic with known probabilities, and a choice under uncertainty when the probabilities are unknown. A Bayesian Markov Chain is proposed for the last type of choice.

<u>Chapter 11</u>: A brief summary of this study, and some general observations and conclusions are presented. In view of the importance of the problems for which this method is intended, the need for further study is discussed. Some of the problems that deserve priority are also mentioned.

CHAPTER TWO

THE NATURE OF STRUCTURAL DESIGN DECISIONS

2.1 INTRODUCTION

This chapter contains an introductory discussion on the nature of structural design decisions, and the major problems involved in this process of decision. Such a study would help deeper understanding of the design problem, the selection of the decision techniques and development of the overall framework of a design method. The importance of the design phase, the nature of the decisions involved and the various techniques in operations research and statistical decision theory available for making the decisions are discussed. The existing design methods, and approaches are also studied to some extent, so as to distinguish the features of the method proposed in this thesis from those of existing ones.

It is emphasized herein that the structural design is a decision under uncertainty. The deterministic approach considers the uncertainties through safety factors, while the classical probabilistic design does not fully consider the uncertainties. It is shown that game theory and statistical decision theory (Bayesian, Minimax etc.), if selected as decision tools, can accommodate uncertainty aspects in a better manner.

2.2 STRUCTURAL SYSTEM

2.2.1 Structural System

In almost all engineered systems, whether it is a building, a huge industrial complex, or even a simple mechanical lever, there exists a structural component or subsystem which is to be designed for operation along with other components of the overall system. The primary function of the structural subsystem is to maintain the functional configuration under various loads and other environmental effects, and to assure adequate safety to its occupants. Without the structure, the material form cannot be preserved. When the overall system is in operation, the hidden structural component is also in constant action with the oncoming environmental 'stresses and strains' to maintain the material form. A general system may involve more than one structural subsystem, and if the performance of one structural element does not any way affect that of another, they can be further isolated and treated separately.

The aim of a structural designer is to equip the structure with enough means to carry on its assigned functions. For him it constitutes the major problem. This aspect is persued here and the structural part of an engineered project is called the system in this investigation. To distinguish it from the overall system, the latter is called the general system. To assure and maintain interdependence with general

system, <u>the interaction of the structure with other components of the overall system (either functionally or mechanically) must always be considered.</u> (Fig.2.1).

2.2.2 <u>System Environment</u>:

Any engineered system is embedded in a set of environments - physical, social, political, economical and technological - herein called the <u>total environment</u>. The total environment influences to some extent, the selection of a particular structural system. However, in its performance, the structural component reacts only with the external physical environment (including the reaction with other components of the general system). Depending upon the effect on the structural system, the physical environment can be broadly classified as follows. (i) That which causes a material damage or change. It involves an environment that causes deterioration of material, or weakening of the strength of the material with which the structure is made. The environment usually gives rise to a chemical action (oxidation, corrosion, radiation etc.). (ii) That which causes a structural disturbance or damage of the material. They include the generalized loading acting on the structure including the effects of settlement, temperature etc. Stresses, strains, inelastic deformations, creep shrinkage etc. are the outcome of this environmental action. The former may also indirectly cause a structural damage

due to reduction in cross section, material damage or due to the reduction in the resistance of the material of the system.

<u>The interaction of the system with its environment causing a structural disturbance is herein called structural action</u>. The pattern of structural action changes with the change in environment, material and structural configuration.

2.3 SYSTEM QUALITIES

The status of the structure in relation to the general system is already defined. The interaction of the structure with the environment has also been discussed. Some of the qualities of the system are discussed below, which are intended to reveal the importance of the design, for the future structural action in an efficient manner.

2.3.1 System Life:

The life cycle of an engineered system consists of two periods (i) an acquisition period which includes all those steps necessary to define the needs, the resources, and to design, test and evaluate the system. (ii) A use or operation period that consists of those items required to operate and maintain the system (73). The structural action also extends over the entire service period, of the general system, (until the system fails or the operation is terminated by obsolescence or end of its designed life).

The acquisition period is further divided into three phases; an administrative decision phase, an engineering design phase, and an execution and evaluation (production) phase. The functional needs and availability of resources are evaluated and a decision is made in phase 1, (i.e. whether the system is feasible or whether it is needed). Wherever new systems are contemplated, it involves both technical and economic factors. In the engineering design phase, the needs and qualities of the system and the resources available are further analysed. A particular system is chosen out of all available alternatives. In the execution and evaluation phase, the creative ideas of the designer are put into practice through construction and/or production techniques, and the performance is evaluated.

The various operations are shown in Table 2.1. The phases are subdivided into stages. From the technical and behavioural point of view, the preliminary design stage is the most important, since all the important decisions involving the technical aspects of design are made in this stage. The type of structure, its dimensions, materials etc. are decided in this stage, along with the designs of other components (17).

Detailed design is the stage in which all the decisions made in preliminary design stage are communicated in the form of drawings, specifications. Detailed dimensions

are prepared, connections are designed, and drawings are prepared. It is followed by production planning.

2.3.2 System Status:

Structural system is at the lowest level of Boulding's (74) heirarchy of systems. They have neither physical growth as that of plants or animals, nor intellectual growth as that of human beings. By themselves they are not self-adaptive. However, structures do show some adaptability and intelligence in the distribution of the load coming over it. Any such capability is infact imposed on them at the time of design. The importance of self adaptability is further discussed in Section 2.8 with the help of relevant examples.

2.3.3 Rationality of the System:

By rationality of the structural system, it is meant the rational action under the environmental influence. A structure, being at the lowest of the levels of systems, does not behave rationally on its own. Any rationality of the system results from a rational choice of the appropriate structure most suited to the situation. The irrationality in behaviour must be taken into account while choosing the system for the situation and must be compensated for. A rational design is the one that accounts for the irrationality of structural behaviour in the most logical way.

The above discussion shows the importance of design to achieve an effective system. The basic concepts of

structural design are discussed below.

2.4 BASIC CONCEPTS OF STRUCTURAL DESIGN

Structural design is essentially an engineering design possessing all the qualities of a creative decision process. Hence it is worthwhile to examine the nature of structural design activity from the viewpoint of systems engineering. In what follows the word 'design' means only the preliminary design.

2.4.1 Design:

The central activity of engineering technology is design. Like most scientific methods, design method is a kind of problem solving. Scientific methods are aimed at to find what exists in nature, whereas design method is a pattern of behaviour employed in inventing things of value which may not as yet exist. Science is analytic, design is 'constructive' or synthetic. However, the two approaches to problem solving have some similarity and hence engineers very often, without conscious knowledge switch over from the design methods to scientific (analytical) methods. Though scientific methods can very much aid a design process, design is essentially creative. Systematic procedures can be developed for this creative synthesis of systems.

2.4.2 Basic design considerations:

Structural design is a decision making process

aiming to develop a structural system optimally. Traditionally structural mechanics offered a framework for the design of systems through analysis. But, it is mainly used to verify whether the structure obeys the basic laws of mechanics in the process of structural action. The stresses, strains, deflections etc. computed by this process of analysis satisfying the laws of mechanics are then checked whether they are admissible. Design has a wider significance than the mere satisfaction of laws of mechanics or the acceptance of a stress level. Design involves decisions in which various quantities of facts, data (subjective or objective) experience, faith, intuition and bias are combined in making a selection from a number of alternatives (75). Hence apart from the laws of mechanics, the following basic considerations are necessary in formulating a structural design process irrespective of the type of structure.

(i) A decision scheme.

(ii) A decision criterion or utility by which merits of various alternatives can be ordered.

(iii) Availability and the nature of input-output information.

(iv) A design methodology and decision models through which the decision can be made.

(v) An acceptance rule by which one of the alternatives is chosen from the available set.

those functional configurations that are structurally realizable are to be selected so that economy can be achieved.

(iii) Choice of Structural Concept: Structurally a truss is a concept different from a beam, though both may be resisting laterally applied loads. Numerous types of structures apparently serving the same structural and functional purpose with different degrees of effectiveness may be known to the designer. A judicious choice considering all the basic design criteria is needed to arrive at the best structural concept suited for the situation out of all available alternatives. For example, a flat roof covering a rectangular area may be designed as one-way or two-way slabs, T-beam and slab system a grid-floor slab, cellular plate, flat slabs with radiating ribs etc. The designer has to choose optimally one out of these alternative structural types.

(iv) Geometric Configuration of the Structure and Type of Connections: The selection of structural concept is followed by the selection of the arrangement of its various members. The arrangement of members can very well influence the optimality of the design by way of proper distribution of loads among the members. Several optimization procedures have been suggested for achieving the optimum truss geometry (55-58). Michell structures are examples of optimal arrangement of skeletal members. Nervi (76) developed among others a flat slab concept with ribs so placed that they

are in the lines of maximum moments. With the use of ferro-cemento forms and precasting methods, economy is achieved. Nervi's designs are characterized by very efficient structural forms.

(v) Selection of the Geometry and Material of the Cross Sections of Members: On the chosen layout, the material may be distributed in the form of proportioning of cross sections. The selection of material and the geometry of the cross sections at all the points on the structure forms the decision in this step. From the structural safety point of view, it is the major decision step, as the value system (to be explained later) for structural efficiency is considered in this step. In addition to an assessment of structural behaviour (similar to stress analysis) and safety, this step is preceded by a load analysis and strength analysis. Chapters 4-10 of this thesis deal with this step of decision in detail.

The five steps discussed above form a scheme of preliminary design. In fact the decisions mentioned above are not new. In the classical structural analysis also, a structure is evolved by making all these decisions. The only difference is that in the classical approach, some decisions are made by the designer mentally (by intuition, judgement etc.) and the remaining through detailed mathematical computation or experimental study. The decisions listed above can be made

in a more systematic way if suitable mathematical decision techniques are used. The modern concepts towards the study of structural design processes lead to this end. The various steps of design are in no way independent of one another and to obtain the decisions at one step, not affecting others or not affected by that in other steps, is practically impossible. The flow diagram shown in Fig. 2.2 indicates that decision at each step is influenced by decision at all other steps resulting in a complex process. The complexity arising out of information feedback, makes stepwise decisions of any sequence not always optimal.

2.4.4 Decision Criteria:

To arrive at an optimal decision considering all alternatives, a basic measure or utility, by which the alternatives can be compared, is necessary. It is mentioned earlier that no such utility appears to have been considered explicitly in the classical analytical approach to design. The modern processes for optimum design make use of a variety of utilities like weight, cost, deflections, energy, amplitude of vibration etc. Among cost, the minimum expected cost criterion of Forsell (45) is perhaps the most accepted one for civil engineering structures. Sawyer (77) and Haider (67) have extended this expected cost criterion to include different modes of failure. The expected cost models, in general, consider not only the cost of the structure but also the expected cost of failure. Thus expected cost C is

$$C = C_s + p_f C_f \tag{2.1}$$

where C_s is the cost of structure, C_f is the cost of failure, and p_f the probability of failure. When more than one failure mode is to be considered, the expected cost of failure may be expressed as given by Haider (67) (see Eq. 1.2).

In most of the design processes, the cost or weight is minimized to achieve arbitrarily specified safety levels. However, safety and cost are two objectives that pull in different directions. With these conflicting objectives, a trade-off between safety and cost is to be sought for. This concept is different from the one mentioned above, in which cost is minimized keeping the safety level fixed. Further, as the structure is to serve as an integral component of the general system, the conformity of the structure to the functional needs must be an added criterion.

The expected cost models mentioned above trade off the cost and safety of the system. A system-effectiveness or cost-effectiveness criterion (incorporating the functional conformity also), is proposed in this investigation. System effectiveness is an integration of system capacities, performance and cost. Blake (68) has proposed a cost-effectiveness model for structural design in which a trade-off between cost, weight and reliability is sought for. The system-effectiveness of a structural system for any situation can be expressed in terms of the following.

(i) The conformity of the structure to the functional needs of the system. It includes function, aesthetics and other values shown in Fig. 2.3.

(ii) The efficiency with which the system performs its structural action. It includes the values in design namely, safety, serviceability and ductility (17).

(iii) Long term economy. (It may be in terms of cost, weight, time, available human skill or use of scarce materials.)
Fig. 2.3 shows the value system in the model.

(i) <u>Conformity of the structure to the functional needs</u>: Though a primary consideration in design is to proportion the structure from its structural point of view, it must always be done considering the structure as a component of the general system. The elaborate arrangements done for the structural purpose must be conformal to the functional needs of the system. The aesthetic, utilitarian and other considerations very often restrict the designer from resorting to very efficient structural systems. Suspension cables, Michell structures, and some forms of shells (e.g. funicular shells) though structurally very efficient, do not get as much place as the less efficient beams, slabs etc. because of either the noncomformity to the functional needs or the difficulties in making them materially realizable.

This criterion, namely conformity of the system to the functional needs is a subjective criterion, and it is generally

impossible to state it quantitatively.

(ii) Structural Effectiveness: A structural design is made, based on the expected behaviour of the structure under loads and other environmental effects. The structure must retain the functional form with adequate safety under the influence of the environment (including the loads). The effectiveness of the system is to be measured in terms of the efficiency with which it performs the structural action. The value system that measures the structural efficiency against various stages of failure is discussed in Chapter 4 in detail. The model developed in Chapter 4 is assumed to take care of the values safety and ductility consistent with economy.

(iii) Long Term Economy: The extent to which economic considerations enter into engineering activity can hardly be overstated. If society is to be benefitted from engineers' creations, they must be solutions which are economically feasible. Cost of a system is one element of value or benefit forgone, in order to secure a greater benefit. It is in effect a negative benefit. Cost is not limited to money, but rather it must include all benefits or desired effects which may have to be sacrificed in order to obtain greater benefit (78). It includes money, time, consumption of scarce resources and use of available human skills. In terms of money, it involves not only the initial cost, but also the cost of maintenance, cost of repairs and cost of money, life or matter lost because of failure (functional, or various stages of structural failure).

In a design the three objectives may be weighed and traded off such that an optimum system-effectiveness (cost-effectiveness) is achieved. Since the value system for functional conformity is subjective, it is not attempted to state them mathematically. A cost-effectiveness model combining cost and structural effectiveness is developed in Chapter 4.

2.4.5 Availability of Input Information:

In addition to a rational design method, a design process requires sufficient information about the input data like loads, strength of material, conditions in use etc. Lack of accurate information may lead to uneconomic or possibly unsafe designs, however, rational the design method may be. Unfortunately a good deal of uncertainty exists in this aspect of the problem. This may be either due to the difficulty in getting exact data or due to the enormous cost involved in the collection of data. Sometimes, the cost of getting more accurate information may far out-weigh the benefits to an individual client. Even gathering the data for a region to include in a code of practice may be very costly, if not impossible. The data so collected when used for an individual design may bring in inaccuracy. Thus, a designer has to work very often with uncertain data. A detailed analysis of the uncertainties is made in Chapter 4. Broadly uncertainties can be classified into three.

(i) Uncertainty associated with generalized loads and loading process. It includes thermal effects, settlement etc.

(ii) Uncertainty associated with structural action and material behaviour.

(iii) Uncertainties of design and constructional origin like that arising from idealizations, assumptions, inaccuracies in the calculations,poor workmanship etc. (called nonmeasurable uncertainties).

In the deterministic design procedure, the uncertainties are taken care of by a factor called safety factor. In many countries, this safety factor is a 'global' value specified by the code writers. However, this penalizes very good designers who are able to get more accurate information, have rigorous supervision and quality control procedures and where the function is such that the loading is reasonably controlled or the possibility of the change of use (or function) in the design lifetime is small. The tabular form of evaluation of safety factors (34) discussed in Chapter 1, is an improvement in this direction which allows some flexibility to the designer to judge the factor by himself for the situation. By this method the judgement can be done on smaller factors rather than on a global basis. Similar safety factors are used in the extended reliability method (27), semi probabilistic limit state approach of CEB (26), and in the optimum design of codes of practice proposed by Lind (30) wherein probabilistic information is put in a deterministic format. Methods are also proposed for the rational evaluation of these safety factors by Ravindra etal (32)

and Knol (35). However, in all these cases an element of empiricism creeps in due to the subjective judgement by which these factors are evaluated. <u>In this thesis a different approach is suggested. The design of structure is to be considered as a decision under uncertainty.</u> This aspect is further emphasized in Section 2.5.4. Decision models like game theory and statistical decision theory are made use of for inelastic structural design using the cost - effectiveness concept.

2.5 SCIENCE OF DECISION MAKING AND STRUCTURAL DESIGN

Structural design is accepted as a decision-making process, and it would be advisable to classify the structural design processes on the basis of the nature of decisions involved. This would help to select the appropriate decision techniques. Fig. 2.4 shows the tree of decision making.

2.5.1 <u>Decision of the Acceptability of a Hypothesis (belief) or of course of Action:</u>

The decision problem involved at any situation may be either about the acceptability of a hypothesis (belief) or of a course of action (79). The decision is to lead the decision maker to a rational belief or rational action. Basically, having a rational belief is different from resorting to a rational action. Decision of the acceptability of a hypothesis is generally of the non-utilitarian type and therefore the acceptance rules of the non-utilitarian type

are applicable in this case (Acceptance rules are discussed in Section 2.7). The non-utilitarian structural design discussed in Chapter One is of the former type. Design in this case may be considered as the formation of the opinion about the safety of the structure. The statement of a safety factor, or a probability of failure is a statement of the degree of belief on the safety of the structure. A structure having a safety factor greater than or a probability of failure less than their respective assigned limits is said to be acceptable. Conceptually, probability of failure may be a more rational belief than factor of safety. The belief of code writers is reflected in the specifications of a design load or a maximum permissible working stress. Experience, experimental evidence, and scientific studies are relied upon to rationalize the belief. The utilitarian design processes belong to the class of the acceptability of course of action. The designer decides which structure he has to choose out of the alternatives.

A slightly different method of decision is proposed in this thesis. The selection of layout of members and the distribution of materials in the layout are treated as the decision of a rational course of action by the structure, and not by the designer. The course of action of a structure is its structural behaviour. Hence this assumption allows the decision of structural behaviour often expressed as force-deformation relations. In a design, designer decides the course of action of the structure and not his action.

Structural action is simulated as a game in which structure is a player. The selection of a structural concept is however treated as the utilitarian decision by the designer. The tree of decision making shown in Fig. 2.4 is for the decisions of the course of action.

2.5.2 Individual and Group Decision Making:

Depending upon the individual or group that makes a decision, decision making is divided into individual decision making, and group decision making (80). A group of people having a common interest or aim is termed here as an individual. Structural design is a group decision process made by an administrator (client), architect, structural engineer, construction engineer and other technologists. Tung Au (15) suggests a heuristic game approach in which the design is treated as a group decision.

In the methodology proposed here, the initial selection of position and functional configuration is a group decision, while the selection of structural concept is a decision by individual. In the other two steps of design, the structure is assumed to decide how it must act (which is decided by the designer on behalf of the structure), and therefore, the decision is an individual decision making process.

2.5.3 Intuitive and Logical Decision:

In Fig. 2.4, the individual decision making is

further divided into intuitive or logical decisions. An intuitive decision is one in which a particular decision is chosen by the designer and tested or analyzed for its acceptance. The classical analytic process of design is an intuitive one. If an optimum design is needed, it leads to an iterative procedure. A logical decision, on the other hand, is the one made by considering all alternatives available, and by choosing one of them as the acceptable one with respect to an optimization criterion in a logical way (81). Naturally a logical decision process is preferable as it gives the optimum decision out of all possible alternatives. The modern operational research techniques have a logical structure.

2.5.4 Decision Under Conditions of Certainty, Risk And Uncertainty:

Depending upon the information conditions with which the decision maker operates, decision may be classified as that under certainty, risk and uncertainty (80). Decision maker is operating under

(a) Certainty if each action is known to lead invariably to a specific outcome,

(b) Risk, if each action leads to one of a set of possible specific outcomes, each outcome occuring with a known probability, and

(c) Uncertainty if each action has as its consequence a set of possible outcomes, but where the

probabilities of these outcomes are completely <u>unknown</u> or are even not meaningful. In this, all actions are considered to be known (i.e. all states are known but their probabilities are not known). It is worth studying under what conditions a structural design falls in each of these classes. This would help to verify whether the information is taken in the proper perspective.

Let us consider that the designer is faced to select a slab out of four possible alternatives 3'', 4'', 5'', and 6'' thick. The load on the slab and strength of material are not known. Any combination of load and strength may occur. Let any such load and strength jointly represent the <u>state of nature</u>. Five such states are assumed to occur as shown in Table 2.2. This is a simple <u>discrete</u> model of a much wider problem. Let us also consider that uncertainty may arise <u>only</u> from these two sources, load and strength. The state of nature that actually occurs is called <u>the true state of nature</u>, which is unknown. Hence load and strength are considered random. The probability with which these states occur are generally <u>unknown</u>. Therefore the decision maker, according to our classification, is operating, under <u>conditions of uncertainty</u>. In most of the practical problems. the knowledge of the probability of the state of nature is incomplete. Let us assign utilities u_{ij} for the ith slab when it is at the jth state of nature.

In the deterministic approach to design, a load and strength are specified by code. In the example shown, it amounts to specifying the state of nature. If the state of nature is specified, each act of selection of slab leads to one and only one outcome, bringing the decision as a decision under certainty. In a non-utilitarian design process, the alternative with maximum safety factor will be chosen. Thus if state 4 is specified, 6" slab will be chosen. In a utilitarian approach that alternative with optimum utility (least cost) will be chosen. Thus <u>a deterministic approach to design, whether it is utilitarian or non-utilitarian, is brought to as a decision under certainty</u>.

In a probabilistic approach to design, it is recognized that any one of the states of nature may occur and the probability of each state (strength and load) is known. <u>When the probability of occurrence of various states are known, probabilistic design is a decision under risk</u>. Here, only the uncertainty in the state of nature is considered. The uncertainty in information, for example the probability distribution, is not considered here. In the non-utilitarian process, an arbitrary value of probability of failure is assumed as the acceptable limit.

However, the accuracy of a decision depends also on the accuracy of the information obtained. If the probability density function of loads and strengths are known <u>accurately</u>,

and no other uncertainty exists, a probabilistic approach is perfectly justified. However, in most cases, it is not possible to get the accurate distribution. Under such conditions, it is more appropriate to consider the various states of nature with unknown probabilities. The structural design in this manner is considered by Turkstra (22); Benjamin (23) and Sexsmith (25). Uncertainties of non-random nature to which much attention is not so far given also, always exist. The variations in timing, sequence, position, direction etc. of the loads are strictly not probabilistic though in some cases they may be expressed probabilistically. The influence of these variables are very important in serviceability (deflection, crack-width etc.) controlled designs. Treating these variations as 'strategic' variations, game theory models can be brought in for decision making. The uncertainties are further classified in Chapter 4.

2.6 DECISION MODELS

Modelling of the system, the environment and system behaviour are essential in structural design decisions. The various modelling processes followed by engineers are given by Krick (12) as shown schematically in Fig. 2.5. Decision models of diverse nature have been developed in the field of operations research. These models can be divided into analytical models and simulation models.

2.6.1 Analytical Models:

These are mathematical models having a scientific treatment of the problem. The various analytical decision models for decisions of diverse nature are shown in Fig. 2.4 and Table 2.3.

(i) Mathematical Programming:- The utility is expressed as a mathematical function of decision variables. All mathematical programming techniques belong to this class. A classification of the various optimization techniques under this class is given in Tables 2.3, 2.4 and 2.5. The mathematical programming methods of structural optimization discussed in Chapter One falls under this group.

(ii) Riskless Choice Based on Preferences:- When the utility is not derivable in mathematical form, mathematical programming techniques are not applicable. The criteria of optimization in overall structural design in general are subjective and a mathematical statement is very often difficult to make. Non-risky utility approaches (like subjective programming (84) have been proposed for solving such problems.

(iii) Risk Decision:- Certain axiomatic approaches using expected value criterion (80, 85) are generally made use of. Stochastic linear programming, (86) and chance constrained programming (87) are also suitable models for

risk decisions.

(iv) Games of Strategy:- Uncertain conditions arise in a decision making process from two sources. Firstly, uncertainty arising from the secret decisions made by an intelligent opponent which may affect the gain or loss of the decision maker. This situation leads to the application of games of strategy (von Neumann's theory of games) as the decision model.

In game theory, it is assumed that each player knows the desires of the other fully, and that each player is rational striving to better his gains. All possible strategies of the opponent must be known to each player.

(v) Statistical Games:- The second type of uncertainty arises from the random states of nature; the probability of which is unknown. When complete ignorance prevails, statistical games proposed by Wald (88) and Blackwell and Girshick (89) offer a conservative estimate.

(vi) Bayesian Decision Processes:- When the probability subjectively assigned a priori can be improved by experimentation, a Bayesian decision process can be advantageously used, which gives a fairly reasonable estimate. Bayesian decision models suggested by Savage (90), Raiffa and Schlaifer (24) and others (91-94) are suitable for uncertainty decisions of this class.

Structural design is treated in this work as a decision under uncertainty. Naturally it would lead to any of the decision techniques; game theory, statistical game (Bayesian process or minimax game). The techniques chosen are discussed in Chapters 3, 7 and 10.

2.6.2 Simulation Models:

Some of the simulation models are shown in Fig. 2.5. The participative models like gaming and the non-participative models like Monte-carlo Method are the most popular simulation techniques.

Warner and Kabaila (95) have proposed a Monte-carlo study of structural safety. One of the difficulties associated with probabilistic method of design is the evaluation, in closed form, of the cumulative distribution of stochastic variables. Numerical integration is very often needed for this purpose. The Monte-carlo Method is a simulation process that can be applied for such situations. The evaluation of region of safety, ultimate strength, factor of safety etc. using probability distribution can be done by this method. The method is illustrated for an axially loaded reinforced concrete column.

2.7 ACCEPTANCE RULES:

Finally, the choice of the proper acceptance rule is as important as the choice of the decision model. By

acceptance rule, we mean a way of stating our preference of utility and thereby the act that appears to be acceptable for the situation. Various acceptance rules under different conditions of decision making are given in Table 2.6. Acceptance rules are of two types. A utilitarian rule explicitly takes account of the gains or losses involved in the acceptance of a hypothesis or course of action. This is mainly for acceptance of course of action. An acceptance rule which does not explicitly take account of the gains or losses involved in the acceptance of a hypothesis or course of action is called Non-utilitarian. These rules are mainly used for the selection of acceptance of a hypothesis (belief). It is not attempted here to describe all the acceptance rules listed in Table 2.6. A detailed account of them can be had from ref. (70). The rules that are used in this thesis for making decisions will be discussed in later chapters along with the formulation of structural design problem. Some of the rules are briefly stated below.

The rules of high probability and high weight state that the hypothesis having high probability and high weightage respectively may be accepted. In structural design, the statement of an acceptable probability of survival as 0.9999 is a rule of high probability. The structure having this probability is chosen and is believed to be the acceptable design.

The rules of maximum probability and maximum weight are relative merit criteria that allow the acceptance of a hypothesis from a given set of alternatives choosing the one with maximum probability or weight respectively.

Likelihood criterion is used when the relative likelihood of events are subjectively decided.

Among the utilitarian acceptance rules, the rules for decision under certainty and risk are well known. Wald's criterion, minimax theorem and Bayesian rule will be discussed in the subsequent chapters. Other rules are defined below.

Given the alternative actions and the possible states of nature, the utility u_{ij} of any action-state combination can be found out. The matrix so formed may be a loss matrix or gain matrix depending upon whether the elements represent loss or gain. The minimax rules that pick out the acceptable act with different degrees of optimism are given below:

(i)	Min. loss	$\underset{i}{\text{Min}}\ u_{ij}$	$[u]$	loss
(ii)	Max min gain	$\underset{i}{\text{Max}}\ \underset{j}{\text{Min}}\ u_{ij}$	$[u]$	gain
(iii)	Max Max gain	$\underset{i}{\text{Max}}\ \underset{j}{\text{Max}}\ u_{ij}$	$[u]$	gain
(iv)	Min Max loss	$\underset{i}{\text{Min}}\ \underset{j}{\text{Max}}\ u_{ij}$	$[u]$	loss

Hurwicz's rule is a combination of min max and max max criteria. A factor β representing the optimism of the decision maker ($0 \leq \beta \leq 1$) is chosen. According to

this rule, that act i for which

$$\max_i \left[\beta \max_j u_{ij} + (1 - \beta) \min_j u_{ij}\right]$$

is chosen. Here $\left[u_{ij}\right]$ is a gain matrix.

Savage's regret rule is a modified form of Wald's criterion. A regret matrix is found out from gain matrix with its each element as the difference of the corresponding element in gain matrix and the maximum in its column. Let $\left[r_{ij}\right]$ be the regret matrix. The choice goes to that act i for which

$$\min_i \max_j r_{ij} .$$

It shows that acceptance rules are not unique except for utilitarian decision under certainty. Depending upon the situation and the optimism a suitable acceptance rule is to be chosen by the decision maker.

2.8 SOME MAJOR PROBLEMS IN STRUCTURAL DESIGN DECISIONS

Some of the major problems that deserve attention in a structural design process are discussed below. Of these problems some are traditional, some newly created.

2.8.1 <u>The Analytical Basis of Structural Mechanics</u>:

Traditionally, structural mechanics developed as an analysis-oriented process. The process of analysis makes use of the constitutive relations as a link between the forces

and the deformations. Constitutive equations are functions of cross sections and material of the system. In modern optimization processes, the variables representing the geometry of cross sections are taken generally as design variables and have to undergo a number of revisions before an optimum set of variables are chosen. Every time a change is made on these variables, constitutive equations undergo a change and the entire analysis is to be repeated especially in statically indeterminate systems. The iterative optimization techniques work on this philosophy.

The operational research techniques have a logical structure different from the analytical structure discussed above. They proceed to select the optimum alternative out of all possible alternatives with respect to some acceptance rule subject to certain constraints. Because of this difference in approach, the fitting in of structural design in the framework of operation research techniques is somewhat complex.

2.8.2 Subjectivity of Needs or Criteria:

Modern operational research techniques demand a quantitative statement of the utility of the alternatives. But most of the functional needs are subjective in nature, and a quantitative statement is difficult. For example, the monetary value of the motivation behind the construction of Taj Mahal or pyramids of Egypt is in no way measurable. It

is this problem of subjectivity, that compels the designer to resort to simulation and other subjective methods. The argument is true for other aspects alike namely aesthetics cost, safety, serviceability etc.

2.8.3 Arbitrariness of Safety Level:

This is a problem which has received the attention of research workers in the past. The statement of the acceptable factor of safety, load factor or probability of failure is quite arbitrary. As discussed earlier, stating an acceptable safety factor or probability of failure is equivalent to choosing the acceptance rule of high probability. This rule itself has no proper quantitative limit and any statement is made arbitrarily. The optimization methods in which failure probability is taken as constraint also have the above mentioned drawback.

One way of obviating this difficulty is to tie down safety with some utility like cost of failure, or cost-effectiveness. According to De Finnetti's concept of subjective probability, an individual choice of any particular action not only depends on the probability of outcome but also on the utility associated with it. The probability of failure concept in design recognizes only the probability of a state as the measure of acceptance. If it is coupled with the associated utility a better model will be obtained and the arbitrariness can be eliminated. Such a model is developed

in Chapter Four. It considers not only one type of failure but many types of failure.

2.8.4 Role of Experience:

Experience in design is an inevitability in the classical iterative approach to design to yield acceptable results quickly. In the modern decision processes also, experience very often becomes a necessity in the search for alternatives as well as in the problem analysis. The decision maker in some cases has to list out all the alternatives before he apply the decision techniques in which experience, knowledge and intuition are good guides. The dependence on experience brings in two difficulties.

(i) When a fresh engineer ventures into the field of structural design, he often gets puzzled as to how he can proceed in the face of lack of experience.

(ii) When the design is to be performed in a new surrounding about which no information is available, the designer has to really tackle a challenging problem.

The heuristic gaming suggested by Tung Au (15) is aimed to educate the designers for absorbing experience and engineering judgement by considering new design situations.

2.8.5 Inevitability of Human Judgement:

At various stages of design decisions, human

judgement is inevitable. It eners in the decision process through the analysis of problem, idealizations, selection of decision models and the acceptance rule, and finally in evaluating the results. The human interference hinders the complete automation of design with the help of computers. Since human skill is costly, it would increase the cost of design also. By making the computer to participate more in judgements involved in routine designs, this problem can partly be solved. This would necesitate the introduction of artificial intelligence in design processes as suggested by Spillers (16).

2.8.6 Need of Modelling:

Modelling of the real system, the environment and system behaviour are esential in structural design. This is done either because the designer is unable to tackle the problem exactly or because of the volume of computation involved in treating the problem in its correct perspective. The decision models for various conditions are discussed earlier. Apart from this, idealization of the system and system behaviour is needed in the decision process. The elastic, elasto-plastic, visco-elastic models etc. are idealized models for material behaviour. Pins, rigid joints, rigid supports etc. replaces the real type of connections and supports. If the model is not judiciously chosen, the real behaviour of the physical system and the behaviour of

model differ considerably and a prediction of the real system through the model is no longer valid.

2.8.7 Diversity in Design Techniques:

Today, with the increased human needs, social changes, and spirit in scientific knowledge, numerous types of structures, numerous structural materials and their composites have come into use. The environmental influences are also varied. Each situation therefore demands separate considerations and modelling. From practical point of view, a unified approach or algorithm irrespective of the diversity in materials, structures or environment makes the design process economical and automation easy. The system engineering approach to structureal design is a welcome step in this direction. Any unified approach must not be at the loss of accuracy.

2.8.8 Introduction of Self-Adaptivity:

By the term self-adaptivity, it is meant that the system must adapt itself to the different conditions to which it is subjected, showing efficiency in each of the conditions. Among the heirarchy of systems only living beings (animate or inanimate) are self adaptive. Self adaptivity, if introduced in structures, is very useful in multiload conditions.

A simple example of a self-adaptive structure is a

suspension cable. Because of its ability to change shape, it adapts itself to loads applied at different points on the cable. Zuk (96) suggests that artificial 'brain' must be supplied to the structure in the form of a built in programmed computer or servomechanism which directs the structure to adjust itself to the various types of loads. For example, when the wind forces are excessive causing lateral deformations to be exceeded beyond a specified value, a servo-mechanism is activated which prestresses the vertical cables thereby restricting further deflections. Several possibilities of such structures, called kinetic structures, are given by Zuk (96). Yao (97) uses the self-adaptive principles in making the structure to be flexible after a certain lateral deformation to reduce dynamic effects under earthquake.

2.9 SUMMARY

The nature of structural design decisions are examined in the light of system engineering and theory of decision making. The structure is treated as the component of an overall engineered system. The importance of design in rationalising the structural behaviour is emphasized. The basic concepts of design are discussed, and the decisions in a structural design are divided into five steps. They are the selection of: a. Position or positions occupied by the structure, b. The functional configuration,

c. Structural concept or type, d. Arrangement of members and types of connection, and e. The geometry and material of members.

A system effectiveness model is suggested as the decision criterion in which structural values, economy and function are traded off. In general terms, structural design is shown as a decision under uncertainty. It is emphasized that decision models for decision under uncertainty are to be used to account for the uncertainties rather than taking uncertainties empirically or by subjective judgement. Further some of the major problems in structural design that deserve the attention are listed.

In the light of the information gathered in this chapter about structural design process, a design methodology for the five steps of decision is proposed in Chapter Three.

CHAPTER THREE

OUTLINE OF THE PROPOSED DESIGN METHODOLOGY

3.1 INTRODUCTION

Design is a creative activity in which the talents of the designer may often outweigh any systematic methodology in achieving better designs. However, a methodology can help the designer to make decisions in a well-organized way and not to miss any alternative by oversight. With this intention, the nature of structural design decisions, and the associated problems were discussed in Chapter 2. Five decision steps were suggested in a design process, and was shown that all the decisions must be treated simultaneously so as to achieve an optimum design. The consideration of all the decisions simultaneously is practically impossible on account of the complexity of the problem. Hence,it is worthwhile to examine in what order the decisions can be taken, and what are the decision models required for each stage of decision. A methodology of design is proposed in this chapter, making use of the ideas developed in Chapter 2. The rationality of a design depends on the interaction of the designer,design methodology and the relevant information of the input and output data. Design-designer relationship is discussed in Section 3.2. The role of experience, learning and judgement in design is discussed in Section 3.3. With this background, the design methodology is proposed in

Section 3.4. An operational gaming model is proposed in Section 3.5 for the selection of the initial selection of site and functional configuration. In Section 3.6, the selection of the structural concept is considered and the use of certain models like checklist, decision tree etc. is described with illustrative examples. Other decision aspects are dealt with in the subsequent chapters. The chapter is intended to present a central framework within which design can be done.

3.2 DESIGN-DESIGNER RELATIONSHIP

Design, being a subjective activity, is more personal in character than science. The role of designer has a greater impact in design, and therefore it is useful to define the design-designer relationship. Structural designer is primarily entrusted with the selection of the structural concept and the decisions in the subsequent two steps namely arrangement of members, and the selection of geometry and material of members. Thus the real responsibility of structural engineer starts only when the selection of functional configuration is over. However, it is to be noted that the structure is an integral part of the overall system. To maintain the interaction with other components and to achieve better economy, the structural designer has to associate with other designers involved in the design of the system. He participates in the selection of the overall system. The role of the structural designer as a decision

maker changes from step to step. It may be considered that three roles are assumed by the designer.

(i) Partner in a Group Decision: Initially, structural designer assumes the role of a partner in a group decision in the functional planning of the system. The decisions involved at this stage of design have greater social, economical, functional and political content. The participation of the structural engineer in this group decision benefits the design in two ways:

(a) He impresses upon other designers about the structural needs in the overall planning of the system. The functional configurations chosen must be consistent with structural feasibility, and efficiency.

(b) He gathers enough information about the functional needs of the system so that, when he selects the structure he can do it keeping the functional needs in view.

(ii) Individual Decision Maker: Once the selection of functional configuration is made, it is the sole duty of the designer to select the structural concept for the system. He has to do this with the three objectives in view, namely functional conformity structural efficiency and economy.

(iii) Decision Adviser: The decisions in the latter two steps following the selection of structural concept are more of a behaviour (action) oriented type. The decisions

are made on the basis of the behaviour of the structure under load. This process can be viewed from a different angle. Instead of the choice of the geometry of members, let us assume that the structure has to decide its course of action to resist the external loads, bringing structure itself as the decision maker. The course of action in this case is the structural action or behaviour. It is a course of action adopted by structure and not by the designer. This approach has specific advantage in dealing with the uncertainty in the information of design data, by considering that structure takes a decision under conditions of uncertainty. The designer in this case makes decisions on behalf of the structure. In other words, he assumes the role of a <u>decision adviser</u>.

3.3 EXPERIENCE, LEARNING AND JUDGEMENT IN DESIGN

In addition to technological skill, experience and ability to make judgement are the two qualities that a designer must possess. These qualities are acquired partly by learning. Even to build up the intuitive ability to evolve new designs for new situations, experience and knowledge can help to some extent. Very often by repeating design with the knowledge obtained in a prior cycle, the design can be improved. This concept is made use of in the method of design proposed in Section 3.4.

3.4 PROPOSED DESIGN METHODOLOGY

A methodology of design is proposed in this section. The discussions made in the preceding sections are taken into account in this proposal.

3.4.1 Decisions Involved:

The decision scheme involved in the structural design is the following:

1. Selection of site or position of the system
2. Selection of the functional configuration
3. Choice of the structural concept
4. Arrangement of members and types of connections
5. Selection of the material and geometry of members i.e. proportioning of the members.

In addition to the major decisions involved as given above, there are some associated decisions, pertaining to the design activity. They are the collection of relevant design data, and its projection throughout the design life, a load analysis, the choice of a model to study the system etc. These decisions are as important as the decisions listed above.

3.4.2 Essential Characteristics of the Design Methodology:

Any design methodology for structural design should have the characteristics discussed in the earlier sections, a summary of which is given below.

(i) The data needed for design must be easily obtained. Only such methods will be accepted by practical designers.

(ii) The design data are generally obtained with different degrees of accuracy. The parameters may be random or non-random. The information may be exact or uncertain. In the case of uncertainty, some data may be known to lie within certain bounds, while others may be completely unknown. According to the nature of information, decisions are classified into decision under certainty, risk or uncertainty. The method of design must be flexible enough to accommodate the design data of various levels of accuracy and yet capable of arriving at reasonably acceptable decisions.

(iii) Functional conformity, structural effectiveness and economy are the needs or criteria in the design of structures. As discussed earlier, the statement of the needs are often subjective, and qualitative. In the estimation of the cost of structure, cost of failure, serviceability requirements etc. considerable uncertainty exists. The method of design must take into consideration the subjectivity and uncertainty in the estimation of the needs. As far as possible, the design must not be very sensitive to the stated needs. Slight variations in the human needs are very common, and any design sensitive to the change in needs will have psychological implications. An absolute economic design sensitive to the design needs may not be as acceptable as a

suboptimal design that is flexible to the changing needs.

(iv) The decision model must be a logical one. It must be capable of considering all available alternatives, and the optimum must be picked out considering all of them.

(v) If past experience on the design is available, it must be made use of. If such experience is not available, the design process must be capable of creating enough information by way of repeated trials so as to evolve an optimal system. However, the trials must be consistent with the cost of design, time available for design and the degree of refinement needed.

(vi) The need of human judgement at every stage of design must be eliminated as far as possible. However, human judgement cannot be fully excluded from the design procedure. The judgement must be incorporated into the design in such a way that it should not interfere with the flow of computation. This is especially needed in the automation of routine design. Judgement may sometimes be fed into the computation as an input data as is done for the decision of the marginal safety factors in Chapter 7.

(vii) The decision models chosen must be consistent with the role of the designer in the various stages of design, discussed in Section 3.2.

(viii) Apart from safety and other structural values, the social, psychological and economic involvement in the design of the system must also be brought into the design process

at least subjectively if possible.

(ix) The design method must be same for any situation irrespective of the structure, loads or design needs; but not at the expense of accuracy.

(x) The structure that is to be designed must be treated as the component of an overall system constantly in action along with other components. Consideration of the interaction of all the components would be helpful in the optimization of the performance of the system as a whole.

3.4.3 Grouping of the Decisions:

The ten requirements of a design methodology listed above are difficult to obtain in a method of design. Further, a simultaneous consideration of the decisions taking all the factors given above is too complex. Hence, a stagewise design is attempted here. The five decisions are grouped into three categories as follows:

(i) A group decision for the selection of site and functional planning

(ii) Choice of the structural concept

(iii) Design of structural elements.

The details of the decisions involved in each group are given in the next section. This grouping of decisions would help to select proper decision models.

3.4.4 Characteristics of the Decisions and the Choice of Decision Models:

The characteristics of the decisions in each group are discussed and the appropriate decision models are chosen.

(i) Group Decision: The interaction of the various requirements in the design of the general system is considered in this decision. Some of the requirements may be of competitive nature, while others may be complimentary to one another. The final decision is a compromise among the various requirements. The data available at this stage of design are totally inadequate for a rigorous mathematical analysis. Most of the requirements are subjective and a quantitative statement may not be possible. The social cultural, psychological and even political implications are significant at this stage of decision. The personal preferences with regard to functional needs and aesthetics may very often outweigh other considerations. No pattern of set rules can be established for human preferences.

A simulation model may be better suited to this stage of decision than any rigorous mathematical model. Attempts have been made to utilize mathematical programming technique for the decisions involved in this group. For example, Schilling (98) makes use of unconstrained gradient method (steepest descent) to find an optimum site of a static missile test stand so that the total cost of construction,

consisting of the cost of allied constructions and that of a safety barrier, is minimized. Aguilar (99) uses a linear programming algorithm for decision making in building planning and real estate development. The decision considers the construction of apartments with zero, one, two, three and four bed rooms both furnished and unfurnished. The problem is to find how many apartments of various type should be built in order to maximize return on investment. The constraints on the solution involve the total amount of money available, the maximum amount to be spent for furnishings, the minimum number of units to be considered and probable preferences for various types of apartments. However, in practical problems, the data may not be as specific as used in these problems due to the subjectivity of the problem.

The heuristic game approach proposed by Tung Au (15) is an elegant model for the decision. According to this model, structural design operation is a game played by a group of designers in a computer simulated environment. The various design requirements are assumed to be in the hands of each of the designers. The various proposals are rated and optimal choice is made. Au uses the method for the entire design activity, starting with the selection of site and ending with the final choice of members (100, 101). In the present formulation, gaming is intended to use for two decisions only, and it is proposed to have a greater involvement of human preferences. The owner of a house may have

preferences between a shell roof and a flat roof, but he will generally be indifferent to the arrangement of beams, the thickness of slabs etc. Hence, the personal preferences of the client or even that of the experts for designs of other components may not be generally carried over to the stage of detailed structural design. Hence the use of simulation model is proposed to be limited for the group decision activity. An operational gaming model is proposed for this decision, and is discussed in Section 3.5.

(ii) Choice of Structural Concept: The decision represents the transition stage from the decision of overall system to the more detailed decision of the structure and its elements. The structural designer is considered to be the sole decision maker. He makes use of the information that he has gathered from the group decision activity, and his own judgement for the selection of the type of structure. The personal preferences of the client and others, the functional needs, structural effectiveness and economy are the bases of decision. The choice is made from a set of alternatives which may include existing types, modifications to the existing types and even new innovations. Experience and creativity are needed to find out new concepts (both structurally and conceptually) for which no methodology can be proposed. However, when the alternatives are listed out, a choice can be made from them in a systematic way. The data available at this

stage are more encouraging than those in the previous stage. The design needs are still subjective. Subjective decision models based on preferences are suitable, if data for choice are not adequate. A checklist method and certain other models of decision are proposed in Section 3.6.

(iii) Design of Structural Elements: The detailed design of the structure chosen is carried out at this stage. Traditionally, structural design referred to this particular stage of decision and considerable attention has been given to this decision. Therefore, today we are in a better position to deal with the design in a systematic manner.

As mentioned before the decision is a behaviour-oriented decision. Structural effectiveness and economy are more important than functional aspects, although functional constraints cannot be neglected. Two decisions are involved.

(i) Arrangement of the members and the connections
(ii) Selection of the material and geometry of members

From economy point of view, the two decisions must be considered together. For any arrangement of members, the cross sections can be designed for safety and economy. Any change in the arrangement of members or connections affect the geometry of members. Also any change in material may affect the type of connections. The data available may be certain, random with known probabilities or uncertain. The needs may be stated quantitatively even though certain amount

of inaccuracy may be involved in their estimation. The decision process as suggested below.may be suitable for this stage.

The structure is assigned with the role of a decision maker, who decides its own course of behaviour under the action of external loads. Since uncertainties of different origin exist in the design, the decision is considered as a decision under uncertainty and appropriate models are chosen. Structure is assumed to be playing a game against nature. The behaviour of the structure is the strategy of structure, and the uncertainty in the loading process is that of the nature. Structure has to decide two types of decisions.

(i) The optimal arrangement of members and types of connections

(ii) The material and geometry of the sections of members.

Both the choices jointly minimize a cost-effectiveness criterion. The minimization is done as follows:

(i) One arrangement of the members together with the connections is chosen. For this strategy chosen a game is played by which the optimal choice of material and geometry is made. (This decision problem is dealt with in the subsequent chapters)

(ii) Games are played for several such arrangements. This may, in some cases, be possible by changing certain

parameters defining the arrangement of members; for example angle of inclination of a truss member. In several cases, no such variable may be available and design has to be done individually for each alternative. That alternative giving the optimum cost-effectiveness criterion is chosen as the optimally arranged one. Fig. 3.2 shows the scheme of decision.

3.4.5 Inter-relationship of Decisions and Flow of Information:

In the preceding section, decisions involved in the three groups are described. However, a decision process having independence of one from another is not possible, as the decision at any stage requires information from all other stages and vice versa. This inter-relationship of decisions make a stage by stage decision of any order not considering the flow of information uneconomical. A method is proposed below to carry out the design in a sequence.

(i) Proceed with the five decisions in the order in which they are presented.

(ii) At any stage, a decision made in a prior stage is not allowed to be violated. In other words, they are taken as constraints. The information required from a decision not yet made is taken empirically. If details of them are available from existing designs, they can be made use of. Otherwise they are chosen arbitrarily.

(iii) At the end of each decision, the empirical data used in prior stages are checked with the data obtained through the decision at the current stage. If the empirical data used is unsatisfactory, a procedure to go back to that step and repeat the decision is followed.

(iv) Proceed upto the last decision as described above, completing one sequence of decision forming one operation in an iterative process.

3.4.6 Search for Optimality Through Learning Cycles:

The design so obtained by a single sequence may not be an optimal one, as empirical data is made use of. This is more so when the design is performed for a new surrounding about which very little data are known. However, improvements can be made by repeated trials of design. Design process is considered for this purpose as a learning mechanism. When one sequence of decisions is completed, enough data is 'learnt'. The information gathered is checked with the empirical data used and the accuracy of design is evaluated. If data obtained through past experience is used, the deviation from optimality may not be much. The design may be repeated with the data gathered. The repeated trials in the framework of a heuristic algorithm may lead to an optimum design. The process requires considerable research work to make it a full-fledged optimization process.

3.4.7 Stopping Rule:

Several trials become necessary to arrive at a theoretical optimum by this way. Further, mere repetition of trials may not guarantee an optimum at all. Therefore, the process of refinement must be stopped at some level of suboptimal design on account of the enormous cost of design or lack of time available. A stopping rule may be proposed to find out at what trial it is economical to stop the design.

Let C_o be the cost of a single sequence of design decision. Let C_1, C_2, ... be the costs of the structures obtained in a sequence of trials made. Let C_1, C_2, ... C_n be the values in n trials. The designer may stop at the nth trial or may proceed to the (n+1)th trial. If he stops at nth trial, he is supposed to spend an amount

$$V_n = \left\{\min_{1 \leq i \leq n} C_i\right\} + n\,C_o \qquad (3.1)$$

If he proceeds to the (n + 1)th trial, the designer is supposed to spend

$$V_{n+1} = \min\left[C_{n+1}, \left\{\min_{1 \leq i \leq n} C_i\right\}\right] + (n+1)C_o \qquad (3.2)$$

If C_{n+1} is less than $\left\{\min_{1 \leq i \leq n} C_i\right\}$, the trial is advantageous for the designer. However, in a heuristic procedure as proposed here, there is a risk involved in this process,

and C_{n+1} may not always be less than $\underset{1\le i\le n}{\text{Min}}\, C_i$. Hence the designer has to decide whether it is worth proceeding to the (n+1)th trial. Considering the sequence of trials as a stochastic phenomenon, stopping rule may be derived by a method similar to the method of optimal persistence policies of MacQueen and Miller (102).

3.4.8 Storage of Information for Future Designs:

The information gathered through an optimal design is the experience gained, by the designer. The ideas that are learnt may be stored for the benefit of future designers. The information gained from the design of all sorts of civil engineering structures may be collected by a central agency and made available for designers in the form of charts, graphs etc. This prior experience and data will help to reduce the volume of computations in the design of structures. In civil engineering construction, mass production of any particular design as done in mechanical design or aerostructures is rare. The cost involved in the refinement of a design is justifiable in the latter cases on account of the mass production as it will lead to considerable saving. On the other hand, the cost of such extensive trials may not be justifiable for a design leading to the construction of only a single structure. The data from prior designs would be very much useful in such cases to reduce the cost of design by cutting down the number of design cycles. This would be

also helpful in evolving codes of practice.

3.4.9 Summary:

Now the various decisions, sequence of decisions, evaluation of the design, storage of experience etc. discussed may be presented schematically as shown in Table 3.1 and Figs. 3.1 and 3.2.

3.5 GAMING SIMULATION

Gaming simulation is extensively used in management and war decisions, urban planning (103), planning education (104) and architectural design (105). It is, in general, very useful for prediction of behaviour and training individuals. Conceptually gaming is different from theory of games, though both processes take into account the conflict in the interests of players. In the latter theory, a formal mathematical structure is developed making assumptions in the behaviour of players. It is assumed that the players are rational, always striving to better their gains. In gaming, no such assumption is made, nor a formal mathematical structure exists.

The proposed decisions of the site and functional configuration are brought within a gaming model. Design is assumed to be a game with a number of players. In general the list of players consists of the client who receives the

benefit, the client who is financier (treated as two players because of the conflict of interests), the society, architect, structural engineer, construction engineer, and any other individuals or groups interested or involved in the decisions. The players may have interests which are conflicting or complementary. The preferences and opinions of the players and the various proposals made by them are put forward in the form of questions and answers. The final decisions may be obtained by arriving at a compromise in the needs and proposals. This process of rating the proposals can be carried out with the help of a computer.

Two separate games are played; one for the selection of site and the other for the choice of functional configuration. No attempt is made in this thesis to study the problem at length.

3.6 CHOICE OF STRUCTURAL CONCEPT

The choice of structural concept as explained earlier has to be made considering several factors ranging from structural efficiency to human preferences. If the objectives with regard to each factor can be mathematically stated, a multifactor optimization procedure may be adopted. Krokosky (70, 72) has used this approach for designing materials for wall systems and for a sandwich panel for structural, thermal, acoustic and other design requirements. However, in many problems, such a mathematical statement is

difficult due to the subjectivity of needs and presence of qualitative information regarding different structural forms. Hence, two methods based on personal preferences are proposed here, which consider the subjectivity involved.

A choice is to be made out of all available alternatives considering all factors giving due weightage for each of them. Shepard (106) after conducting a number of experiments, reports that the ability of human being to arrive at an overall evaluation by weighing and combining or trading-off all of the separate attributes (factors) at the same time is difficult as compared to evaluation of one factor at a time. Hence, it is always convenient to have judgement on smaller aspects and a method for finding the relative merits for the global factors from these partial judgements. This approach helps in the systematic selection of structural concepts out of available alternatives. It aids the experienced designer for tackling new design situations or as a check to his intuitive solution for known situation. However, it is very helpful to systematize previous knowledge and also to train engineers in making intelligent decisions. The basic concept involved in all subjective decisions is that, given two alternatives, a decision maker is able to express his preferences or indifference between the two. This may be consistent at a smaller level of decision than at a global level. A numerical characterization of preferences is made by assigning utilities to each of the alternatives

which represent the preferences in terms of numbers.

3.6.1 Multi-attribute Selection of a Structural Concept by Checklist Method:

Checklist method or decision tables is a formal framework for guiding the designers' thoughts. They are not substitutes for creative thinking, nor can give optimal decisions automatically. The method is extensively discussed by Eder (13), Matousek (107) and Alger and Hays (108). The essentials of a checklist method are the following:

(i) The various alternatives to be considered are listed out. All eligible structures existing, or new must be considered.

(ii) A list of influencing factors that are relevant to the problem must be set up. Care should be taken not to leave any factor by oversight. A general list consisting of all the factors that are associated with the choice of structural concept must be made available. Matousek (107) presents a large number of such lists for mechanical engineering design. The relevant factors may be selected by the designer from such a general list.

(iii) Each factor j is given a weightage w_j such that

$$\begin{aligned} & w_j \geq 0 \\ & \sum_j W_j = 1 \end{aligned} \qquad (3.3)$$

(iv) For ith alternative and jth factor a utility u_{ij} is assigned. The numerical characterization may be factual if enough data exists or based on qualitative preferences which are transferred to numbers. Thus, if ith alternative is preferred to jth one, a greater utility is assigned to ith alternative than that of jth one, and vice versa.

(v) The rating U_i of the ith alternative is

$$U_i = \sum_j W_j \; u_{ij} \tag{3.4}$$

(vi) The alternative having optimum rating U_i is chosen as the best alternative. However, those alternatives in the vicinity of optimum must not be left not considered.

Table 3.2 shows the decision table for the choice of the optimum action. The decision table approach is illustrated below by means of an example of a roof system.

Illustrative Example: Consider the choice of a roof for a dining hall 60' x 60' size. The live load on the roof is taken as 40 lbs/sq.ft. The decision set consists of all types of roofs that can be thought of. Two constraints are imposed on the choice as follows:

(i) The roof should be flat, and

(ii) The area is to be column-free.

The two constraints together restrict the choice to a permissible set of flat roofs, which is a subset of all roofs. All such roofs that the designer can think of must be

listed out. Let them be listed as follows.

a. One way slab

b. Two way slab

c. Beam and girder system

d. T-beam floor (one way beams)

e. Grid floor (rectangular)

f. Grid floor (diagonal)

g. Cellular plate construction

h. Precast concrete floor system.

Let the attributes or factors based on which the choice is to be made be the following:

1. Structural efficiency
2. Cost
3. Aesthetics
4. Function
5. Ease of construction
6. Fire resistance
7. Heat insulation
8. Acoustics
9. Lighting

The weight of each attribute is chosen as follows. Let the weight of item 1 be 20. The weightage of other factors can be put in numbers relative to that of 20. Let them be 20, 20, 10, 10, 5, 4, 3, 2, 1 respectively for the factors in the order given above. Now they can be reduced to fractions

such that the sum is equal to 1.0. Thus the weight for item 1 is $\frac{20}{75} = 0.267$. The weights of the various factors are shown in Table 3.3. Now, utilities are assigned to each combination of alternative and attribute for the particular conditions under consideration. The utilities are assigned based on the preferences of the designer. A greater number is assigned to the one most preferred. For consistency, numbers between 1 and 10 are assigned. Utilities are given in Table 3.3 for the conditions of constructing the hall in Kanpur. The numbers given in Table 3.3 are for illustration only. The rating U is computed and entered in the last column of the Table. From the rating it can be seen that the choice goes to the grid floor of the rectangular type as it has a higher rating.

3.6.2 Choice of Multiple Component Systems:

The checklist method given above helps to select a particular element out of all alternative elements for many attributes. Very often a structural designer has to select a harmonious combination of various elements of a single structure. For example, the selection of the structural concept of a bridge involves the selection of the deck, the type of supports for the deck, the piers and foundations. The selection is optimum only when the combination is optimum rather than each component is individually optimum.

(a) Decision Tree Approach: In general, there is strong

interdependence among components and a choice is to be made for a combination. A decision tree approach discussed by Eder (13) is one method for this decision. According to this method a tree of alternative combinations of the components is prepared. For example, in the bridge problem, let there be 4 types of bridge decks, 4 types of supports for the deck, 3 types of piers and 3 types of foundations. Each alternative structure may be obtained by combining components selecting suitably from each group. Thus, in the particular case 144 alternative combinations can be thought of. Of these, some combinations may be deleted, as they are either impossible or not worth considering. Considering each branch of the remaining tree as an alternative, checklist method may be applied as described before. An approach similar to this was developed by Murthy (54) in the section on design for cost-effectiveness. The problem tackled is the synthesis of available structural form and available material (3 in each case) for designing a beam for a minimum weight, cost and reliability. For the 9 combinations of 3 materials with 3-structural form, utility factors are given based on previous experience or subjective factors for each of the 3 criteria. Then subjective probabilities are associated for each choice (combination) giving the desired outcome in terms of weight, cost, reliability. The global utility of a design combination is found by taking the sum of the products of probability and utility associated with each choice or decision, and

then the alternative having the maximum global utility is chosen.

(b) Riskless Choice Based on Preferences (Subjective Programming): The decision tree approach very often become impracticable and inconsistent due to the large number of alternatives to be considered and due to the excercise of judgement on a relatively global level rather than at component level. Hence, certain simplified procedures may become, feasible, if an assumption is made as follows. If the components have utilities based on preferences, the utility of any combination of these components is the sum of the utilities of the components. This assumption called additivity assumption leads to the application of subjective programming given by Aumann (84) and other methods of riskless choice developed by Adams and Fagot (109).

Subjective programming problems deal with decisions without any objectively measurable utilities. The decisions are therefore made based on subjective preferences. The preferences are transferred to numerical utilities. This judgement is done for each component with regard to each attribute or factor. The method is similar to the axiomatic utility theory of Von Neumann and Morgenstern (85) for situaions involving risk, a version of which is used in Chapter 7. The decision methods give a formal procedure to synthesize the decisions at a smaller level to a global decision. The method of subjective programming is illustrated below with

respect to a structural design problem.

Let the structure consists of three component systems A, B and C. Let A_1, A_2, ... A_ℓ, B_1, B_2, ... B_m, and C_1, C_2, ... C_n be the alternatives available that are admissible.

Let each component is to be selected considering say k factors or attributes as described in checklist method. Let $(a_1^{\ 1}, a_2^{\ 1}, \ldots a_k^{\ i})$ be the factors under consideration. The alternative A_i can be considered as a point in a k dimensional space with co-ordinates $(a_1, \ldots a_k)$. Each alternative defines a point in the space. The utility among the alternatives can be considered as the weighted sum of utilities of such point in k space. Thus

$$U(A_i) = w_1\, u(a_1^{\ i}) + w_2\, u(a_2^{\ i}) + \ldots + w_k\, u(a_k^{\ i}) \qquad (3.5)$$

If A_i and A_j are two comparable elements in the set A

$$A_i \succsim A_j \quad \text{implies} \quad U(A_i) \geq U(A_j)$$

Preferential order exists among each class of elements. Between two classes of elements say roofs and walls, no such preferential order exists. Hence

$$A_i \not\succsim B_j \quad \text{or} \quad B_j \not\succsim A_i \quad \text{for all } A_i \text{ in A and } B_j \text{ in B.} \qquad (3.6)$$

For consistency requirements, the following assumptions must

hold good.

(i) Transitivity: $A_i \succsim A_j$ and $A_j \succsim A_k$ implies $A_i \succsim A_k$ (3.7)

In terms of an example, if

grid floor $\succsim$ cellular plate, and
cellular plate $\succsim$ T-beam floor, then
grid floor $\succsim$ T-beam floor.

(ii) Reflexivity: $A_i \succsim A_i$ (3.8)

(iii) Completeness: If every class of elements are comparable, the decision set is complete. In the problem it is assumed that each class of elements have preference ordering. In subjective programming this assumption is not a necessity.

(iv) Additivity: The additivity assumption says that the utility of a combination of elements is the sum of the utilities of their components. Thus

$$\text{if} \quad A_i \succsim A_j, \quad A_i + B_k \succsim A_j + B_k \tag{3.9}$$

and

$$U(A_i + B_k) = U(A_i) + U(B_k) \geq U(A_j) + U(B_k) = U(A_j + B_k) \tag{3.10}$$

Case 1. Additivity Assumption is Valid: This assumption is valid if the decision maker excercises the preferences based on the utilities of individual components and is indifferent to any manner of combining them. If this

common in the addition of components in structural systems. This interdependence may be taken care of by modifying the utilities of the components by suitable judgement or weightage factors obtained by judging the relative performance of the components in the overall system, and adding them.

Thus

$$U(A_i + B_j) = \alpha\, U(A_i) + \beta\, U(B_j) \qquad (3.11)$$

where α and β are two weightage factors that modify the utilities of the components. The factors may be visualized as numbers that modify the worth of a component in the midst of a set of other components. For consistency, α and β may be chosen such that their sum is 1.0. A similar procedure can be adopted if there are more than two components. The utilities of all admissible combinations must be obtained in this way, and the one combination with highest utility is to be chosen as the best alternative. However, the alternatives having utilities in the vicinity of the maximum may also be considered in further decisions.

In order to be consistent in the judgement of the factors for various alternatives, a procedure as described below may be followed for getting weightage factors.

Firstly, basic gradation factors are to be chosen to represent the relative weightage that one gives to the utilities of the various components of the system in terms of required system attributes. For example, a parapet wall

may not be given as much weightage as for a roof or foundation in terms of cost etc. This gradation is with respect to basic importance of components and is the same for all alternatives. Suppose, for a three component system, g_A, g_B, g_C be the weightage for components A, B and C. They are evaluated by a tabulation method as shown in Table 3.5.

Secondly, the <u>interaction factors</u> that modify the utilities of components (as derived in Tables 3.4 and 3.5) for the interaction with adjacent components are chosen. The weightage factor is the product of gradation factors and interaction factors. The interaction factors differ for different combination of components. The method of choice is illustrated in Table 3.6. At the time of excercising the preferences, the decision maker proposes how he will modify the utilities by means of a multiplication factor, if the component interact with an adjacent element. If there is no change, he enters 1.0 in Table 3.6. If the utility is increased, a number greater than 1 is to be chosen. In expressing these factors, the interaction of the same component with other alternatives of the interacting component must be kept in view so that the preference is consistent. After assigning such numbers, the product of all the numbers that belong to each component is found out as shown in Table 3.6. Finally, the numbers are reduced to fractions such that fractions of the components in a system add to 1.0.

Alternatively, a given combination may be chosen as the datum and rated on the basis of weightage given to system attributes as shown in Table 3.5. This procedure is repeated for all combinations of components comprising the system, and factors are given for each of system attributes relative to those given for the above 'datum'. Weightage factors are to be obtained for each component, separately for each combination similar to Table 3.5. This procedure, although more rational, is lengthy.

Finally, the weighted sum of utilities for each alternative is found out.

Illustrative Example: Consider the design of the dining hall taken up in Section 3.6.1. Let, in addition to the roof, vertical load bearing members and the foundations are also to be chosen. Suppose the decisions are to be made with respect to the nine factors discussed in the example considered in Section 3.6.1. In the case of foundation, the unwanted factors like aesthetics, lighting etc. can be eliminated by choosing the weightage as zero for all of them. The alternatives in each group can be listed as follows:

1. Types of roofs (only a selected list is given here).

   B_1 Grid floor (rectangular)
   B_2 Beam and girder system
   B_3 Grid floor (diagonal)
   B_4 Cellular plate

2. Vertical load bearing members.

   C_1 Wall system (load bearing brick wall)
   C_2 Column system (with partition walls brick or precast concrete)
   C_3 Rigid frame (with partition walls brick or precast concrete)
   C_4 Truss supports (with partition panels of asbestos or G.I sheets).

3. Foundations

   D_1 Spread footing
   D_2 Pile foundation
   D_3 Raft foundation

The utilities U(B) may be taken as those in Table 3.3 and the utilities U(C) and U(D) are shown in Table 3.4. The gradation factor and factor for component interaction are calculated in Tables 3.5 and 3.6 respectively. The final weighted addition is done in Table 3.7. As the table is self-explanatory it is not attempted here to describe the computations involved. The choice goes to the combination of grid floor + column + spread footing (B_1 C_2 D_1) though by additive composition without considering interaction the preference is for grid floor + wall + spread footings.

3.6.3 <u>Discussion</u>:

The methods mentioned above for choice of structural concept are not exhaustive. Other available decision theory methods may be developed for the decision. The methods given above are only to illustrate the division of judgement to a level of smaller factors and the synthesis of these

judgements to a global decision.

Subjective programming and checklist method are identical methods. The former considers the selection of a multiple component system under a restricted condition of additivity. The additivity assumption is to be used with care in structural design problems. The decision tree approach does not have this drawback as the utility for each alternative combination is obtained. For the example of dining hall design discussed earlier, if 9 attributes are there for each combination, 432 utilities are to be assigned in the form of a 48 x 9 matrix. In the method using the weighted utility factors, the decisions involved are the selection of 99 utilities for the 3 sets of components and 144 weightage factors. Thus in certain problems, the latter approach may have some saving in the number of decisions to be made. Another feature of the latter method is that it involves decisions at a smaller level than that in a decision tree approach. The utilities are decided for each component separately with respect to each of the components.

3.7 SUMMARY:

A design methodology consistent with the modern concepts of engineering design , and theory of decision making is suggested. It is not intended to study all the aspects of the problem, as it is too wide and is beyond the scope of a single thesis. Methods are proposed for the

decisions at each step and for combining the operations in the framework of a learning mechanism. Of these decisions, checklist and other methods for selection of structural concept are explained through examples.

In the subsequent chapters, the last stage of decision, namely the selection of material and geometry of cross sections, is treated as a decision problem under uncertainty and is studied in detail.

CHAPTER FOUR

UNCERTAINTY, FAILURE MODES AND COST-EFFECTIVENESS

4.1 INTRODUCTION

The choice of the material and member cross sections for a given layout of members is considered in the present chapter and the following six chapters. Of these, the present chapter and the next two chapters are devoted to the development of necessary background information and design methods for the more-detailed formulation presented in Chapters 7-10. As the design is considered as decision under uncertainty, a detailed study of the different types of uncertainties is very helpful. With this intention, the uncertainties in the design information are classified in Section 4.2. In the inelastic design, several modes of failure including unserviceability and different degrees of damages are to be considered. When a random loading is present, all these failure modes are probable. Different failure modes under static and quasistatic conditions of loading are discussed in Section 4.3. A cost-effectiveness model that trades off cost of the structure to the effective ness measured in terms of expected cost of failure is discussed in Section 4.4. Cost-effectiveness is the specific objective criterion for structural optimization.

4.2 UNCERTAINTIES

It is emphasized in Chapter two, that the decision maker is operating under conditions of uncertainty in making structural design decisions. The uncertainties were classified broadly into three types and are further divided as shown in Fig. 4.1. The word uncertainty refers to only the uncertainties in the information of the phenomena. Another type of uncertainty arising from random phenomena (called objective uncertainties) is considered as a risk situation, because the decisions under random states can be made with risk.

4.2.1 Uncertainty Associated With Loads and Loading Process:

A correct assessment of loads and their effect on structure is very much essential in ensuring a safe and economic design. The effect of a load is to cause a structural disturbance leading to the development of internal stresses, strains, deflections, cracks etc. The loads acting on the structure may be of varied nature ranging from instantaneous impact loads to sustained loads acting throughout the life of structure. In general, loads are dynamic in character in the sense that they change from time to time, both in magnitude and the manner of application. In spite of the dynamic nature, depending upon the manner of application, loads may have three different effects.

1. Static effect
2. Fatigue effect
3. Dynamic effect

The so called static and quasistatic loads of very few number of repetitions cause only static effect, and hence in structural analysis they are treated as static loads only. The loads that may cause fatigue effect without any appreciable dynamic oscillations are the following.

1. Repeated cycles of quasistatic loads.
2. Dynamic excitation of loads in which the resonance amplification is not important or can be eliminated. The elimination occurs as follows(110): (a) When the structure is of large mass and is under applied transient load patterns of discrete excitation frequency. (b) By making the structure to have natural frequency different from excitation frequency. (c) By reducing the intensity of excitation by providing vibration insulators. Failure occurs within the small deformation range and is generally by the propagation of cracks. The type of fatigue failure include dynamic fatigue due to load, creep fatigue and thermal fatigue. Failure of structure under dynamic loads occurs generally with large deformations. It may occur before sufficient number of cycles to cause a fatigue failure.

<u>As a first attempt, the uncertainties associated with the first type of loads namely the loads having only</u>

static effect are considered here.

Uncertainties associated with the quasistatic loads are of two types: Uncertainties of statistical nature, and uncertainties of strategic or non-random nature.

(a) Uncertainties of Statistical Nature: The maximum magnitude of the load that acts on the structure may be a random quantity, for which a probability density function may be thought of. The probabilistic approach to design presupposes the knowledge of such a function. If the density functions of loads are known, it amounts to knowing the probability of the state of nature as discussed in Chapter Two, and the problem of uncertainty does not exist. Instead the decision reduces to a decision under risk. However, very often, one may not be able to get the true density function that represents the randomness of the variable. In such cases, the states of loading are of unknown probabilities. One may be able to reduce the uncertainty by minimizing the measurement errors in the experiment and also by increasing the number of observations. Economic considerations may however rule out the efforts to reduce uncertainty by resorting to larger number of more refined experiments. Also, a control on the randomness of loads by controlling the magnitude is difficult especially for wind loads, and other types of abnormal loads. By conducting studies on loads acting on structures, it may be possible to specify two bounds to the

actual density function. The exact distribution may be unknown within these bounds. This unknown state of probabilities of loads is one kind of uncertainty and called uncertainty of the statistical type.

(b) Uncertainties of Strategic or Nonrandom type: The statistical description of a load helps to assess the probability of any magnitude of the load. But it does not say in what manner the load is applied on the structure. The strategic concept concerns this problem.

In many problems of structural design, the designer, even if he knows the probability of extreme values of the individual load components, may not have a priori knowledge of the manner in which these loads combine or follow each other. For a bridge, for instance, the individual dynamic loads, wind loads, snow loads etc. may have well-specified extreme values or well-specified distribution functions, while the order of the application and their precise values at a given instant are not known. Prager and Symonds(111) discuss such uncertain states, for elasto-plastic structural analysis.

If the structure is linearly elastic, the uncertainty regarding the timing and sequence of the loads does not create difficulty. The effects of the various types of loads may be studied separately and the most unfavourable combination of loads may then be obtained for each element

of the structure by superposition. In such cases the path of loading has no influence. But, for inelastic materials and for materials in which cracks develop as a function of force level, the sequence of loading affects the behaviour especially the deformations. Horne (112) points out that 'when stress-strain or load-displacement relations are non-conservative, the final state after deformation may only be derived from initial state if account is taken of the sequence of loads, displacements, stresses and strains throughout the structure'. The inelastic and time-dependent deformations are infact non-conservative.

These variations are called <u>strategic</u> in this investigation as they involve the tactical skill rather than a chance mechanism. The statistical nature of any path of loading is not considered. In other words, all paths of loading are assumed to be equally likely. The strategic uncertainties can be listed as follows.

1. <u>Position</u>: The point of the application of the load may be uncertain. This sort of uncertainty is common in bridges and other structures on which moving loads are acting.

2. <u>Direction</u>: The direction in which the force is acting may be unknown. Ex. Wind load.

3. <u>Timing</u>: The time at which a particular load is applied is considered uncertain.

4. Sequence: The sequence in which the loads are acting may be unknown.

Many other uncertainties associated with the repeated application of loads, dynamic effect, impact etc. are not considered in this first attempt to formulate inelastic structural design problem under uncertainty.

The designer is engaged in the design activity with these uncertainties in the information of the loads and loading process. From the point of view of structure, it may be said that the structure resists a load, the moves of which load are thoroughly unknown to structure. This concept is made use of in formulating the structural action game.

4.2.2 Mathematical Representation of Uncertainties in Loading

Consider that m loads $\underline{w} = \{w_1, w_2, \ldots\ldots w_m\}$ are acting on the structure.

(a) Statistical Uncertainties: The probability density functions of loads are assumed to be unknown. Let us assume as a first approximation that the distribution belongs to a class of given functions say normal distribution. Also let the mean of the distributions of the m loads are known, and the standard deviations are unknown. Thus

$\underline{\mu} = \{\mu_1, \mu_2, \ldots\ldots \mu_n\}$ are the means which are known,

and $\underline{\theta} = \{\theta_1, \theta_2, \ldots\ldots \theta_m\}$ are the deviations which are

unknown. However, it is assumed that the range within which they lie are known. Thus

$$\theta_i^L \leq \theta_i \leq \theta_i^U \quad , \quad i = 1, 2, \ldots, m \qquad (4.1)$$

The bounds are to be obtained through experiments, and codified. As more and more information is obtained through experiments, the bounds can be brought closer and closer.

(b) <u>Strategic Uncertainties</u>:

Case 1. <u>Only Sequence of Loading is Important</u>: This case arises in inelastic materials. Let us consider that all the loads vary with respect to a common arbitrary variable '$\bar{s}$'. Then the loading process can be considered as an <u>unknown function of the variable</u> '$\bar{s}$', called the stage variable.

$$w_i = w_i(s) \quad , \quad i = 1, \ldots\ldots, m \qquad (4.2)$$

Any sequence of loading can be obtained by suitable combination of such functions. Also an instantaneous application as well as a sustained load can be expressed as the two special cases; a dirac delta function and a constant function as in Fig. 4.2.b.

Case 2: <u>Both Sequence and Timing are Important</u>: When the material of the structure has time-dependent behaviour like creep, timing of the load also becomes important. This can be easily taken by considering time t as the stage variable in Eqn. 4.2. Thus

$$w_i = w_i(t) , \quad i = 1, \ldots\ldots m \tag{4.3}$$

Case 3. Change in Direction is Important: This phenomenon occurs in the case of cyclic loads. Change in direction can also be accommodated in the Eqs. 4.2 and 4.3.

Case 4. Change in Position is Important: This is a very common phenomenon occurring when movable loads are present. In fact, the effect is equivalent to that of a multiple load condition. That is the same load acting at two positions can be considered as two separate load conditions which can not occur simultaneously. A moving load can also be simulated by this way considering the loading as a multiple load condition with finitely many load positions Fig. 4.2.c.

4.2.3 Uncertainty Associated With Structural and Material Behaviour:

The second type of uncertainty (Fig.4.1) arises from the structural and material behaviour. For purposes of design, the cross sections of the structure cooperate in resisting the external loads. The behaviour of the cross sections enters in the design computations through the force-deformation relations, while the 'cooperation' of the cross-sections in resisting the loads appears through equilibrium (which shows how loads can be shared by cross sections), and compatibility conditions. The uncertainties can also be classified into two categories on this basis.

(i) Uncertainty in the behaviour of cross sections. It consists of all uncertainties associated with the force-deformation relations of cross sections.

(ii) Uncertainty in the assessment of equilibrium and compati-bility conditions.

(i). Uncertainty in the Behaviour of Cross-Sections: A single cross section may have as many as six force-deformation relations, when it acts as a part of the structure in the most general case; three moment-curvature relations and three force-displacement relations. Let us refer all of them as force-deformation relations (F-x). Each force-deformation relation may be a function of one or more deformations. An uncertainty arises from the idealizations of these relations in eliminating some of the unimportant variables. The interaction of the constitutive equations affects the behaviour and failure modes of the cross section, which are generally expressed as failure theories. An uncertainty may also exist in the definition of the failure theories.

The most important and influencial uncertainty is that arising from the randomness of the force-deformation relations. The classical probabilistic design considers the probability distribution of strength of the material, stiffness, cross sectional dimensions etc. separately. This is convenient in an elastic design. If the material behaviour is non-linearly

inelastic or elastic, probability distributions of strengths and stiffnesses at various levels of stresses are needed. This aspect has not attracted much attention in the past. Ferry Borges (113) has described an approximate method for nonlinear materials, which will be discussed in Chapter 10. This problem is pursued further as the investigation in this thesis is related to nonlinear materials. The random relation may be considered as a stochastic phenomenon in which the section changes from one state of force to another under the influence of external loads. The transition from one state to another may be considered as a statistical phenomenon.

Let F^1, F^2, ... F^N be the N states of forces that may be occupied by the cross section. Let $p_{ij}(y)$ be the transitional probability at any deformation level y with which the force at the cross section changes from F^i to F^j. (i, j = 1, ... N). This process is said to constitute a <u>Markov chain</u> if the probability of transition to future states depends only on the state presently occupied by the system and not on the history of the system prior to entering that state (92).

$$\text{i.e.}\quad p(F^{N+1} \mid F^1 \ldots\ldots F^N) = p(F^{N+1} \mid F^N),\ N=1,2,.. \quad (4.4)$$

The N x N matrix p_{ij} is called transition probability matrix. It is such that

$$p_{ij} \geq 0, \quad i, j = 1, \ldots\ldots N \qquad (4.5)$$

$$\sum_j p_{ij} = 1, \quad i = 1, \ldots\ldots\ldots N \qquad (4.6)$$

The matrix, which has the properties given above, is called a stochastic matrix. The stochastic matrix is generally unknown in which case the problem is to be treated as a decision under uncertainty. By repeated experiments and statistical studies it may be possible to obtain the stochastic matrix. The randomness of structural behaviour and the associated uncertainties (unknown probabilities) are dealt with in Chapter 10.

(ii) Uncertainty in Equilibrium and Compatibility Equations:

The equilibrium equations and compatibility conditions are obtained by making idealizations and assumptions in the behaviour of members, connections etc. Uncertainty exists in these idealizations and assumptions. If the cross sections of the structure are taken as the basic elements for measurement of forces and deformations, the equilibrium equations or compatibility conditions represent the combined action or cooperation of two or more sections.

4.2.4 Uncertainty Arising from Imperfect Knowledge, Information, Inaccuracies in Computations etc.

The third type of uncertainty arises from the overall imperfect knowledge, information and inaccuracies in design, errors in construction and operation etc. These uncertainties are more of subjective nature and a scientific

schematically in Fig. 4.4. It is not an exhaustive list of failure modes. Only those modes that are relevant to the thesis are presented and discussed. Three major modes of structural failure occur in a practical design.

1. Failure associated with quasistatic loads of abnormal intensity.
2. Failure resulting from fatigue.
3. Failure due to dynamic effect.

In this work, only the first type of failure is considered.

Failure is classified into functional failure (unserviceability) and structural failure, the former is generally followed by the latter in the first type of failure given above. In a fatigue type of failure, structural failure due to fatigue can occur even before the functional failure (excessive deflections, cracking etc.) occurs (110).

4.3.1 Functional Failure:

From serviceability point of view, conservation of stable structural form is the primary requirement of design. The failure to fulfill this need arises from one or more of the following.

1. Unwanted deflections
2. Excessive oscillations under load
3. Excessive permanent deformations, cracking etc.

The magnitudes of the loads within which the structure must be serviceable can be called as limit value or in the

terminology of Comite Europeen du Beton (CEB) (26) as characteristic value of the load. When the loads are within the limits, the structure is said to be in a normal load condition.

Generally, a functional failure need not be considered as the loss of safety. But in certain cases the loss of serviceability is to be considered as the end of life of structure, in which case the two types of failure are assumed to coincide.

The loss occurring due to unserviceability depends upon the type of structure and the use to which it is put in. For example, when unserviceability governs the complete failure of the system

$$C_{sc} = C_F$$

where

C_{sc} = loss due to unserviceability

C_F = Total investment in the general system + consequences of unserviceability if any.

In buildings, an excessive deflection, minor cracks etc. may have some psychological influence on the individuals occupying the building, and the feel of security may be lost. This may bring down the rental value of the building, and the monetary return consequently reduces. If the building is used by the owner himself, the loss of happiness caused by the

unserviceability cannot be measured in terms of money. In certain cases the cost of repair by putting additional supports, or through additional constructions can be considered as the loss due to the unserviceability. The prescription of realistic limits on deflection etc. at the service load or working load is important especially in long span bridges. In U.S.A., total deflection is restricted to $\frac{1}{750}$ of span while the same in Europe is about $\frac{1}{400}$ span. This sort of specification also affects economy significantly.

4.3.2 Structural Failure:

The structural failure may be of different types leading to different extent of financial loss, a classification of which is shown in Fig. 4.3. They include

1. Repairable damages.
2. Partial collapse in which a part of the structure has failed with or without warning.
3. Total collapse with warning.
4. Total collapse without warning.

1. Repairable damages: This is a case in which one or more members of a structure fails structurally; not neccssarily leading to the immediate collapse of the structure. If the part of the structure affected can be unloaded and the failed member replaced, structure can be restored to its full use. The cost of damage C_d is the sum of the cost of repair incurred and the loss of income during

the repair time. The feasibility of repair depends on the importance of the member in the overall system, the statical redundancy and the repairability of the members. The failure of a member may be due to:

a. failure of its connection with other members,

b. brittle failure of its cross sections leading to fracture, or

c. ductile failure or yielding.

A member is said to have brittle failure, if one or more critical force-deformation relations of the section is not available to take up the additional forces caused by the increase of external load at any stage. A member is said to have ductile failure, if one or more cross sections of the member have force-deformation relations that ceases to bear additional force, yet continue to deform (i.e. a plastic hinge is formed).

2. Partial Collapse: Sometimes, a part of the structure may fail, which can be reconstructed in order to restore the full use of the system. Failure may occur instantaneously without enough warning or may be gradual. The loss incurred in such a failure C_p is the cost of reconstruction and the loss of life or property if any.

3. Total Collapse Without Warning: The term collapse represents the complete failure of the system or a part of the system. It differs from damage. However, damage and collapse

coincide in the following cases.

a. In statically determinate systems, failure of any member leads to complete collapse.

b. In statically indeterminate system, failure of certain members can lead to complete collapse.

c. In a statically indeterminate system of redundancy r, the simultaneous failure of r+1 force-deformation relations of the members causes complete collapse of the structure as no time is available for unloading and repairs.

d. If all members are brittle or unstable the failure of one member may sometimes accelerate the process of collapse. No prior warning is available in all the four types of failure. Such a design is termed as <u>weakest link</u> design. The loss of failure C_c is the total investment in the system at the time of collapse plus the cost C_L due to loss of life and properties if any minus the cost C_R salvaged. Thus

$$C_c\,(t) \;=\; C_{IN}\,(t) + C_L\,(t) - C_R\,(t) \qquad (4.7)$$

The cost of failure C_c is shown as a function of time. It is a function of time of failure. For example, the loss may be more if a structure fails immediately after construction than if it fails after fifty years, as some of the investment have already been recovered during this period. At any time, cost $C_{IN}(t)$ will be the cost invested minus the return so far obtained. If the system is insured, the money recoverable

through insurance in case of failure may be deducted and the premium upto the time t added.

4. <u>Total Collapse With Warning</u>: A collapse with warning can at least save the loss of life and property. This concept of design is termed as '<u>failsafe</u>' design. Warning before failure can be achieved by means of the following precautionary measures.

a. Adequate degree of redundancy

b. Avoiding too many critical sections

c. Providing enough ductility

The cost of failure C_W

$$C_W(t) = C_{IN}(t) - C_R(t) \tag{4.8}$$

4.4 COST-EFFECTIVENESS

In Chapter 2, it is suggested that the decision criterion for structural design must be the system-effectiveness or cost-effectiveness measured in terms of functional conformity, structural efficiency and long term economy. Under such a criterion, structural design becomes a utilitarian process and the arbitrariness of safety level is not a problem. In the conventional nonutilitarian design as well as in the 'reliability constrained' optimum design, safety level is arbitrarily specified by a preassigned safety factor or probability of failure. In Section 2.8.5, the inevitable draw back of this definition of safety, and the superiority of a

utility-based safety definition have been discussed in detail. Foresell (45), in 1921, postulated the need of a cost-based design. In 1940, Kjellman (114) suggested a generalized cost criterion. The generalized cost C_T is defined as

$$C_T = C_s + \sum_i p_f C_f \tag{4.9}$$

where C_s = is the initial cost of construction plus the capital required to cover the maintenance less the value of construction when no more in use.

p_f = probability of one type of failure,

C_f = cost due to this type of failure.

Sawyer (77) has proposed a similar criterion for nonlinear design of reinforced concrete structures. Haider (67) discusses in detail an expected worth function similar to that given in Eq. 4.9. This model has been reviewed in Section 1.4. Under certain restricted conditions, this model is related to the weight of the system. Blake (68) has suggested a cost-effectiveness model by which cost, reliability and weight may be traded-off optimally. The common practice of using the same factors of safety for many different structural parts of a system, regardless of weight, cost, reliability, and function is inconsistent and dispensed with. Instead, a consistency in the trade off between

effectiveness and reliability is proposed. The effectiveness of a system for a reliability $R = 1$ is taken as U_1. The effectivenss for $R = 0$ is taken as U_0. The effectivenss U for any R is given as a linear equation

$$U = R(U_1 - U_0)$$

If S is the size or strength of the system, the ideal S for maximum effectivenss is given by

$$\frac{dU}{dS} = 0 = (U_1 - U_0)\frac{dR}{dS} + \frac{d(U_1 - U_0)}{dS} R \quad (4.10)$$

or

$$-\frac{1}{R}\frac{dR}{dS} = \frac{1}{(U_1 - U_0)} \frac{d(U_1 - U_0)}{dS} \quad (4.11)$$

The left hand side of above equation gives a trade off between size and reliability and that on the right side between effectiveness of the whole system and size. The significance of a trade-off between cost-effectivenss and weight is also illustrated through an example of shield structure for protecting a spacecraft from mateoroids. A cost-effectiveness criterion that takes into account many possible failure modes and their probabilities of occurrence is presented in this section. This is derived from an analogy with a mechanical system.

4.4.1 <u>Utility:</u>

Utility is a concept derived from the traditional

value theory. Utility means usefulness, the satisfying of a need. A decision or an outcome has high utility when it satisfies the need as efficiently, as it can with the available resources. For a given problem all outcomes must be measured on one scale of utility so that they can be compared. The value of an outcome or the measure of its utility is the result of a subjective evaluation by the decision maker. In general, utility can be divided into two categories (115).

i. Utility Without Natural Measure: There are cases where utility cannot be measured quantitatively. Innovation, manager performance and development, worker performance and attitude, public responsibility are some of the examples in management science. State of health, pleasure, or pain etc. of individuals are also not measurable in terms of utility.

ii. Utility having a natural measure.

The conventional concept of utility was based on the assumption that individuals are rational. That is, all men being rational would behave in the same way under similar circumstances. The modern empirical concept assumes that utilities being subjective can differ despite an identical set of external circumstances. The modern view point is due to the axiomatic approach to utility suggested by Von Neumann and Morgenstern (85).

4.4.2 Cost-Effectiveness and Utility:

The cost-effectiveness analysis is essentially

an economic analysis in which the resources and benefits are traded off such that the human needs are satisfied with limited resources. When resources exceed the needs, an economic analysis is not needed. It is a principle of scarcity which is the central concept in economics. When resources are limited, they may be allocated in the best way to achieve maximum benefits. Utility is the basis of comparison of cost and effectiveness of the system, and therefore it plays a central role in any cost-effectivenss analysis.

Cost-effectiveness model is not without any draw backs. Kazanowski (116) has listed about two dozen fallacies and misconceptions regarding cost-effectiveness.

4.4.3 Costs and Benefits:

Costs that are predictions of the future, can only be estimates. Estimates of future values are done on the basis of past experience. However, a certain element of uncertainty always exists regarding the accuracy of cost estimation. This uncertainty is more and the estimation is less reliable, when the time that a cost is incurred is more remote. In economics, this is illustrated with the concept of a planning horizon (78), similar to the physical one. In planning horizon, it is the time in the future to which ones visibility is limited. Generally, 5 to 10 years is put as a limit to the future visibility.

Benefit is more of subjective nature and it can not be measured always in terms of currency. Hence in many natural systems, the benefit derived may have a utility without natural measure.

4.1.4 A Cost-Effectiveness Model for Structures:

A principle of a cost-effectiveness model for structural systems can be established by an analogy with a power generating system. It is shown in this section that the generalized cost model given by Eq. 4.9 is a good cost-effectiveness criterion for structural design. Consider a power generating plant, as in Fig. 4.5.a, in which work is the input and energy the output. A part of the work is dissipated in the system. The amount of dissipation of work measures the efficiency of the system. When efficiency is 100%, dissipation is zero and output is equal to input. To increase the efficiency, the cost of the plant is to be increased. Hence in an optimal system, cost of the plant and efficiency are to be traded off.

An analogous argument can be made on an economic system, shown in Fig. 4.5.b in which money is the input, and output may be money or commodities. However, there is a difference in this case that the economic system may dissipate or generate money.

Now, consider the structural system on which the safety of the overall system depends. Let the cost of

structure be $C_s(t)$. At any time t the cost $C_s(t)$ is the initial cost of the structure and the cost of maintenance (<u>of structural parts only</u>) upto the time t.

Consider the general system, of which the structure is a part, as an economic system, as shown in Fig. 4.5.c. The input cost $C_{IN}(t)$ on the overall system of which structure is a sub-system is the total investment in terms of money at the time t. It is different from the cost of structure and it includes the cost of both structural elements and nonstructural elements. The output may be measurable in terms of money or it may be a benefit in the form of comfort, safety etc.

The system is dealt only with respect to the structural behaviour. The structure as a component of the system can not create wealth as an economic system. But if it fails, it causes a loss both in terms of the cost and benefit (consequences of failure of the function for which the overall system is intended). Depending upon the type of failure the input cost may be fully or partially lost. There may be a loss in the benefit in terms of consequences of failure. The cost of a particular failure is therefore the total 'dissipation' of cost in terms of the loss in the input cost and loss in the output benefit. With regard to structure, when there is no structural failure it is considered that there is no dissipation of cost and the system is 100 percent efficient (effective). The cost of

failure is thus a measure of the effectiveness of structural system just as dissipation of energy is a measure of the efficiency of the power generating plant. Since different failure modes may occur with their own probability of occurrence the total expected cost of failure is taken as a measure of the effectiveness of the structural system.

Mathematically, if C_c is the cost of collapse,

$$C_c = \text{Loss in input} + \text{loss in benefit.}$$

$$= C_{IN}(t) - C_R(t) + C_L(t)$$ in which $C_{IN}(t)$ = system cost, $C_R(t)$ = salvage value and $C_L(t)$ is human and economic consequences of failure of a particular mode. If p is the probability of occurrence of this failure $p \times C_s$ is the expected loss. If k such failure modes can occur with probabilities p_i and cost of failure $C_i(t)$, expected cost of failure $C_F(t)$ is

$$C_F(t) = \sum_{i=1}^{k} p_i \; C_i(t) \qquad (4.12)$$

The significance of cost of failure can be visualized through an example. Consider the design of a water tank for industries which depend on water for its production processes. The cost of failure of the water tank not only involves the cost of replacement of the same, but also the economic losses which come out as a result of the failure. The present approach of specifying a safety level arbitrarily does not directly take this consequences of failure into account.

If the effectiveness is to be high, the expected cost of failure is to be minimum. Hence expected cost of failure is a negative of the effectiveness. To have a higher effectiveness or lower expected cost of failure, a higher safety level is required and the cost $C_s(t)$ of structural system also increases. Hence, cost of structure and the expected cost of failure are to be traded off in an optimal structural system. In other words, the total cost $C_T(t)$ given by

$$C_T(t) = C_s(t) + C_F(t) \tag{4.13}$$

should be minimum. Substituting Eq. 4.12 in Eq. 4.13, one gets

$$C_T(t) = C_s(t) + \sum_{i=1}^{k} p_i \; C_i(t) \tag{4.14}$$

For an optimum system $C_T(t)$ is a minimum. Since it represents the cost of the structure and the effectiveness of the system (both measured in terms of currency), it is called a cost-effectiveness criterion. The model developed here is the same as that proposed by Kjellmann (114) and Sawyer (77). Haider (67) has derived an expected worth model in which an expected worth function is to be maximized instead of the minimization of the expected cost of failure. For a single mode of failure it represents Forsell's Law. The derivation presented herein gives a physical explanation to the expected cost criterion and clarifies the various terms involved. Further, the criterion is derived as a function of time so that the effect of time of failure can

also be taken into account. Further, it agrees with general concept of cost-effectiveness analysis of systems (78). An analogous representation is that of an expected worth model which is maximized. The worth of the system is represented as benefit minus the costs as functions of time.

4.4.5 A Simplified Version of the Cost Effectiveness Model:

The cost-effectiveness model presented above is too complex to apply in a practical problem because of the following reasons. The cost of structure C_s, and the costs of failure C_i are to be estimated accurately to find an acceptable optimum. As discussed before, some uncertainty exists in the estimation of these quantities as they are the costs to incur in a future date. The uncertainty increases with the increase of the gap between the time of estimation and the time for which it is estimated. Apart from this, a true estimation of the cost of structure is possible only when the design is completed.

If the design is very sensitive to the slight variations in the estimated initial cost and cost of failure, obviously the solution will not be optimum. The cost of structure may be usually very small compared to the overall cost of failure since cost of failure includes cost of non-structural components also, which may be very high. Hence, Turkstra (33) assumes that the level of safety is not very sensitive to the variations in the initial cost

of structure. Considering this aspect the Eq. 4.14 can be rewritten as follows:

$$C_T(t) = C_s(t)\left[1 + \sum_i p_i\, K_i(t)\right] \qquad (4.15)$$

where

$$K_i = \frac{C_i(t)}{C_s(t)}$$

The factors K_i represent the cost of ith mode of failure as a fraction of the cost of structure. A reasonably accurate estimate of these factors may be made initially for purpose of design. This is justified due to the low sensitivity of safety level to the slight variation in the initial cost. Such estimation of K_i is more reasonable than arbitrarily specifying the level of safety. Further, it always gives a way of checking the estimation on the completion of design substituting back the actual costs involved.

Another advantage of Eq. 4.15 is that it gives a convenient way of splitting the optimization as follows into stages:

Step I Minimize: $1 + \sum_i p_i\, K_i(t)$ for assumed parameters $K_i(t)$.

Step II Minimize: $C_s(t)\left[1 + \sum_i p_i\, K_i(t)\right]$ keeping p_i constant.

In Chapter 10, it is shown, that for a suboptimal design, these two steps of design are quite enough. However, if a greater precision is needed an iteration procedure may

be adopted as follows. After one sequence of design, with assumed values of $K_i(t)$, accuracy of the data is checked. If it is not accurate, the new values of K_i are substituted and the design is repeated. Such a cycle would lead to an optimum design.

.5 SUMMARY

The uncertainties arising in the structural design decisions are classified and some of them are mathematically stated. The various failure modes are discussed. Unlike in the classical design, where only one or two modes of failure are considered, many possible modes of failure depending upon the true state of the random load are taken into consideration.

A cost effectiveness criterion of the form

$$C_T(t) = C_S(t) + \sum_i p_i \; C_i(t)$$

is developed from an analogy with a mechanical system. A simplified version of the equation is also proposed for suboptimal designs by stepwise minimization.

The uncertainties discussed in this chapter are considered in the design formulation proposed in Chapters 7-10. Failure modes and cost-effectiveness model are made use of in the methods proposed in Chapters 6 and 9.

CHAPTER FIVE

INELASTIC STRUCTURAL DESIGN UNDER CERTAINTY AS OPTIMAL CONTROL PROBLEM

5.1 INTRODUCTION

In this chapter, it is intended to develop a direct method of inelastic design of structures by which the non-linear force-deformation relations of the cross sections can be obtained as the output. This design is done for given paths of loading, the magnitudes of the loads not exceeding the serviceability limit state values. Such a design would not only satisfy the laws of motion, but also the design constraints like strength against failure, serviceability etc. for the given path of loading. Knowing the force-deformation relations that the structure needs, various cross-sections involving different materials can be chosen such that they possess the same set of relations. This process of design differs from the conventional approach in which force-deformation relations like moment-curvature relations are taken a priori as the link between forces and deformations to analyze the system. The design problem is formulated as an optimal control problem. Brief reviews of the literature on optimal control theory are given by Oldenburger (117) Paiewonsky (118) and Athans (119). This formulation is analogous to the classical variational mechanics problems (120).

The basic assumptions and concepts are presented in Section 5.2, while in Section 5.3 the problem is formulated. The subsequent three paragraphs are devoted for the solution aspects, the illustrative examples for time-independent and time-dependent behaviour respectively. A general case in which loads have instantaneous jumps in between smooth variations with time is presented in Section 5.7. The discussion in Section 5.8 contains certain mathematical derivations to show the validity of the formulation and a special discussion on the method and some merits of the method. No uncertainty, as discussed in Chapter 4, is considered in this formulation. Also random nature of loads and material behaviour are not considered. In fact, the methods presented in this and the following chapter are intended to be used when uncertainty is absent, and to pave way for the more detailed formulation in the subsequent chapters in which uncertainties are considered. Since the design is done for a given path of loading, it is called a design under certainty, or a deterministic condition. Further, the method is essentially for serviceability controls, and no economic considerations are made in this process. This aspect of the problem is taken up in Chapter 6.

Nonlinear analysis of structures is a field of study that has picked up momentum in recent period. Theory of plasticity and variational mechanics are also being studied. The present formulation derives concepts from all these branches of structural mechanics and attempts to have a

direct format of design. Optimal control theory offers the basic framework for this formulation. Optimal control formulation of structural design has been attempted earlier by Haug and Kirmser (69). The method is formulated to design a beam of homogeneous isotropic elastic material for minimum weight. The length along the beam is taken as the stage variable and deflection as state variables. The dimensions of the beam cross sections are the control variables. The weight of the beam is the performance index. The problem finally turns out to be an optimal control problem with inequality constraints on state variables. It is then solved by a calculus of variations approach. The present formulation is for inelastic materials and minimizes an energy index, as it is intended to cover only serviceability requirements by this method. This formulation can be used to select inelastic materials and their cross sections for a serviceability controlled design for a given loading process. The most serious loading process can be found by a game formulation as shown in Chapter 8.

5.2 BASIC ASSUMPTIONS AND CONCEPTS OF THE METHOD

5.2.1 Assumptions:

The following assumptions are made in the formulation:

1. The arrangement of members and types of connections between the members of the structure are assumed to be known and remain unaltered. By design, it is meant the selection

of the shape, dimensions and material of the members, i.e. the design of their cross sections.

2. The connections between members of the structure are so strong that no failure of them occurs before the member cross sections fail.

3. Loads are quasistatic. They may vary in magnitude in any manner, but the variation is so slow and non-repetitive that fatigue and dynamic effects are negligible, but at the same time not instantaneous causing any impact effects.

4. The upper and lower bounds within which the magnitudes of the loads may vary are deterministically defined a priori. The lower bound may even be negative causing a change in direction. But in no case, the bounds are violated for the formulation presented in this chapter.

5. Displacements are assumed to be small, so that the effect of any change in geometry can be ignored. This would allow the superposition of deformations.

6. The total deformation of any cross-section of the structure is considered as the sum of both the time-dependent and time-independent deformations.

7. Material is assumed to be homogeneous.

8. No temperature effects are included. Deformation is isothermal.

9. Poisson's effect is neglected.

5.2.2 Basic Concepts of the Formulation:

It is mentioned earlier that the purpose of the present design procedure is to select the member cross sections. The behaviour of the cross sections, when they participate in the total structural action may be represented by the force-deformation relations. Depending upon the type of structure, nature of loads etc., as many as six force-deformation relations may become necessary in 3-dimensional system to fully represent the response of each cross section. They include three moment-curvature, two shear-deflection and one axial force-axial strain relations. It is supposed that the knowledge of the relations that are needed for safety, is sufficient to design the cross sections, and at least one cross section of a suitable material possessing these characteristic relations can be chosen. Hence, the design of cross section is considered here as equivalent to the design of force-deformation relations required at all sections to resist the external load such that certain design requirements are fulfilled. The force-deformation relations obtained by the solution of the problem are called task curves, or structural action curves, as they represent the task to which the section is put in, when it acts as a part of the structure.

This method of design differs from the conventional approach in which such relations are utilized for analysis rather than obtained as outputs of the design. The method is primarily intended for inelastic design of structures,

though elastic case can also be obtained as a degenerate case. One of the differences between an elastic and an inelastic structural action is the non-uniqueness of deformation for loading and partial or complete unloading. In the case of elastic design, there is one-to-one correspondence between forces and deformations. Super position of forces is valid for linear elastic materials. Stresses and strains due to individual loads can be obtained separately, and the critical conditions obtained by adding them properly. In an inelastic design, this advantage is lost. The deformation at any state of the loading depends on the entire history of loading. An inelastic design therefore holds good only for a given load path. The load path is assumed to be known a priori. In practice, such an assumption is not valid. The more general case of uncertain loading process is considered in Chapter 8 where a method is developed extending the concepts involved in this chapter.

5.3 FORMULATION OF THE PROBLEM

5.3.1 A Brief Outline:

The design problem is considered as an optimal control problem in which the structure is assumed to be a controlled or programmed system. A controlled system has its transformation from one state to another governed by a set of controls through predefined equations of state (121). Under a given external loading, the deformed state of the structure

is controlled by the force-deformation relations of its cross sections. The controls namely force-deformation relations are to be obtained such that

(i) a set of constraints pertaining to equilibrium and compatibility conditions and serviceability requirements is satisfied.

(ii) a performance index which is a measure of the energy potential of the system is a minimum. (Physical significance of the performance index is given in Section 5.8.2.)

5.3.2 Stage Variables:

Optimal control problem is a multistage decision process (121, 122). A stage variable, usually time, is taken to denote the sequence of the process. In structural design problem, a stage variable 's' is chosen which may be time t itself when timing of the structural action is important on account of the time-dependent nature of material or may be arbitrary when time is unimportant. When the material of the structure is free from any time-based deformation the effect of applying the load in an interval of one year or one second is the same. As the principle of super-position is not valid, all the loads may be assumed to be acting together. A practical way of analyzing the problem is to increment the loads in small steps. This incremental process of loading can be expressed as the function of an arbitrary variable s , which starts from $s = 0$ and continues to $s = S$. The variable

s renders a basis by which the sequence of loading can be represented.

5.3.3 State Variables:

Consider that a structure is subjected to loads, fixed in position, but the magnitudes of which vary with the stage variable s. The variation of load with s is illustrated in Chapter 4. The structure, under the loads, deforms; the deformed state also changes with s. Fig. 5.1a illustrates the change in the deformed state of a two bar truss with $s = 0, 1, 2, 3, \ldots$ At any stage s, the deformed state of the system is fully known, if the deformations of all its cross sections are known. The deformations at any s may be measured with reference to the undeformed coordinates at $s = 0$ (also called the material coordinates or Lagrangian Coordinates). If the state of the system can be defined by finite number of variables, the system is called a discrete system. On the other hand, if the definition of the system state requires infinite number of variables, the system is called a distributed parameter system. For example the deformed state of trusses made of uniform bars and homogeneous material, can be defined in terms of the axial strains of the bars, and hence the axial strains constitute the state variables.

In the present formulation, it is assumed that the deformed state of both distributed and discrete systems can

be represented by finite number of state variables. In distributed systems, (for example a beam or plate) the deformations of cross sections chosen at regular intervals as in a finite difference grid will form the finite number of state variables. The deformations of other points may be obtained by interpolation. In the example of two bar truss shown in Fig. 5.1, the axial strain of the members AC and BC may be taken as the state variables.

Let $\underline{x} = \{x_1, \dots x_n\}$ denote the n <u>state variables</u> belonging to N cross sections which are chosen to define the deformed state at any s. Any change in the deformed state causes a change in the state vector $\underline{x}$. The change that occurs in $\underline{x}$ can be considered as the motion of a point in an n-dimensional Euclidean Space E^n. The path $\underline{x}(s) = \{x_1(s) \dots x_n(s)\}$ traced by this point in the space E^n with s is called the <u>path of state vector</u>. $\underline{x}$ is always a real-valued vector with possible negative components. The n-dimensional space is called deformation space $\underline{Y}$.

At s = 0,

$$\underline{x}(0) = (\underline{x})^0 \equiv \{x_1^0, x_2^0, \dots, x_n^0\} \tag{5.1}$$

The vector $(\underline{x})^0$ represents any residual deformations of manufacturing origin. When such deformations are not causing any internal stresses, they can be neglected.

Hence, $$(\underline{x})^0 = \{0, \dots 0\} \tag{5.1a}$$

At $\quad s = S,$

$$\underline{x}(S) = (\underline{x})^f = \left\{x_1^{\,f}, \ldots, x_n^{\,f}\right\} \tag{5.2}$$

Any any $s = s$, let

$$\underline{x} = \underline{y} + \underline{z} = \left\{y_1 + z_1, \ldots, y_n + z_n\right\}, \tag{5.3}$$

because of the assumption 6 in Para 5.2.1. An element of the above vector can be written as follows:

$$x_i = y_i + z_i, \quad i = 1, \ldots, n \tag{5.4}$$

where y_i = time-independent component of ith deformation generally caused by load alone.

z_i = time-dependent component of ith deformation (creep) when the material has no time-dependent behaviour $\underline{z}$ is zero, and therefore $\underline{x}$ can be replaced by $\underline{y}$.

In many problems, time may enter as an additional variable in order to define the time of failure etc. (cost-effectiveness criterion is a function of time). In such cases, the vector $\underline{x}$ may be augmented by taking t as an additional state variable, which may lie between zero and the proposed service life T of the structure. The value of T ranges from years in buildings and bridges to fractions of a minute in the case of rockets. Then, $\underline{X} = \left\{\underline{x}, t\right\} \in E^{n+1}$ represents the new state vector.

5.3.4 Loads and Loading Functions:

Let the structure be acted upon by m loads with magnitudes $\underline{w} = \left\{w_1, \ldots, w_m\right\}$. Let these loads be chosen at any s from

$$\underline{w}(s) = \left\{w_1(s), \ldots, w_m(s)\right\} \tag{5.5}$$

given a priori. Fig. 5.2.a and b illustrate the variation of load with s or t. The m dimensional space to which $\underline{w}$ belongs is called Load space $\underline{W}$. The Functions 5.5 controls the process of loading on the structure and may be called the state path $\underline{w}(s)$ of the controller. Let the load w_i vary, as follows:

$$W_i^L \leq w_i \leq W_i^u, \quad i = i, \ldots, m \tag{5.6}$$

The upper bound W_i^u of the ith load may be the limit value within which the structure is to be serviceable. This may be similar to the characteristic values of loads defined by CEB (26). The lower bound W_i^L may be even negative indicating a change in the direction, in which case it is the limit within which the structure is to be serviceable under the reversed loading. If no reversal of load occurs, W_i^L may be taken as zero, thus making

$$0 \leq w_i \leq W_i^u, \quad i = 1, \ldots, m \tag{5.6a}$$

In the present chapter, it is assumed that Eqs. 5.5 and 5.6

are given.

A problem arises when loads are expressed as functions of time t. Loads, in practice, may not always be continuous functions of time. They may also be applied intermittently on the structure. The duration of such loading may be so small that the loads can be considered as instantaneous (yet impact effects are negligible). This would give a loading function as in Fig. 5.2.c. The effect of such instantaneous loads is to cause a change in $\dot{\underline{y}}$ without causing a change in $\underline{z}$. This type of discontinuous change in state variables is referred to as jump motion of $\underline{x}$ in optimal control theory. In between such jumps, $\underline{x}$ will be a continuous function of t. In Para 5.3, loads which are continuous functions of t are considered. The complex case arising due to jumps in state variables is considered in Para 5.7.

5.3.5 Control Variables and Control Laws:

The variables that control the change of state variables are called control variables. The loads are one such set of controls. The variation of the control with s or t is called a control law. In other words, the control variables at any s, are chosen from the control laws. The controls are said to be admissible, if they are bounded.

In addition to the control by loads, the deformation of a structure is controlled by its force-deformation relations. Hence the variables that define these relations are taken as

$\underline{\alpha} = \left\{\alpha_1, \ldots, \alpha_n\right\}$ form another set of controls. They are chosen from the laws

$$\underline{\alpha}(s) = \left\{\alpha_1(s), \ldots, \alpha_n(s)\right\} \tag{5.9}$$

$\underline{\alpha}(s)$ are called the tangent modulus at any $\underline{F}(s)$, or the slope of $\underline{F}(y)$ with respect to $\underline{y}$.

Knowing the two sets of controls $\underline{v}(s)$ and $\underline{\alpha}(s)$, the force-deformation relations can be obtained as given in Section 3.11.

5.3.6 Constraints on Control Variables:

(a) Equilibrium Equations: The admissibility of controls $\underline{v}$ is governed by the equilibrium equations. At any state $\underline{x}$, the internal force $\underline{F}$ in the structure must be in equilibrium among themselves and with the externally applied loads $\underline{w}$, or $\underline{v}$ must be in equilibrium with $\frac{d\underline{w}}{ds}$. As finite number of cross sections have been chosen to define the structure, the necessary equilibrium equations can be obtained by considering the equilibrium of finite elements cut out of the body. Let, in all, p such equations of the form

$$R_i(\underline{x}, \underline{F}, \underline{w}) = 0, \quad i = 1, \ldots, p \tag{5.10a}$$

can be obtained. The equations, in large deformation theory when written with reference to the undeformed coordinates will be function of the deformations $\underline{x}$. However, under the assumption of small deformations,

$$R_i(\underline{F}, \underline{w}) = 0, \quad i = 1, \ldots, p \qquad (5.10b)$$

In matrix notation, the above equations can be written as

$$\left[a_{ij}\right] \left[F_j\right] + \left[b_{ik}\right] \left[w_k\right] = 0 \quad \begin{array}{l} j=1,\ldots,n \\ k=1,\ldots,m \\ i=1,\ldots,p \end{array} \qquad (5.10c)$$

when $\underline{v}(s)$ form the control variables F_j and w_k may be replaced by $v_j(s)$ and $\frac{dw_k}{ds}$ respectively. Writing in terms of $\underline{v}$ and $\frac{d\underline{w}}{ds}$, Eq. 5.10b becomes

$$R_i(\underline{v}, \frac{d\underline{w}}{ds}) = 0, \quad i = 1, \ldots, p \qquad (5.10)$$

in matrix form

$$\left[a_{ij}\right] \left[v_j\right] + \left[b_{ik}\right] \left[\frac{dw_k}{ds}\right] = 0 \qquad (5.10d)$$

In statically determinate systems, the number of linearly independent equations p will be equal to the number of unknowns $\underline{F}$, and hence $\underline{F}$ can be uniquely solved in terms of $\underline{w}$. Hence for a given loading function, $\underline{F}$ is also known and hence $\underline{F}$ need not be considered as unknown functions of s. In statically indeterminate systems, on the other hand, the number of equations may not be sufficient to solve all the n unknowns. The difference $n-p$ will be equal to the statical redundancy of the system.

According to structural mechanics, any set of forces, that satisfy equilibrium equations among themselves and with external loads, are said to be <u>statically complete</u> (123).

(b) Admissibility of $\underline{\alpha}(s)$: $\underline{\alpha}(s)$ may be a piecewise continuous variable with discontinuity occurring whenever a change in the sign of $\underline{v}$ occurs. As $\underline{\alpha}(s)$ represents the slope of force-deformation relations, it has great influence on the shape of the optimal force-deformation relations finally obtained. Therefore, constraints may be imposed on $\underline{\alpha}$ so that the final result agrees with the shape of practically available force-deformation relations. The desired behaviour idealization can be partly achieved by this way.

Fig. 5.3 shows the usual pattern of force-deformation relations of inelastic materials. To have such shape and to satisfy irreversibility conditions of plastic deformation, a constraint on $\underline{\alpha}(s)$ for the continuous portion is imposed as follows:-

$$\frac{d\alpha_i}{ds} \leq 0 \quad , \quad i = 1, \ldots, n \qquad (5.11)$$

It states that the curvature of the force-deformation relation is zero or negative. The equality sign refers to the linear elastic condition. Further, any required minimum nonlinearity in force-deformation relation may be introduced by restating the above condition as

$$\frac{d\alpha_i}{ds} \leq -|\nu(F)\ v(s)| \qquad (5.11a)$$

where $\nu(F)$ is to be specified depending upon the nonlinearity

required. In addition to the above condition, upper and lower bounds are also given to $\underline{\alpha}$ as follows

$$0 < \alpha_i \leq \alpha_i(0) \quad , \quad i = 1, \ldots, n \qquad (5.12)$$

The initial slope $\alpha_i(0)$ is also a variable to be governed by Eq. 5.25 given later. The lower limit is to eliminate inelastic instability conditions due to yielding. In the case of nonlinear elastic materials, the values of $\underline{\alpha}$ are retraced when any section is unloaded instead of taking a new set of values. Hence the constraint Eq. 5.11 may be modified as follows.

$$\frac{d\alpha_i}{ds} = -|v(F)w(s)|, \text{ for } F_i(s)\, v_i(s) \geq 0 \qquad (5.13)$$

If unloaded at $F_i = F_i^0$ and $s = s^0$, putting $s' = s$, $0 < s < s^0$ and $s'' = s$, $s^0 < s$. $\alpha(s') = \alpha(s'')$, when $F(s') = F(s'')$

The admissible region of control $\underline{\alpha}(s)$ is shown in Fig. 5.4. The bound stated above is too complex to be taken in practical problems. A constraint of the type given by Eq. 5.11 is difficult to incorporate in solution by usual methodss in control theory. Hence certain simpler approaches are suggested in Section 5.4 to overcome this difficulty.

5.3.7 <u>State Equations or Differential Constraints</u>:

The equations that govern the change or motion of state variables are called <u>state equations</u>. In structural

behaviour, they govern the change in force and deformation with s. They are of the form

$$\dot{x}_i = \frac{dx_i}{ds} = f_i\ (\underline{x}(s),\ \underline{F}(s),\ \underline{v}(s)\ \underline{\alpha}(s)),\ i = 1, \ldots, n \qquad (5.14)$$

The first order equations can be solved only when the form of the function f_i and the variables $\underline{F}(s)$, $\underline{v}(s)$, and $\underline{\alpha}(s)$ are known. The initial values $x_i(0)$ of x_i ($i = 1, \ldots, n$) are given by Eq. 5.1. The functions $\underline{f}$ depend upon the <u>type of behaviour idealization</u> chosen for the material of the structure. The various idealizations and the corresponding functions are given below.

(a) Time-independent behaviour.

In this case the component $\underline{z}$ vanishes.

Hence,

$$\frac{dx_i}{ds} = \frac{dy_i}{ds} = \frac{v_i}{\alpha_i}\ ,\quad i = 1, \ldots, n \qquad (5.15)$$

The equation given above assumes that lateral effect of deformation is negligible (Poisson ratio is zero).

When material is linearly elastic, α_i in Eq. 5.15 may be replaced by $\alpha_i(0)$. Thus

$$\frac{dx_i}{ds} = \frac{dy_i}{ds} = \frac{v_i}{\alpha_i(0)}\ ,\quad i = 1, \ldots, n \qquad (5.16)$$

(b) Inelastic behaviour, time-dependent.

In this case $s = t$. From Eq. 5.4,

$$\frac{dx_i}{dt} = \frac{dy_i}{dt} + \frac{dz_i}{dt}, \quad i = 1, \ldots, n \tag{5.17}$$

The component $\frac{dy_i}{dt}$ may be expressed by Eq. 5.15 or 5.16 depending upon whether the nonlinear or linear idealization is made.

$$\frac{dz_i}{dt} = g_i(\underline{x}, t, \underline{F}), \quad i = 1, \ldots, n \tag{5.18}$$

$$= a_i |F_i|^{\gamma} \operatorname{sgn} F_i \quad \text{for steady state creep} \tag{5.19}$$

where a_i, and γ are constants depending on the material to be used. Substitution of Eq. 5.14 and 5.18 in Eq. 5.17 results in the following:

$$\dot{x}_i = \frac{v_i(s)}{\alpha_i(s)} + g_i(\underline{x}, t, \underline{F}), \quad i = 1, \ldots, n \tag{5.20}$$

The phenomenon of creep recovery is not considered in this equation. It is assumed that the entire creep deformation is irrecoverable. The assumption agrees with the strain-hardening law of creep. The form of state equation is chosen through the idealization one makes. Even then, the Eqn. 5.14 cannot be solved without knowing the values of $\underline{v}(s)$ and $\underline{\alpha}(s)$.

Another set of state equations related to the force-state variables may be written as

$$\frac{dF_i}{ds} = v_i, \quad i = 1, \ldots, n \tag{5.21}$$

5.3.8 Constraints on State Variables:

(a) Serviceability Constraints:

Serviceability requirements offer some constraints on the state variables as follows:

(i) Deflection Control: The structure is to be serviceable at the normal load condition that is under consideration. One way of stating the serviceability requirement is that the deflections anywhere in the structure should not exceed pre-assigned allowable limits throughout the service life of the structure. In Fig. 5.1b, the condition, that the deflection of point C should not be more than 1 inch, horizontally and 1 inch vertically, is shown. The deflection at any point on the structure may be expressed in terms of the unknown state variables. If the deflection is equated to the limit value, the resulting equation would define a hypersurface in the space E^n. All such surfaces defining deflections of all critical sections define a manifold in the space E^n dividing it to an acceptable region $\underline{\bar{A}}$ and an unacceptable region. The boundary $\underline{\bar{B}}$ may be defined by

$$\underline{Q}(\underline{x}) = 0 \tag{5.22}$$

The surface so obtained for a two bar truss is shown in Fig. 5.1.c. If the state variable is within the boundary $\underline{\bar{\bar{B}}}$, the corresponding solution is generally acceptable from serviceability point of view. The statement in this form does not clearly represent the requirements. A structure with

highly rigid members obtained at a higher cost may not be as acceptable as those designs that have a lesser cost and at the same time deflections reach the boundary. Accepting this, the serviceability constraints may be stated as follows:

$$Q(x) \leq 0 \tag{5.23}$$

<u>The path of state variable $\underline{x}(s)$ touches the boundary $\bar{\underline{B}}$ at least once during the interval 0-S.</u> This requirement will be discussed further under transversality condition in this section. Thus if $\underline{x}'$, $\underline{x}''$ etc. are points in the path of deformation that belong to the boundary $\bar{\underline{B}}$, then

$$Q(\underline{x}') = Q(\overline{\underline{x}''}) = 0 \tag{5.23a}$$

(ii) <u>Control of Permanent Deformations and Crack Widths</u>: It is very often required to control the crack width in reinforced concrete members, and the permanent deformations in inelastic structural members. The permanent deformation undergone by any inelastic member is the difference of the total deformation and the elastic component. For member with only one active force-deformation relation, the permanent deformation is given by

$$x_i^P = x_i - \frac{F_i}{\alpha_i(0)} \quad , \qquad i = 1, \ldots, n \tag{5.24}$$

where $\alpha_i(0)$ = the initial slope or stiffness. The constraint on permanent deformation may be stated as follows:

Equations 5.26 define a manifold $\underline{\bar{C}}$ in the playing space E^n and restrict that the state variables must always move along this manifold. In statically determinate problems, such a constraint may not be necessary to solve the problem.

If such a manifold exist, $\underline{x}$ lies only on that part of $\underline{\bar{C}}$, passing through origin and intersected by $\underline{\bar{B}}$. The deformations that are compatible among themselves and with boundary are said to be kinematically complete (123).

(c) Returnability and Transversality Conditions: The condition of returnability states that the state variable, if it reaches the boundary $\underline{\bar{B}}$, can be brought back to the admissible state space.

In structural design problems, the state variable is returnable whenever the control $\underline{v}$ can take negative values. If $\underline{v}$ is positive, the state variable may or may not be returnable.

Transversality conditions are constraint equations to be satisfied by the costate variables in the process of solution. The conditions give the necessary initial values to the first order differential equations in costate variables. In general, structural design problems are of the free right end type which states that no constraints are imposed on the state variables $\underline{x}^f$. Under this condition, the costate variables will have zero final values at s=S. If $\underline{\bar{B}}$ is a surface to which the state variable $\underline{x}^f$ belongs, the transversality

condition states that the costate vector must be orthogonal to the surface at $\underline{x}^f$.

If the returnability conditions are not satisfied, the boundary surface $\overline{\underline{B}}$ may be considered as the surface offering transversality conditions, assuming that the state variable terminates at the boundary $\overline{\underline{B}}$ at $s = S$.

According to the serviceability requirements, the state variable must touch the boundary $\overline{\underline{B}}$ at least once. If the state variable is returnable after the contact with the boundary, the end point $\underline{x}^f$ of state variable may be in the interior. Under such conditions, the problem may be treated as a right end free problem, provided the equality constraints given by Eq. 5.26 are absent.

5.3.9 Performance Index:

So far the variables have been defined and the constraints have been specified. A performance index is to be assigned based on which various controls can be rated, and the optimal control can be chosen. The quantity may be either minimized or maximized depending upon whether it is a measure of loss or gain. In this formulation, performance index is to be minimized. The performance index is of the integral form

$$P(\underline{x}, s, \underline{F}, \underline{v}, \underline{\alpha}) = \int_0^S G_0(\underline{x}(s), s, \underline{F}(s), \underline{v}(s), \underline{\alpha}(s), \underline{w}(s))d \tag{5.27}$$

where P = performance index for the period 0–S of loading, and

G_0 = The change in index in an interval ds, due to the change in $\underline{F}(s)$, $\underline{w}(s)$ and $\underline{x}(s)$. In the present formulation, P is considered to be the energy potential of the system, when it changes from the state at $s = 0$ to state at $s = S$. At any s, it consists of the energy potential for the load on that instant and the energy lost in the interval 0 to s. This is further explained in Section 5.8. Hence, G_0 represents the change in energy potential in an interval ds for the entire volume of the structure.

$$\text{Let } G_0 = G_{01} - G_{02} \tag{5.28}$$

where G_{01} = the change in the internal energy for the entire volume of structure in ds.

G_{02} = the sum of the change in external work and complementary work done by all the loads $\underline{w}$ in an interval ds. The significance and measurement of G_{01} and G_{02} are illustrated in Fig. 5.6 for a two bar truss.

(i) Calculation of G_{01}: Consider a structure with linear elements only (trusses, beams, frames etc.). Consider an interval s to $s + ds$ in which all the forces, loads, stiffness and deformations undergo changes. Let u_{ij} be the change in energy of the ith force-deformation relation (belonging to jth section)

$$u_{ij} = F_i(s) \cdot \frac{v_i(s)}{\alpha_i(s)}, \quad \text{for time independent behaviour} \qquad (5.29)$$

$$= F_i(t) \cdot \frac{v_i(t)}{\alpha_i(t)} + F_i(t) \cdot \dot{z}_i \, dt,$$

$$i = 1, \ldots, K, \; j = 1, \ldots, N$$

for time-dependent behaviour where K = the total number of force-deformation relations at any section j. For the jth cross section, the change in energy is given by

$$e_j = \sum_{k=1}^{K} u_{kj}, \quad j = 1, \ldots, N. \qquad (5.30)$$

The summation is done over K, the total number of force-deformation relations at jth section. Then for truss

$$G_{01} = \sum_{j=1}^{N} e_j L_j \qquad (5.31)$$

and for other linear systems like beams,

$$G_{01} = \sum_{j=1}^{N} \int_0^{\Delta L_j} e_{j,j-1}(\xi) \, d\xi$$

where ΔL_j is the length between two sections j and $j-1$.

(ii) <u>Calculation of G_{02}</u>: The change in external work + complementary work done by the ith load is

$$e_i = w_i(s) \frac{d\Delta_i(s)}{ds} + \Delta_i(s) \cdot \frac{dw_i(s)}{ds}, \quad i = 1, \ldots, m. \qquad (5.32)$$

where $\Delta_i(s)$ is the deflection or rotation in the direction of $w_i(s)$. $\Delta_i(s)$ is a function of $\underline{x}(s)$. Now,

$$G_{02} = \sum_{i=1}^{m} e_i \qquad (5.33)$$

In the case of distributed loads, the energy for an infinitesimal length of loading can be calculated, and it is then integrated over the length of loading to get the total value of G_{02}.

5.3.10 Optimal Path and Optimal Controls:

The paths $\underline{x}^*(s)$ of the state variables from $(\underline{x})^0$ at $s = 0$ to some final value $(\underline{x})^f$ at $s = S$ that, when adhered to, minimizes the performance index P subject to the constraints is called the optimal path. Similarly $\underline{F}^*(s)$ is the optimal path of $\underline{F}$. The control laws $\underline{v}^*(s)$ and $\underline{\alpha}^*(s)$ associated with the minimum value of the performance index P^* and the optimal path $\underline{x}^*(s)$ are called the optimal control laws. P^* is single valued. However, it does not mean that $\underline{x}^*(s)$ is single-valued for a given path of loading. There may be many paths $\underline{x}^*(s)$ that have the same P^* or there may not be any. Corresponding to each $\underline{x}^*(s)$ there may be one set of $\underline{v}^*(s)$ and $\underline{\alpha}^*(s)$.

5.3.11 Force-deformation Relations from Optimal Control Laws:

The optimal path $y_i^*(s)$ can be obtained as follows:

$$y_i^*(s) = (y_i)^0 + \int_0^s \frac{v_i(s)}{\alpha_i(s)}\, ds., \quad i = 1, \ldots, n \quad (5.34)$$

$y_i^*(s)$ can also be obtained by eliminating $z_i^*(s)$ from $x_i^*(s)$. Eliminating s from $F_i^*(s)$ and $y_i^*(s)$, F_i^* is obtained as a function of $\underline{y}^*$

$$F_i^* = F_i^*(y_i^*) , \quad i = 1, \ldots, n \tag{5.35}$$

These are the task curves defined earlier. That is, these curves are the force-deformation relations needed to satisfy the serviceability requirements and laws of motion. If the structure is designed such that its cross sections have force-deformation relations identical to these task curves, the structure is safe and serviceable for the given load history or path. Cross sections may be chosen such that they possess the same relations under normal conditions. The method of selection of cross sections is described in Chapter 10.

5.3.12 Statement of the Problem:

Now, the optimal control formulation of the structural design problem can be formally stated as follows:

Given

s = stage variable,

$\underline{F} = \{F_1, \ldots, F_n\}$, (Forces), and

$\underline{x} = \{x_1, x_2, \ldots, x_n\}$, state variables (deformations),

$\underline{w}(s) = \{w_1(s), \ldots, w_m(s)\}$, given functions of s(load),

$\underline{v} = \{v_1, \ldots, v_n\}$ (rate of change of force), and

$\underline{\alpha} = \{\alpha_1, \ldots, \alpha_n\}$ as control variables (stiffnesses),

determine

$$v_i^* = v_i^*(s) \quad , \qquad i = 1, \ldots, n,$$

$$\alpha_i^* = \alpha_i^*(s) \quad , \qquad i = 1, \ldots, n,$$

$$F_i^* = F_i^*(s) \quad , \qquad i = 1, \ldots, n, \quad \text{and}$$

$$x_i^* = x_i^*(s) \quad , \qquad i = 1, \ldots, n,$$

so that

$$P^*(\underline{v}^*, \underline{\alpha}^*) = \min_{\underline{v},\ \underline{\alpha}} \int_0^S G(\underline{x}, \underline{F}, \underline{v}, \underline{\alpha}, \underline{w})\,ds \tag{5.27}$$

subject to the constraining conditions,

$$\text{(i)} \quad \frac{dF_i}{ds} = v_i \quad , \qquad i = 1, \ldots, n \tag{5.21}$$

$$\text{(ii)} \quad \frac{dx_i}{ds} = f_i(\underline{x}, \underline{F}, \underline{v}, \underline{\alpha}), \quad i = 1, \ldots, n \tag{5.13}$$

$$\text{(iii)} \quad R_i(\underline{v}, \frac{dw}{ds}) = 0, \qquad i = 1, \ldots, p \tag{5.10}$$

$$\text{(iv)} \quad K_i(\underline{x}) = 0, \qquad i = 1, \ldots, q \tag{5.26}$$

$$\text{(v)} \quad Q_i(\underline{x}) \leq 0 \tag{5.23}$$

$$\text{(vi)} \quad Q_i(\underline{x}') = Q_i(\overline{\underline{x}''}) = 0 \tag{5.23a}$$

$$\text{(vii)} \quad |x_i^p| \geq d_i, \quad \text{for } F_i(s)\ v_i(s) \geq 0 \tag{5.25}$$

$$\leq \bar{d}_i', \quad \text{for } F_i(s)\ v_i(s) \leq 0$$

$$i = 1, \ldots, n, \quad \text{and}$$

$$\text{(viii)} \quad \frac{d\alpha_i}{ds} \leq 0 \text{ or } -|\beta(F)v(s)|, \quad i = 1, \ldots, n \tag{5.13}$$

and the initial conditions

$$F_i(0) = (F_i)^0, \text{ and} \tag{5.7}$$

$$x_i(0) = (x_i)^0, \quad i = 1, \ldots, n \tag{5.1}$$

Some of the constraints may be satisfied automatically while some others may be absent depending upon the nature of the problem.

5.4 SOLUTION OF THE PROBLEM

5.4.1 Methods of Solution:

The structural design problem as formulated above is analogous to an optimal control problem of the fixed time, right end free type with state variable constraints. The interval 0-S is fixed, and the final value $(\underline{x})^f$ of $\underline{x}$ at $s = S$ are neither fixed, nor constrained by equations except in certain specific cases discussed earlier. The problem may be solved with the help of well-developed analytical or computational techniques in optimal control theory. The analytical method essentially makes use of the necessary conditions for the existence of solution. The most popular analytical techniques are

(i) Pontryagin's Maximum Principle (121)

(ii) Bellman's Dynamic Programming (124)

(iii) Variational Approach (122, 125-127, 128)

(iv) Functional Analysis (129).

Pontryagin's principle is well-suited for problems

with bounded control, especially when the admissibility condition of controls is free of state or stage variables. Dynamic programming method is analogous to the Hamilton-Jacobi formulation in calculus of variation and makes use of a partial differential equation. Rozonoer (130) has established the correspondence of the maximum principle and dynamic programming with classical variational mechanics. Variational approach gives necessary conditions for bounded control problems under certain assumptions of continuity and differentiability in which the control variables are constrained by algebraic equation containing state and stage variables.

The computational techniques rely on numerical methods with or without the use of necessary conditions. They include

(i) gradient techniques (due to Bryson and Denham and also due to Kelly),

(ii) conjugate gradient method (131),

(iii) generalized Newton-Raphson method (132),

(iv) direct sensitivity method (133), and

(v) second variation method (134).

In addition, many other methods for special types of problems are also available for computation. The computational methods are divided into direct methods in which a search for solution is directly made and indirect methods in which a two point boundary value problem resulting from the

application of necessary conditions is solved.

5.4.2 Necessary Conditions:

The necessary conditions for the existence of a solution due to Berkovitz (122, 128) for various conditions are briefly presented here. The conditions are later applied to solve some problems. In this section time t is used as the stage variable in place of s to keep the generality of the problem.

(a) Assumptions:

(i) $\underline{\alpha} = \{\alpha_1, \ldots, \alpha_n\}$ are piecewise continuous and has piecewise continuous first and second derivatives.

(ii) The path of $\underline{x}^*$ is in the interior of state space, it intersects with $\underline{B}$ only once.

(iii) $\underline{f} = \{f_1, \ldots, f_n\}$ are of class C'' with respect to $\underline{x}$.

(iv) $G_0 =$ a function of class C''.

(v) $\theta^j(t, \underline{x}, \underline{\alpha}) \geq 0$, $j = 1, \ldots, r$ is a constraint on control variables. $\theta^j(t, \underline{x}, \underline{\alpha})$ is of class C''

(b) Necessary Conditions of Problems Without State Variable Constraints:

Consider Hamiltonian H in the form

$$H(t, \underline{\alpha}, \underline{x}, \lambda_0, \underline{\lambda}) = \lambda_0 G_0(t, \underline{x}, \underline{\alpha}) + \underline{\lambda}\, \underline{G}(t, \underline{x}, \underline{\alpha}) \tag{5.36}$$

where $\lambda_o \geq 0$ and $\underline{\lambda}(t)$ are costate variables (also known as adjoint state variables). $\underline{\lambda}(t)$ is an n-dimensional vector. Let $\underline{\mu}(t) \leq 0$ be an r-dimensional vector continuous on the interval 0-T. If $\underline{x}^*(t)$ is an optimal path, $\underline{\alpha}^*(t)$ is optimal control, then the necessary condition states that along $\underline{x}^*$ $\lambda_o, \underline{\lambda}(t)$ never vanishes, and the following conditions are satisfied.

(i) $\frac{d\underline{x}}{dt} = \frac{\partial H}{\partial \underline{\lambda}}$, with $\underline{x}(0) = (0, 0 \ldots, 0)$ (5.37)

(ii) $\frac{d\underline{\lambda}}{dt} = -\left\{\frac{\partial H}{\partial \underline{x}} + \underline{\mu}\, \frac{\partial \theta}{\partial \underline{x}}\right\}$, with $\underline{\lambda}(T)$ is to be defined by transversality condition at $s = S$. (5.38)

(iii) $\frac{\partial H}{\partial \underline{\alpha}} + \underline{\mu}\, \frac{\partial \theta}{\partial \underline{\alpha}} = 0$ (5.39)

(iv) $\mu^i\, \theta^i = 0, \quad i = 1, \ldots, r.$ (5.40)

(v) For every $(t, \underline{x}^*, \underline{\alpha}^*, \lambda_o, \underline{\lambda})$ of $\underline{x}^*$ and every $\underline{\alpha}$ such that $\underline{\alpha} = \underline{\alpha}(t)$ for some $\underline{\alpha}$ that is admissible,

$H(t, x^*, \underline{\alpha}, \lambda_o, \underline{\lambda}) \geq H(t, x^*, \alpha^*, \lambda_o, \underline{\lambda}).$ (5.41)

(vi) at the end point transversality condition holds

$\underline{H}\, T - \underline{\lambda}\, \underline{x}^f = 0$ (5.42)

(vii) At each point of $\underline{x}^*(t)$ let $\bar{\theta}$ denote the vector formed from $\underline{\theta}$ by taking those components of $\underline{\theta}$ that vanish at that point. Let

$\underline{e} = (e^1, \ldots, e^m)$ be a non-zero solution of the linear system

$$\frac{\partial \bar{\theta}}{\partial \underline{\alpha}}\, \underline{e} = 0$$

at a point of $\underline{x}^*(s)$. Then

$$\underline{e}^T \frac{\partial^2 [H + \underline{\mu}\,\underline{\theta}]}{\partial \alpha^2} \cdot \underline{e} \geq 0 \quad \text{at this point} \tag{5.43}$$

Finally, H is a continuous function along $\underline{x}^*$.

<u>Special Cases</u>:

(i) <u>$B^i(t, \underline{x}) \leq \alpha_i \leq A^i(t, \underline{x})$, $i = 1, \ldots, m$</u>: A^i and B^i are of class C'' in state space.

When $A^i > B^i$, at each point of $\underline{x}^*(t)$

$$\frac{\partial H}{\partial \alpha^i} \begin{cases} \geq 0 & , \text{ if } \alpha_i^* = B^i \\ = 0 & , \text{ if } B^i < \alpha_i^* < A^i \\ \leq 0 & , \text{ if } \alpha_i^* = A^i , \end{cases} \tag{5.44}$$

The constraints can be written as

$$\begin{aligned} A^i(t, \underline{x}) - \alpha_i &\geq 0 \\ \alpha_i - B^i &\geq 0 \end{aligned} \tag{5.45}$$

(ii) <u>$\alpha_i \leq A^i(t, \underline{x})$</u>: In this case

$$\begin{aligned} \frac{\partial H}{\partial \alpha_i} &= 0 \quad , \text{ if } \alpha_i < A^i \\ \frac{\partial H}{\partial \alpha_i} &\leq 0 \quad , \text{ if } \alpha_i = A^i \end{aligned} \tag{5.46}$$

Similar statement holds good for $\alpha_i \geq B^i$.

(iii) <u>A^i and B^i are constants</u>: In this particular case, the

results are equivalent to that of maximum principle.

Choice of λ_o: An optimal curve $\underline{x}^*(t)$ is said to be normal if no sets of multipliers with $\lambda_o = 0$ occurs.

If the minimizing curve is normal, then the multipliers can be chosen so that $\lambda_o = 1$, and with this choice of λ_o, the multipliers are unique.

If $\lambda_o = 0$, no optimal path exists in the vicinity of that curve.

(c) Necessary Conditions for Problems With Equality State Variable Constraints:

Let $K(\underline{x}, t) = 0$ be the equality constraint.

Let

$$\phi(t, \underline{x}, \underline{\alpha}) = \frac{\partial K(\underline{x}, t)}{\partial t} + \frac{\partial K(\underline{x}, t)}{\partial \underline{x}} \underline{f}. \qquad (5.47)$$

$K(\underline{x}, t)$ is a piecewise smooth boundary each piece of which is defined by a relation $K_i(t, \underline{x}) \geq 0$. Let $\underline{x}^*(t)$ be lying on the surface $\underline{\bar{C}}$. Along $x^*(t)$,

$$\phi(t, \underline{x}^*, \underline{\alpha}) = 0 \qquad (5.48)$$

Let

$$H(t, \underline{x}, \underline{\alpha}, \lambda_o, \underline{\lambda}) = \lambda_o G_0 + \underline{\lambda} \cdot \underline{f} \qquad (5.49)$$

The necessary condition states that along x^* there exists

(i) a constant $\lambda_o \geq 0$

(ii) a continuous n-dimensional vector $\underline{\lambda}(t)$ such that $(\lambda_o, \underline{\lambda}) \neq \left\{0, \frac{\partial K}{\partial \underline{x}} \rho\right\}$ where ρ is arbitrary.

(iii) r-dimensional vector $\underline{\mu}(t) \leq 0$ continuous except perhaps at values of t corresponding to corners of $\underline{x}^*$.

(iv) a function ν with same continuity properties as $\underline{\mu}$.

Along $\underline{x}^*$ the following holds good.

1. $$\frac{\partial \underline{x}}{\partial t} = \frac{\partial H}{\partial \underline{\lambda}} \tag{5.50}$$

2. $$\frac{\partial \underline{\lambda}}{\partial t} = -\left\{\frac{\partial H}{\partial \underline{x}} + \underline{\mu}\frac{\partial \underline{\theta}}{\partial \underline{x}} + \nu \frac{\partial \phi}{\partial \underline{x}}\right\} \tag{5.51}$$

3. $$H(t, \underline{x}, \underline{\alpha}) \geq H(t, \underline{x}, \underline{\alpha}^*) \tag{5.52}$$

4. $$\frac{\partial H}{\partial \underline{\alpha}} + \underline{\mu}\frac{\partial \underline{\theta}}{\partial \underline{\alpha}} + \nu \frac{\partial K}{\partial \underline{x}} \; \frac{\partial \underline{f}}{\partial \underline{\alpha}} = 0 \tag{5.53}$$

5. $$\mu^i \, \underline{\theta}^i = 0 \quad , \quad i = 1, \ldots, r. \tag{5.54}$$

6. $$\underline{e}^T \; \frac{\partial^2 (H + \underline{\mu}\, \underline{\theta} + \nu \frac{\partial K}{\partial \underline{x}} \cdot G)}{\partial \underline{\alpha}^2} \; \underline{e} \geq 0 \tag{5.55}$$

(d) Problem With Inequality Constraints: In addition to equality constraints, inequality constraints may be present in the problem. In structural design problem inequality constraints are introduced by the serviceability requirements. In such cases, part of the trajectory may lie on the boundary and part in the open region.

The trajectory that is in the open region obeys the necessary conditions given in part (b) of this section. The

trajectory that lies on the boundary obeys the conditions stated in part (c) of this section. Thus surface $\underline{B}$ given by

$$Q(\underline{x}) \leq 0$$

represents the constraint. When inequality sign is applicable the path is in the open region and when equality sign is applicable, the path is on the surface $\underline{B}$.

(e) <u>Corner Conditions</u>: Every pair of adjoining sections of an optimal trajectory may intersect at a point where it is not smooth. Such a point is called a corner or junction. The conditions to be satisfied at this point are called junction conditions, corner conditions or jump conditions. At such points, co-state variables may take a jump. Four types of junctions may be identified. They are:

(i) A jump from open region on to the surface and return to the region again by costate variables.

(ii) A jump from open region on to the surface and thereafter the point moves along the boundary.

(iii) A point that moves on the boundary surface jumps in to the region.

(iv) A costate point moves from one portion of smooth surface to another.

The four types are schematically shown in Fig.5.7. The conditions to be satisfied at such a corner are the

following. Let t be the time of jump. The trajectory before t is denoted by B and that after t is denoted by A. The values just before t are denoted by a minus sign and that after t by a plus sign. Two mutually exclusive conditions are stated for $\underline{\lambda}$. Thus

$$\text{(i)} \quad \underline{\lambda}_B^- = \underline{\lambda}_A^+ - \nu_A^+ \cdot \frac{\partial Q}{\partial \underline{x}} \tag{5.56}$$

$$\text{(ii)} \quad \underline{\lambda}_B^- + \nu_A \cdot \frac{\partial Q}{\partial \underline{x}} = 0$$

The condition for λ_o is

$$\lambda_{OB} = \lambda_{OA} \tag{5.57}$$

Hamiltonian H takes a jump

$$H^- = H^+ + \nu_A \cdot \frac{\partial Q}{\partial t} \tag{5.58}$$

when $Q(\underline{x})$ is dependent on t last term of right hand side changes. Hence

$$H^- = H^+ + \left(\frac{\partial Q}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial Q}{\partial t}\right) \tag{5.59}$$

Eq. 5.58 can be expanded using Eq. 5.56 and Eq. 5.57 as follows:

$$\lambda_0 (G_0^- - G_0^+) + \underline{\lambda}^+ (\underline{f}^- - \underline{f}^+) = 0$$

$$\lambda_0 (G_0^- - G_0^+) + \underline{\lambda}^- (\underline{f}^- - \underline{f}^+) = \nu_A^+ \left(\frac{\partial Q}{\partial \underline{x}} \underline{f}^+ + \frac{\partial Q}{\partial t}\right) \tag{5.60}$$

(f) <u>Discontinuities in G_0, $\underline{f}$ and $\underline{\theta}$</u>: Discontinuities in

the functions G_0, $\underline{f}$ and $\underline{\theta}$ give certain jump properties. This case may arise in the structural design problem in the equations $\underline{f}$, G_0 and $\underline{\theta}$ due to a change in the sign of control variable $\underline{v}$. If the trajectory of $\underline{x}$ at the discontinuity occurring at t_2, $x_2(t)$ is not tangent to any dividing plane, then the condition can be stated as follows:

$$(H^+ - H^-)dt_2 - (\underline{\lambda}^+ - \underline{\lambda}^-)d\underline{x}_2 = 0 \qquad (5.61)$$

$\underline{\lambda}$ and $\underline{\mu}$ at such points need not be continuous.

(g) Pontryagin's Maximum Principle: Of the several analytical methods, maximum principle due to Pontryagin is one popular technique. The maximum principle finds application not only in optimal control theory, but also in inventory theory, production theory, capital theory and growth theory. One difference of Pontryagin's principle from that presented earlier is that it is applicable when the control $\underline{\alpha}$ belongs to a control region $\mathcal{A}$, which is constant and independent of $\underline{x}$, $\underline{F}$, or t.

For a fixed time free right end problem the maximum principle may be stated as follows:

Problem:

Find an optimal control $\underline{\alpha}(t) \in \mathcal{A}$ a set of admissible controls such that

$$P^*(\underline{\alpha}^*) = \underset{\underline{\alpha} \in \mathcal{A}}{\text{Min}} \int_0^T G_0(\underline{x}, \underline{\alpha}, t)dt \qquad (5.64)$$

subject to the constraint

$$\frac{dx_i}{dt} = f_i(\underline{x}, \underline{\alpha}, t) \quad , \quad i = 1, \ldots, n \tag{5.65}$$

with $x_i(0) = 0$

<u>Statement of the Principle</u>: Assume a Hamiltonian

$$H = \lambda_0 \cdot G_0 + \sum_{i=1}^{n} \lambda_i f_i \tag{5.66}$$

where $\lambda_0(t)$ and $\lambda_i(t)$ are called <u>costate variables</u> or <u>adjoint state variables</u>.

For a fixed time free right end problem

$$\lambda_0(T) = -1 \tag{5.67}$$

and

$$\lambda_i(T) = 0 \quad , \quad i = 1, \ldots, n \tag{5.68}$$

Therefore Eq. 5.66 can be written as

$$H = - G_0 + \sum_{i=1}^{n} \lambda_i f_i \tag{5.69}$$

The <u>maximum principle</u> states that a control $\underline{\alpha}^* \in \mathcal{A}$ is optimal if for every t, $0 \leq t \leq T$,

$$H^*(\underline{\alpha}^*) = \max_{\underline{\alpha} \in \mathcal{A}} \left\{ - G_0 + \sum_{i=1}^{n} \lambda_i f_i \right\} \tag{5.70}$$

The optimal control maximizes the Hamiltonian. This is a necessary condition for nonlinear system.

Optimal Paths . ... The optimal paths of state variable are given by

$$\frac{dx_i^*}{dt} = \frac{\partial H^*}{\partial \lambda_i}, \quad i = 1, \ldots, n \tag{5.71}$$

with $x_i(0) = 0$

The optimal paths of costate variable are given by

$$\frac{d\lambda_i^*}{dt} = -\frac{\partial H^*}{\partial x_i} \tag{5.72}$$

with $\lambda_i(T) = 0$

(h) Analogy of Pontryagin's Principle with Variational Mechanics: Rozonoer (130) has shown the analogy between Pontryagin's Maximum principle and the classical variational mechanics. The function H is analogous to the Hamiltonian. The costate variables $\lambda(t)$ are analogous to the impulses. Pontryagin's principle leads to two equations given by Eqs. 5.71 and 5.72. These two equations are similar in form with the canonical equations of Hamilton in analytical mechanics.

5.4.3. Application to Structural Design Problem:

Due to the complexity of the nature of the problem, arising from state variable constraints and discontinuities in state equations, the methods using necessary conditions, are not feasible for all types of problems. In

the most general form, the problem can be solved with the aid of computational techniques for which special algorithms may be developed. However, certain simple problems can be solved with the help of necessary conditions which may give an insight into the problem. The application of necessary conditions to solve such problems is considered in Sections 5.5 and 5.6. The following points may be noted.

(i) The equations, functions etc. in structural design problem are simple in form. Hence the application of necessary conditions is straightforward. However, discontinuities in the functions may occur wherever there is a change in sign of the control variable $\underline{v}$. Jump conditions may be applied at such points. If there is no change in the sign of $\underline{v}$, the boundary $\underline{\bar{B}}$ is intersected by the state path only at the final time t, and $\underline{\bar{B}}$ may be considered for transversality condition.

(ii) The constraints of the type given by Eqs. 5.11, 5.25, and 5.23a are not very common in usual control theory problems. The first two equalities impose special types of constraints on control variables, while the third condition insists that the path of state variable $\underline{x}$ must intersect certain surface $\underline{\bar{B}}$ at intermediate values $\underline{x}'$, $\underline{x}''$. Special type of algorithm may be developed for problems with such type of constraints. However, in certain practical problems, especially statically determinate system it may not be very difficult to eliminate the complexity. Some simplified approaches may be adopted

as discussed later.

(iii) The solution process requires the following manifolds and equations:

(a) Manifold $\bar{\underline{C}}$ due to compatibility conditions.

(b) Manifold $\bar{\underline{B}}$ due to serviceability constraints.

(c) Equilibrium equations.

For a given structure the manifolds may be obtained in terms of the state variables $\underline{x}$ and the equilibrium equations in terms of $\underline{v}$ and $\frac{d\underline{w}}{ds}$.

(iv) The behaviour idealization is to be considered in the following constraints:

(a) State equations given by Eq. 5.14.

(b) Constraint on $\underline{\alpha}$ given by Eqs. 5.11 - 5.13.

(c) Constraint given by Eq. 5.25.

Depending upon the class of materials available for construction, the idealization may be made and the equations may accordingly be defined.

5.4.4. Simplification of Constraints on Control Variables:

In statically determinate structures the initial slope $\underline{\alpha}(0)$ and the final value y^c are important in any force-deformation relation, and the shape of the curve of intermediate values of $\underline{y}$ may not be very important. In such problems, Eqs. 5.11, 5.12 and 5.25 may be replaced as

follows

$$\left|\frac{\underline{F}^c}{\underline{y}^c-\underline{d}}\right| \geq \underline{\alpha} \geq \left|\frac{\underline{F}^c}{\underline{y}^c}\right| , \quad \text{for } \underline{F}(s)\,\underline{v}(s) \geq 0$$

$$\underline{\alpha} \geq \left|\frac{\underline{F}^c}{\underline{y}^c-\underline{d}'}\right| , \quad \text{for } \underline{F}(s)\,\underline{v}(s) \leq 0 \tag{5.73}$$

where

$\underline{y}^c$ = the maximum value of deformation undergone by the force-deformation relation which represents a point on the boundary surface $\bar{\underline{B}}$. $\underline{F}^c$ = the forces that correspond to the value $\underline{y}^c$, which can be solved in terms of the load $\underline{w}$. Hence, for any assumed value of $\underline{y}^c \in \bar{\underline{B}}$, the bounds to $\underline{\alpha}$ become known constants. In this form, Pontryagin's maximum principle may be used to solve the problem. By taking repeated trial values $\underline{y}^c$, it may be possible to arrive at an optimum solution. In this particular method, the force-deformation relations will be linear with different loading and unloading paths. Any nonlinear curve on the conservative side with $\alpha(0) \leq \left|\frac{\underline{F}^c}{\underline{y}^c-\underline{d}}\right|$ may be chosen for design.

5.4.5 Reformulation of the Constraints:

Method 1: The differential inequality constraints given by Eq. 5.11 may be replaced by algebraic constraints as follows: During first loading,

$$\alpha_i \geq \alpha_i(0)\left[1 - \vartheta_i\,|F_i|\right]$$

and during subsequent unloading and reloading

$$\alpha_i \geq \alpha_i(0) \left[1 - \frac{\vartheta_i}{2} \left|F_i - F_i'\right|\right] \tag{5.74}$$

where F_i' is the force at which the current loading or unloading is started. Sign of F_i' is to be appropriately entered in the equations given above. Using above algebraic form the constraints Eqs. 5.11 and 5.12 may be replaced as follows. For the first loading,

$$\alpha_i(0) \geq \alpha_i \geq \alpha_i(0)\left[1 - \vartheta_i \left|F_i\right|\right] \tag{5.75a}$$

and for the subsequent unloading and reloading

$$\alpha_i(0) \geq \alpha_i \geq \alpha_i(0)\left[1 - \frac{\vartheta_i}{2} \left|F_i - F_i'\right|\right] \tag{5.75b}$$

When constraints are taken in the form $\theta(t, \underline{x}, \underline{\alpha}, \underline{F}) \geq 0$ as given above, necessary conditions derived by Berkovitz through variational approach may be made use of for solution. The problem of tension bar is solved for nonlinear behaviour using this condition.

Method 2: The differential inequality constraints given by Eq. 5.11 may be replaced by equality constraints as follows:

$$\frac{d\alpha_i}{ds} = -\left|\vartheta_i(F_i)\, v_i\right|, \quad i = 1, \ldots, n \tag{5.76}$$

with proper limits assigned for $\vartheta_i(F_i)$. ϑ_i $(i = 1, \ldots, n)$ may be taken as a new set of control variables treating α_i $(i = 1, \ldots, n)$ as state variables. However, $\underline{\alpha}$ must

remain as continuous function of s for the interval under consideration. Whenever unloading occurs, $\underline{\alpha}(s)$ become discontinuous functions of time. Also, transversality conditions are to be obtained at the terminal time or initial time. This approach deserves further consideration.

Method 3: Some of the constraints can automatically be satisfied if Ramberg-Osgood functions as used by Kaldjian (136) for nonlinear material under loading and unloading, are used to represent the force-deformation relations. The Ramberg-Osgood function is of the form

$$y = \frac{F}{\alpha}\left[1 + d^{*}\left|\frac{F}{F_M}\right|^{r-1}\right] \tag{5.77}$$

where α, d^*, r and F_M are parameters defining the curve. Keeping α as variable and adjusting d^*, and r so as to incorporate the conditions given by Eqs. 5.11, 5.12 and 5.25, the problem can be simplified considerably. The three constraints can be dispensed with and the minimization reduces to the minimization with respect to n parameters $\underline{\alpha}$. In this method, a direct minimization of the performance index may be attempted instead of trying to use the necessary conditions in optimal control theory.

5.5 APPLICATION TO THE DESIGN OF STRUCTURES WITH TIME-INDEPENDENT BEHAVIOUR

The method formulated in Section 5.3 is applied to

structures with time-independent behaviour. In this particular case, the following changes in the equations derived in Section 5.3 may be noted.

(i) The state variable $\underline{x}$ may be replaced by $\underline{y}$ as the time dependent part $\underline{z}$ of $\underline{x}$ is zero.

(ii) The stage variable is s. If time is important in the formulation, it may be taken as a state variable with the state equation

$$\frac{dt}{ds} = 1 \tag{5.78}$$

(iii) The component $g(t, \underline{x}, \underline{F})$ in the state equation given by Eq. 5.21 is zero. Hence,

$$\frac{dy_i}{ds} = \frac{v_i}{\alpha_i} , \quad i = 1, \ldots, n \tag{5.79}$$

(iv) The performance index rate G_0 is also free of energy due to time-dependent behaviour.

5.5.1 Statically Determinate System:

Further simplification in the procedure may be obtained in statically determinate systems as follows:

(i) The number of equilibrium equations given by Eqs. 5.10 can uniquely solve the forces $\underline{F}$ in terms of the given load $\underline{w}$. Hence the variable $\underline{v}$ ceases to be a control vector. Hence, only a set of n control variables $\underline{\alpha}$ need be found out.

(ii) The state variable constraints Eq. 5.26 are not to be considered in the solution of the problem, as statically determinate problems can be solved without compatibility equations.

(iii) Performance index may be simplified by substituting Eq. 5.10.

5.5.2 Example of Tension Bar:

The method of design using the optimal control formulation is illustrated in this section through the design of a tension bar. The same bar is solved for different loading and material conditions to illustrate the method of solution. The results are further analyzed in Section 5.8.

Consider a tension bar 100 inches long as shown in Fig. 5.8.a or 5.9.a. It is required to find out the cross section and material of the bar, or alternatively the force-deformation relations (task curve) satisfying the following conditions:

(i) Maximum allowable displacement of the free end is 1 inch.

(ii) Permanent plastic deformation allowable is $d = 0.001$ in/in.

Case 1: Load w is applied on the bar as shown in Fig. 5.8.b. The maximum magnitude of the load is 4 tons.

Material of the bar is elastic: This case is shown for illustrating the method.

s = stage variable (arbitrarily chosen)

F = axial force in the member in tons is the state variable in force space.

y = axial strain of the bar is the state variable in deformation space.

v = control (known in this case and hence v is not to be treated as a control variable)

α = control variable (the stiffness of the member).

Constraints:

(i) $w = s$ is a known function

(ii) $\frac{dw}{ds} = 1$

(iii) Eq. 5.10 (equilibrium equations) may be written as

$$F = w,$$

Hence,

$$F = s$$

$$\frac{dF}{ds} = v = 1$$

(iv) Because of elastic idealization, the state equation given by Eq. 5.14 becomes

$$\frac{dy}{ds} = \frac{v}{\alpha} = \frac{1}{\alpha} \quad , \quad \text{with } \alpha(0) = \alpha = \alpha(s)$$

(v) $|y^p| = 0$. No plastic deformation.

(vi) Since deflection should not exceed one inch, y is constrained by

$$y \leq \frac{1}{100} = 0.01$$

(vii) Performance index is

$$P(\alpha) = \text{Min} \int_0^4 (F. \frac{v}{\alpha} \times 100 - w \times 100 \times \frac{v}{\alpha} - 100y.\frac{dw}{ds}) \, ds$$

Simplifying with the aid of (iii) given above

$$P(\alpha) = \underset{\alpha}{\text{Min}} - \int_0^4 100 \, y \, v \, ds$$

The boundary $\bar{\underline{B}}$ is shown in Fig. 5.8.c. Since there is no reversal of loading the state variable is not returnable once it enters on $\bar{\underline{B}}$. Hence it is considered that $\bar{\underline{B}}$ is intersected by the path of y at $S = 4$. The problem may be treated as a fixed time free right end type (The path is terminated at an infinitesimal distance before reaching $\bar{\underline{B}}$). Hamiltonian is chosen in this problem such that it is maximized.

$$H = \lambda_0 (-100 \, y \, v) + \lambda_1 \frac{v}{\alpha} + \lambda_2 v.$$

$$\lambda_0 = -1$$

$$H = 100 \, y \, v + \lambda_1 \frac{v}{\alpha} + \lambda_2 v.$$

$$\frac{d\lambda_1}{ds} = -\frac{\partial H}{\partial y} = -100\ v, \quad \text{with}\ \lambda_1(4) = 0 \ \text{(since free right end)}$$

$$\frac{d\lambda_2}{ds} = -\frac{\partial H}{\partial F} = 0 \quad , \quad \text{with}\ \lambda_2(4) = 0.$$

Hence solving for λ_1, λ_2

$$\lambda_1 = 400\ v - 100\ v\ s$$

$$\lambda_2 = 0$$

$$H = 100\ v\ y + 400\ \frac{v^2}{\alpha} - \frac{100\ v^2 s}{\alpha}$$

$$= 100\ y + \frac{400}{\alpha} - \frac{100\ s}{\alpha}$$

Since α is constant over s, the state variable y is linearly varying with 's'. Though the problem can be solved by observation that α corresponds to $y(4) = 0.01$ gives maximum value to the Hamiltonian it is attempted to substitute various values of α and find out the one that maximizes H. This is only to maintain the uniformity in procedure with the methods adopted in subsequent problems. The iteration is shown in Table 5.1. From this table the optimal value of α is obtained as <u>400</u>. In this particular case Hamiltonian is constant over s. The corresponding force-deformation diagram is shown in Fig. 5.8.d. The minimum value of P^* is shown by the shaded area in Fig. 5.8.d.

Case 2: Load w is applied on the bar as shown in Fig. 5.8.b. The maximum magnitude of the load is 4 tons. Material is inelastic: The equations for this particular case can be written as follows:

$$\text{(i)} \quad \frac{dy}{ds} = \frac{v}{\alpha}$$

$$\text{(ii)} \quad \frac{dF}{ds} = v = \frac{dw}{ds}$$

$$\text{(iii)} \quad y \leq 0.01$$

$$\text{(iv)} \quad |y^p| = \left|y - \frac{F}{\alpha(0)}\right| \leq 0.001$$

$$\text{(v)} \quad \frac{d\alpha}{ds} \leq -|(F)\, v(s)|$$

$$P = -\int_0^4 100\, y\, v\, ds.$$

Minimize P subject to the constraints given above.

Solution: Since the returnability condition is not satisfied, the state variable y may be allowed to reach the boundary only at $S = 4$, at $w = F = 4$ tons. Substituting $F_{max} = 4$ in the constraint (iv),

$$\alpha(0) \leq 444.$$

Using the reformulation proposed in Section 5.5, let us replace the constraint (v) by

$$\alpha(0) \geq \alpha \geq \alpha(0)\left[1 - \frac{1}{50} F\right]$$

The inequality fits in with the constraints chosen by

Berkovitz (described in Section 5.4) for control variables. The necessary condition can be formulated as follows:

Taking $\lambda_0 = 1$

$$H = -100\ y\ v + \lambda_1 \frac{v}{\alpha} + \lambda_2\ v \tag{a}$$

Using

$$\theta_1 = \alpha - \alpha(0)\ (1 - \frac{F}{50}) \geq 0, \quad \text{and}$$

$$\theta_2 = (\alpha(0) - \alpha) \geq 0 \quad ,$$

in Eq. 5.40,

$$\mu_1 \left\{\alpha - \alpha(0)\ (1 - \frac{F}{50})\right\} + \mu_2 \left\{\alpha(0) - \alpha\right\} = 0 \tag{b}$$

By Eq. 5.38

$$\frac{d\lambda_1}{ds} = -\left(\frac{\partial H}{\partial y} + \mu_1 \frac{\partial \theta_1}{\partial y} + \mu_2 \frac{\partial \theta_2}{\partial y}\right)$$

$$= 100\ v, \quad \text{with } \lambda_1(4) = 0.$$

Solving the differential equation

$$\lambda_1 = (-400\ v + 100\ v\ s) \tag{c}$$

$$\frac{d\lambda_2}{ds} = -\left(\frac{\partial H}{\partial F} + \mu_1 \frac{\partial \theta_1}{\partial F} + \mu_2 \frac{\partial \theta_2}{\partial F}\right)$$

$$= -\mu_1 \frac{\alpha(0)}{50}, \quad \text{with } \lambda_2(4) = 0 \tag{d}$$

Using Eq. 5.39

$$\frac{\partial H}{\partial \alpha} + \mu_1 \frac{\partial \theta_1}{\partial \alpha} + \mu_2 \frac{\partial \theta_2}{\partial \alpha} = 0$$

$$-\lambda_1 \frac{v}{\alpha^2} + \mu_1 - \mu_2 = 0 \qquad \text{(e)}$$

$$\mu_2 = \mu_1 - \lambda_1 \frac{v}{\alpha^2}$$

Substituting in (b) and simplifying,

$$\mu_1 = \frac{50\lambda_1 v(\alpha(0) - \alpha)}{\alpha^2 \alpha(0) F}$$

Substituting in (d)

$$\frac{d\lambda_2}{ds} = -\frac{\lambda_1 v(\alpha(0) - \alpha)}{\alpha^2 F}, \quad \text{with } \lambda_2(4) = 0.$$

Now the problem can be stated as follows.

Find an $\alpha^* = \alpha^*(s)$ such that

$$H(\alpha^*) = \text{Min}\left\{ H(\alpha) \right\}$$

$$= \underset{\alpha}{\text{Min}}\left\{ -100\, y\, v + \lambda_1 \frac{v}{\alpha} + \lambda_2 v \right\} \qquad \text{(a')}$$

such that

$$y^f = 0.01 \qquad \text{(b')}$$

$$\frac{dy}{ds} = \frac{v}{\alpha}, \quad \text{with } y(0) = 0 \qquad \text{(c')}$$

$$v = 1$$

$$\frac{d\lambda_2}{ds} = -\lambda_1 \frac{v(\alpha(0) - \alpha)}{\alpha^2 F}, \; \lambda_2(4) = 0 \qquad \text{(d')}$$

$_1 = -400\ v + 100\ v\ s$, and (e')

$\alpha(0) \leq 444$

This is a two point boundary value problem that can be solved numerically as described below. The method followed here is not a standard procedure, and is only a trial and error procedure. The method cannot be generalized to all problems of this category.

(i) Assume a function $\alpha = \alpha(s)$ such that $\alpha(0) \leq 444$.

(ii) Since v is known, (c') may be integrated to find the final value y^f.

(iii) Several functions $\alpha = \alpha(0)\left[1 - \frac{F}{50}\right]$ are assumed and the one that gives a final value y^f of y as 0.01 is taken as the lower limit. The upper and lower limits of $\alpha(s)$ are found out in Table 5.2.

(iv) Within the bounds assume any $\alpha(s)$ such that

$$\alpha(0) \geq \alpha \geq \alpha(0)\left[1 - \frac{F}{50}\right]$$

(v) Substitute $\alpha, \lambda_1, v, \alpha(0)$, and F in (d') and integrate $\frac{d\lambda_2}{ds}$. The values of λ_2 for any assumed $\alpha(s)$ is entered in Table 5.3. Substituting the values in (a'), H is calculated.

(vi) Find the α^* at $s = 0, 1, 2, 3,$ and 4 that minimizes H at these points. The force-deformation relation corresponding to the optimum H is shown in Fig. 5.8.f. The minimum value of

performance index is equal to the shaded area in Fig. 5.8.f. This case illustrates how the constraints and the Hamiltonian together give the force-deformation relations.

Case 3: Load w is applied on the bar as shown in Fig. 5.9.b. The maximum magnitude of the load is 4 tons. Material is inelastic: This case is intended to illustrate the loading and unloading phenomena and the incorporation of jump conditions in the process of solution. The essential equations for this loading are the following:

(i) $\frac{dy}{ds} = \frac{v}{\alpha}$

(ii) $\frac{dF}{ds} = v = \frac{dw}{ds}$, with $F(0) = 0$. Since $w(s)$ is a known function of s, F can be solved directly in terms of w. Hence $F(s) = w(s)$.

(iii) $|y^p| = |y - \frac{F}{\alpha(0)}| \leq 0.004$, for $v(s) \geq 0$

≥ 0.002, for $v(s) \leq 0$

(iv) $y \leq 0.01$

(v) $\frac{d\alpha}{ds} \leq -\frac{1}{5}|v|$, for loading

$\leq -\frac{1}{10}|v|$, for unloading.

(vi) $0 < \alpha \leq \alpha(0)$

$$P = -\int_0^8 100\, y\, v\, ds.$$

The allowable permanent deformation of 0.004 is arbitrary and is taken to **show** greater dissipation of energy. The boundary $\bar{\bar{B}}$ is given by

$$y = 0.01$$

In this case of load, since v can take negative values, the returnability condition is satisfied. Hence, $y(8)$ need not be on the boundary $\bar{\bar{B}}$. Further, Eq. (i) shows that y monotonically increases until $y(4)$ and then monotonically decreases. Hence, $y(4)$ may be considered as the point that belongs to $\bar{\bar{B}}$. Also, $y(4)$ is a point of discontinuity or corner point. Corner conditions are to be applied at $y(4)$. The limit or boundary of F is given by $F = 4$. Referring to Fig. 5.9.c, it may be noted that the state variables (F, y) has to reach the point B in the two dimensional space $F \times y$. The differential inequality constraint (v) and the constraint (vi) may be replaced as follows:

$$\alpha(0) \geq \alpha \geq \alpha(0)\left[1 - \left|\frac{F}{5}\right|\right], \quad \text{for loading}$$

$$\alpha(0) \geq \alpha \geq \alpha(0)\left[1 - \left|\frac{F-4}{10}\right|\right] \quad \text{for unloading.}$$

The equations of necessary conditions may be written as follows:

<u>For $s = 0$ to 4</u>.

$$H = -100\, y\, v + \lambda_1 \frac{v}{\alpha} + \lambda_2\, v$$

$$y(4) = 0.01$$

$$|y - \frac{F}{\alpha(0)}| \leq 0.004$$

$$\mu_1 \left\{\alpha - \alpha(0) \left[1 - |\frac{F}{5}|\right]\right\} + \mu_2 \left\{\alpha(0) - \alpha\right\} = 0$$

$$\frac{d\lambda_1}{ds} = 100\ v\ , \quad \text{with}\ \lambda_1(4) = \lambda_1^-(4)$$

$$\frac{d\lambda_2}{ds} = -\frac{\lambda_1\ v\ (\alpha(0) - \alpha)}{\alpha^2 F}\ , \quad \text{with}\ \lambda_2(4) = \lambda_2^-(4)$$

$$\frac{dy}{ds} = \frac{v}{\alpha}\ , \quad \text{with}\ y(0) = 0.$$

For $s = 4$ to 8

$$H = -100\ y\ v + \lambda_1 \frac{v}{\alpha} + \lambda_2\ v$$

$$\mu_1 \left\{\alpha - \alpha(0)\ (1 - |\frac{F-4}{10}|)\right\} + \mu_2 \left\{\alpha(0) - \alpha\right\} = 0$$

$$|y - \frac{F}{\alpha(0)}| \geq 0.002$$

$$\frac{d\lambda_2}{ds} = -\frac{\lambda_1\ v\ (\alpha(0) - \alpha)}{\alpha^2 |F-4|}\ \mathrm{sgn}|F-4|, \quad \text{with}\ \lambda_2(8) = 0$$

λ_1 is plotted in Fig. 5.9.d.

Corner Condition: Let the variables just ahead of $s = 4$ be indicated by a negative sign and that after $s = 4$ by a positive sign. Following Berkovitz's (128) corner conditions given by Eqs. 5.56 to 5.60.

$$\lambda_0^- = \lambda_0^+$$

$$\bar{\bar{\lambda}}_1 = \overset{+}{\lambda}_1 - \upsilon_{A1}\frac{\partial Q}{\partial y} = \overset{+}{}_1 - \upsilon_{A1}$$

$_{A1}$ is chosen such that $\upsilon_{A1} \geq 0$ and $\bar{\lambda}_1 + \upsilon_{A1}\frac{\partial Q}{\partial x} \neq 0$

Let us choose $\upsilon_{A1} = \overset{+}{\lambda}_1$ so that $\bar{\lambda} = 0$. Similarly, the multiplier $\overset{+}{\lambda}_2$ is positive and hence to have $\upsilon_{A2} \geq 0$, $\bar{\lambda}_2 = 0$.

The solution is carried out as in Case 2 inserting the corner conditions at $s = 4$. The computations are shown in Tables 5.4 and 5.5. The finally obtained force-deformation relation is shown in Fig. 5.9.e. The shaded area in this figure represents the minimum performance index P subject to the constraints. The value of P^* is positive and represents the energy dissipated.

Case 4: General cyclic loads: In Case 3, the effect of loading and unloading has been studied. In a similar manner, the effect of reversed loading and reloading can be studied. Fig. 5.10 shows the type of loading and the corresponding force-deformation relations. The shaded areas indicate the minimum value P^* in each case of loading.

5.5.3 Example of Two Bar Truss:

The example of tension bar does not fully illustrate the use of Hamiltonian in the solution of the problem. This is because the state variable is fully defined

at two points one at $s = 0$ and one on the boundary. When the state variable can intersect the boundary at more than one point, the choice of the optimum path will be complex. In such a case optimization of Hamiltonian helps to pick out the optimal path or paths. This is illustrated by means of the example of a two bar truss subject to two loads w_1 and w_2 as shown in Fig. 5.11.a. The loading functions $w_1(s)$ and $w_2(s)$ are shown in Fig. 5.11.b. Serviceability requirements are:

(i) Joint C should not deflect more than 1 inch either vertically or horizontally.

(ii) The permanent deformation in any member should be less than 0.001 in/in.

Variables:

y_1, y_2 = axial strains in members AC and BC are state variables in $\underline{\bar{Y}}$ space.

F_1, F_2 = axial forces in members AC and BC respectively are state variables in $\underline{\bar{F}}$ space.

α_1, α_2 = stiffnesses of members are control variables.

$w_1(s), w_2(s)$ = given loading functions.

From equilibrium equations

$$F_1(s) = \frac{w_1(s) + w_2(s)}{\sqrt{2}}$$

$$F_2(s) = \frac{w_1(s) - w_2(s)}{\sqrt{2}}$$

The paths $F_1(s)$, $F_2(s)$ of $\underline{F}$ are shown in Fig. 5.11.c.

$$F_{1max} = 6.363 \text{ tons (tensile)}$$

$$F_{2max} = 3.535 \text{ tons (tensile)}$$

$$= 2.828 \text{ tons (compressive)}$$

The boundary within which the point C can move is shown in Fig. 5.11.d. From strain-displacement relations

$$\Delta_v = 100 \left(\frac{y_1 + y_2}{\sqrt{2}}\right)$$

$$\Delta_h = 100 \left(\frac{y_1 - y_2}{\sqrt{2}}\right)$$

where $_v$ and $_h$ are the vertical and horizontal deflections of joint C. By assumption

$$\Delta_v \leq 1'' \quad ; \quad \Delta_h \leq 1''$$

The boundary $\bar{\underline{B}}$ can be obtained corresponding to the equality sign. Thus

$$y_1 + y_2 = \pm 0.01414$$

$$y_1 - y_2 = \pm 0.01414.$$

These lines define an enclosed area in the $\bar{Y}$ space as shown in Fig. 5.11.e. It shows that the vector $\underline{y} = (y_1, y_2)$ can move only within the boundary abcd. Since y_1 takes only positive strain, only the portion abc is accessible by $\underline{y}$.

$$P = \int_0^{12} \left[100\ F_1 \frac{v_1}{\alpha_1} + 100\ F_2 \frac{v_2}{\alpha_2} - \frac{d}{ds}\left\{ w_1 \Delta_v + w_2 \Delta_h \right\} \right] ds.$$

substituting equilibrium equations and simplifying

$$P = - \int_0^{12} (y_1\ v_1 + y_2\ v_2)\ 100\ ds.$$

Constraints

$$\frac{dy_i}{ds} = \frac{v_i}{\alpha_i} \quad , \quad i = 1,\ 2$$

$$\frac{dF_i}{ds} = v_i \quad , \quad i = 1,\ 2$$

$$|y_i^p| = |y_i - \frac{F_i}{\alpha_i(0)}| \leq 0.001, \quad \text{for loading}$$
$$= 0,\ i = 1,2\ , \quad \text{for unloading.}$$

$$\alpha_i(0) \geq \alpha_i \geq \alpha_i(0) \left[1 - |\frac{F}{20}|\right], \quad \text{for loading}$$

$$\alpha_i = \alpha_i(0)\ , \quad \text{for unloading}$$

boundary $\bar{B}$ is given by $y_1 + y_2 = \pm\ 0.01414$ and $y_1 - y_2 = \pm\ 0.01414$.

Solution

$$H = -100\, y_1\, v_1 - 100\, y_2\, v_2 + \lambda_1 \frac{v_1}{\alpha_1} + \lambda_2 \frac{v_2}{\alpha_2} + \lambda_3\, v_1 + \lambda_4\, v_2$$

$$\frac{d\lambda_i}{dt} = 100\, v_i\ , \quad \text{with } \lambda_i\,(12) = 0\ , \quad i = 1,\ 2$$

λ_1 and λ_2 are calculated and plotted against s in Fig. 5.11.f. incorporating appropriate conditions at corner points.

$$\frac{d\lambda_j}{dt} = -\ \left(\frac{\partial H}{\partial F_i} + \mu_1 \frac{\partial \theta_1}{\partial F_i} + \mu_2 \frac{\partial \theta_2}{\partial F_i}\right), \quad \begin{array}{l} i = 1,\ 2. \\ j = 3,\ 4 \end{array}$$

$$\theta_i(y,\ \alpha) = \alpha_i - \alpha_i(0)\ (1 - \left|\tfrac{F}{20}\right| \geq 0, \text{ for 1st loading}$$

$$\alpha_i = \alpha_i(0)\ , \quad i = 1,\ 2 \text{ unloading.}$$

The total range of stage variable is divided into three parts 0-4, 4-8 and 8-12, since 4, and 8 are stages at which corner conditions are to be satisfied. Proceeding as in Case 3 in the example of tension bar, the force-deformation relations satisfying the conditions and minimizing P can be obtained. In this problem, $\underline{y}$ can intersect with the boundary $\bar{\underline{B}}$ at several points. Hence, the point corresponding to the minimum Hamiltonian may be chosen as the acceptable point. The computations are shown in Tables 5.6 and 5.7. The optimum force-deformation relations obtained are shown in Figs. 5.11.g and h.

5.5.4 Application to Statically Indeterminate Structures:

The design method formulated in Section 5.3 is applied in this section to statically indeterminate structures. The following features may be noted.

(i) The equilibrium equations given by Eq. 5.10 are not sufficient to uniquely solve the forces in terms of the loads. Hence, for any load vector $\underline{w}$, there can be many force vector $\underline{F}$ satisfying the Eq. 5.10. The choice of proper forces $\underline{F}$ (or $\underline{y}$) is also a decision problem, in addition to the choice of stiffness.

(ii) The compatibility conditions given by Eq. 5.26 are also to be taken into consideration as a state variable inequality constraint.

The method is illustrated for a parallel three bar system with a single load w and a three bar truss under two loads w_1 and w_2.

5.5.5 Three Bar System:

Consider a three bar system as shown in Fig. 5.12.a. It is subjected to a load w that varies as shown in Fig. 5.12.b as a function of s. Let the maximum deflection of the rigid block must not exceed 1".

<u>Variables</u>

y_1, y_2, y_3 = axial strains in bars 1, 2, 3 (state variables)

F_1, F_2, F_3 = axial forces in bars 1, 2, 3 (state variables)

$\alpha_1, \alpha_2, \alpha_3$ = stiffnesses of bars 1, 2, 3 (control variables)

v_1, v_2, v_3 = load rates in bars 1, 2, 3 (control variables)

Equilibrium equation is given by

$$F_1 + F_2 + F_3 = w$$

Admissible region of state space is defined by the planes

$$y_1 = y_2 = y_3 = \frac{\Delta}{100} = \frac{1}{100}$$

The admissible region is shown in Fig. 5.12.c. Compatibility constraint is given by

$$y_1 = y_2 = y_3 = \frac{\Delta}{100} .$$

This is represented by a straight line in the $\bar{\underline{Y}}$ space as shown in Fig. 5.12.c.

$$P = \int_0^4 \left[100 \left\{F_1 \frac{v_1}{\alpha_1} + F_2 \frac{v_2}{\alpha_2} + F_3 \frac{v_3}{\alpha_3}\right\} - w \frac{\dot{\Delta}}{100} - \frac{\Delta}{100} \frac{dw}{ds}\right] ds$$

Using equilibrium and compatibility conditions P can be simplified in this problem as

$$P = - \int_0^4 \left\{100(y_1 v_1 + y_2 v_2 + y_3 v_3)\right\} ds$$

Case 1. Material of bars is elastic:

$\alpha_1(s) = \alpha_1$, $\alpha_2(s) = \alpha_2$, $\alpha_3(s) = \alpha_3$, where α_1, α_2, α_3 are stage-invariant parameters. The equations obtained by means of necessary conditions are the following:

(i) $$H = -100(y_1 v_1 + y_2 v_2 + y_3 v_3) + \lambda_2 \frac{v_2}{\alpha_2} + \lambda_3 \frac{v_3}{\alpha_3} + \lambda_1 \frac{v_1}{\alpha_1} + \lambda_4 v_1 + \lambda_5 v_2 + \lambda_6 v_3$$

(ii) $y_1(4) = y_2(4) = y_3(4) = 0.01$

(iii) $\frac{d\lambda_i}{ds} = +100 v_i$, $\lambda_i(4) = 0$, $i = 1, 2, 3$

(iv) $\frac{d\lambda_i}{ds} = 0$, $\lambda_i(4) = 0$, $i = 4, 5, 6$

(v) $\frac{dx_i}{ds} = \frac{v_i}{\alpha_i}$, $i = 1, 2, 3.$

(vi) $v_1 + v_2 + v_3 = \frac{dw}{ds} = 10$

(vii) $\frac{v_1}{\alpha_1} = \frac{v_2}{\alpha_2} = \frac{v_3}{\alpha_3} = \frac{\Delta}{100}$

By a trial and error procedure, taking suitable combinations of α_i and v_i ($i = 1, 2, 3$), Hamiltonian H may be calculated. It can be seen that for any admissible strategy that satisfy (ii), (vi) and (vii) the Hamiltonian is constant equal to -10. Hence, a unique set of control

is not obtained. In other words, there are m
of optimal control that give the same minimum
Hamiltonian. The alternatives may include sir
and three bar systems. Fig. 5.12.d gives the
force-deformation relations. The choice of ar
F_1, F_2 will fix up the third and also the com
deformation relations.

The problem shows that a statically i
structure made of elastic material may not be
solution under a single load condition when c
requirements are taken into account. It is s
that a further imposition of cost-effectivene
makes the choice unique and also shown that s
indeterminate systems are better than determi
when a warning before failure is required.

Case 2. <u>Material is nonlinear</u>: In this case
are imposed as follows:

$$\alpha_i(0) \geq \alpha_i \geq \alpha_i(0)\left[1 - \frac{1}{500}|F|\right]$$

$$\left|y_i - \frac{F_i}{\alpha_i(0)}\right| \leq 0.001, \quad i = 1, 2,$$

Let $\theta_i = \alpha_i - \alpha_i(0)\left[1 - \frac{1}{500}|F|\right] \geq 0,$ i

From the necessary conditions described in S

following equations are obtained:

(i) $\frac{d\lambda_i}{ds} = 100\, v_i, \quad \lambda_i\,(4) = 0, \quad i = 1, 2, 3$

(ii) $\frac{d\lambda_4}{ds} = -\mu_1 \frac{1}{500}\, \alpha_1(0)$

(iii) $\frac{d\lambda_5}{ds} = -\mu_2 \frac{\alpha_2(0)}{500}$

(iv) $\frac{d\lambda_6}{ds} = -\mu_3 \frac{\alpha_3(0)}{500}$

(v) $\mu_1 \left\{\alpha_1 - \alpha_1(0)(1 - |\frac{F_1}{500}|)\right\} + \mu_2 \left\{\alpha_2 - \alpha_2(0)(1 - |\frac{F_2}{500}|)\right\}$

$+ \mu_3 \left\{\alpha_3 - \alpha_3(0)(1 - |\frac{F_2}{500}|\right\} = 0$

(vi) $\mu_i = \frac{\lambda_i\, v_i}{\alpha_i^{\;2}}$, $i = 1, 2, 3$

(vii) $\alpha_i \leq \alpha_i(0), \quad i = 1, 2, 3.$

(viii) $y = y_1 = y_2 = y_3 \leq 0.01$ or $\frac{v_1}{\alpha_1} = \frac{v_2}{\alpha_2} = \frac{v_3}{\alpha_3}$

(ix) $|y_i - \frac{F_i}{\alpha_1(0)}| \leq 0.001, \quad i = 1, 2, 3$

(x) $F_1 + F_2 + F_3 = w$ or $v_1 + v_2 + v_3 = \frac{dw}{ds}$

Conditions (viii) and (x) are used to eliminate variables by substitution. The problem is solved by a trial and error

procedure. Of these, three trials are shown in Tables 5.8 and 5.9. The trials are with one bar, two bar, and three bars. The choice of any three control variables will fix the values of all other variables. Hence, the control variables are to be chosen such that Hamiltonian is minimum subject to the inequality constraints. It is seen in the case of nonlinear materials also that the solution of a statically indeterminate system may not be unique under single load conditions. Several combinations can give same serviceability requirements. The choice will become unique only if further conditions are imposed. This procedure (taking serviceability constraints) considerably reduces the number of choices and by a resorting to a cost-effectiveness analysis a unique solution may be obtained.

5.5.6 Example of a Three Bar Truss:

The above examples illustrate the method of solution for a variety of cases. The solutions also illustrate many features of structural design, some of them are discussed in Section 5.8.5.

An example of a three bar truss is considered in this section. The necessary conditions for the existence of solution are applied and the equations are obtained. Equilibrium equations and compatibility conditions are not used as in the previous problem to directly eliminate the variables.

Instead, they are introduced as constraints using appropriate multipliers. The problem is developed in this way to illustrate the general characteristics of the problem. The problem in this form can be solved only with the help of an efficient numerical method.

The three bar truss, to be made of in-elastic material. is shown in Fig. 5.13.a. It is subjected to two loads w_1 and w_2, the functions of which are given in Fig. 5.13.b. The serviceability requirement is that the joint C should not deflect more than 1" horizontally and 1" vertically. The three bars are of equal length (100" each). Also the permanent deformation should not exceed 0.001 in/in.

<u>Variables</u>:

$\underline{y} = y_1, y_2, y_3$ represent the axial strains in the members 1, 2, 3.

$\underline{F} = F_1, F_2, F_3$ represent the forces in the members which are taken as the state variables.

$\underline{\alpha} = \alpha_1, \alpha_2, \alpha_3$ the stiffnesses forming another set of controls.

$v = v_1, v_2, v_3$ the controls governing state variable $\underline{F}$.

These are the 12 basic variables of the problem. (Even in a nonlinear analysis, it is required to consider all these variables.)

The problem may be formulated as explained in the earlier examples. Only the final equations obtained by applying the necessary conditions described in Section 5.4 are given here.

$$H = 100\left(F_1 \frac{v_1}{\alpha_1} + F_2 \frac{v_2}{\alpha_2} + F_3 \frac{v_3}{\alpha_3}\right) - 100\, w_1 \frac{v_2}{\alpha_2}$$

$$- \frac{100\ w_2}{\sqrt{2}} \left(\frac{v_1}{\alpha_1} - \frac{v_3}{\alpha_3}\right) - 100\, y_2 \frac{dw_1}{ds} - \frac{100}{\sqrt{2}} (y_1 - y_3)$$

$$\frac{dw_2}{ds} + \lambda_1 \frac{v_1}{\alpha_1} + \lambda_2 \frac{v_2}{\alpha_2} + \lambda_3 \frac{v_3}{\alpha_3} + \lambda_4 v_1 + \lambda_5 v_2 + \lambda_6 v_3$$

$$\frac{dv_i}{ds} = \frac{v_i}{\alpha_i}, \quad i = 1, 2, 3$$

$$\frac{dF_i}{ds} = v_i, \quad i = 1, 2, 3$$

$$\phi_1 = \frac{v_1}{\alpha_1} + \frac{v_3}{\alpha_3} - 1.414 \frac{v_2}{\alpha_2} = 0,$$

$$v_2 + \frac{v_1}{\sqrt{2}} + \frac{v_3}{\sqrt{2}} - \frac{dw_1}{ds} = 0,$$

$$v_1 - v_3 - \sqrt{2} \frac{dw_2}{ds} = 0.$$

$$\mu_1 \left\{\alpha_1 - \alpha_1(0)\left(1 - \left|\frac{F_1}{50}\right|\right)\right\} + \mu_2 \left\{\alpha_2 - \alpha_2(0)\left(1 - \left|\frac{F_2}{50}\right|\right\}\right. +$$

$$+ \mu_3 \left\{\alpha_3 - \alpha_3(0)\left(1 - \left|\frac{F_3}{50}\right|\right)\right\} = 0 \quad \text{for loading.}$$

$$\frac{d\lambda_1}{ds} = + \frac{100}{\sqrt{2}} \frac{dw_2}{ds} \quad , \lambda_1(8) = 0$$

$$\frac{d\lambda_2}{ds} = + \frac{100}{\sqrt{2}} \frac{dw_1}{ds} \quad , \quad {}_2(8) = 0$$

$$\frac{d\lambda_4}{ds} = - 100 \frac{v_1}{\alpha_1} - \frac{\mu_1 \alpha_1(0)}{50} \text{ sgn } F_1 \; , \; \lambda_4(8) = 0$$

$$\frac{d\lambda_5}{ds} = - 100 \frac{v_2}{\alpha_2} - \frac{\mu_2 \alpha_2(0)}{50} \text{ sgn } F_2 \quad , \; \lambda_5(8) = 0$$

$$\frac{d\lambda_6}{ds} = - 100 \frac{v_3}{\alpha_3} - \frac{\mu_3 \alpha_3(0)}{50} \text{ sgn } F_3 \; , \; \lambda_6(8) = 0$$

$$100 \frac{F_1}{\alpha_1} - 100 . \frac{w_2}{\sqrt{2}\,\alpha_1} + \frac{\lambda_1}{\alpha_1} + \lambda_4 + \frac{\upsilon_1}{\alpha_1} + \frac{\upsilon_2}{\sqrt{2}} + \upsilon_3 = 0$$

$$100 \frac{F_2}{\alpha_2} - 100 \; w_1 + \frac{\lambda_2}{\alpha_2} + \lambda_5 - \upsilon_1 \; 1.414 \frac{1}{\alpha_2} + \upsilon_2 = 0$$

$$100 \frac{F_3}{\alpha_3} + \frac{100 \; w_2}{\sqrt{2}\,\alpha_3} + \frac{\lambda_3}{\alpha_3} + \lambda_6 + \frac{\upsilon_1}{\alpha_3} + \frac{\upsilon_2}{\sqrt{2}} - \upsilon_3 = 0$$

$$- 100 \frac{F_1 \; v_1}{\alpha_1{}^2} + \frac{100 \; w_2}{\sqrt{2}} \frac{v_1}{\alpha_1{}^2} - \frac{\lambda_1 \; v_1}{\alpha_1{}^2} + \mu_1 - \upsilon_1 \frac{v_1}{\alpha_1{}^2} = 0$$

$$- 100 \frac{F_2 \; v_2}{\alpha_2{}^2} + 100 \; w_1 \frac{v_2}{\alpha_2{}^2} - \lambda_2 \frac{v_2}{\alpha_2{}^2} + \mu_2 + \upsilon_1 \; 1.414 \frac{v_2}{\alpha_2{}^2} = 0$$

$$\frac{d\lambda_3}{ds} = - \frac{100}{\sqrt{2}} \frac{dw_2}{ds} \quad , \quad \lambda_3(8) = 0$$

$$- 100 F_3 \frac{v_3}{\alpha_3^2} - \frac{100 w_2}{\sqrt{2}} \frac{v_3}{\alpha_3^2} - \lambda_3 \frac{v_3}{\alpha_3^2} + \mu_3 - \nu_1 \frac{v_3}{\alpha_3^2} = 0$$

$\alpha_i \leq \alpha_i(0)$ for loading

$\quad = \alpha_i(0)$ for unloading

$\left| y_i - \frac{F_i}{\alpha_i(0)} \right| \leq 0.001$ for loading

$\quad \geq 0.001$ for unloading

$y_2 \leq 0.01$

$y_1 + y_3 \leq \pm 0.01414$

$y_1 - y_3 \leq \pm 0.01414$

The problem as stated above can be solved only with the help of an efficient algorithm. The solution of the three bar truss for elastic case is illustrated in Table 5.8. One trial of the solution is shown. Several such trials are necessary to arrive at an optimal solution. The values of α_1, α_2 and α_3 are assumed in a trial. The variables v_1, v_2, and v_3 are solved using the constraints obtained from equilibrium and compatibility conditions. The Hamiltonian is calculated for the assumed values of α_1, α_2, α_3. In successive trials α_1, α_2 and α_3 are to be changed. The one set of assumed values of α_1^*, α_2^* and α_3^* that gives the minimum H^* is to be chosen as the optimal solution.

5.6 APPLICATION TO THE DESIGN OF STRUCTURES WITH TIME-DEPENDENT BEHAVIOUR

5.6.1 General:

The extension of the problem formulated in Section 5.3 to structures with creep effects represented by simple creep laws is straight forward. The serviceability requirements can be satisfied in this case taking the creep effects also. If the creep rate is a function of the applied force and time without involving any higher order differential co-efficient of forces or deformations with respect to time, the method can be applied without any change. However, the following assumptions are to be made:

(i) The loading function is continuous with respect to time.

(ii) No creep recovery is considered. Creep deformations are considered irreversible.

(iii) Whenever creep is considered, the materials of the structure is decided beforehand so that the creep constants can appropriately be taken.

(iv) No structural failure due to creep deformation occurs during the life time. Eq. 5.18 gives the pattern of creep equation as

$$\frac{dz_i}{dt} = g_i(t, F) \quad , \quad i = 1, \ldots, n. \qquad (5.18)$$

In the case of steady state creep, the state equation may be written in the form

$$\frac{dx_i}{dt} = \frac{v_i}{\alpha_i} + a_i \, |F_i|^{\gamma} \operatorname{sgn} F_i$$

It agrees with the strain-hardening law of creep. Other equations may also be used to represent the creep characteristics of materials. The following points may be noted:

(i) t is the stage variable in place of s.

(ii) x is to be taken as the state variable in place of y.

(iii) P may be calculated taking the additional energy change due to creep deformation.

(iv) The material may be elastic or inelastic

5.6.2 <u>Example of a Simply Supported Beam with Creep Effects:</u>

Consider a simply-supported beam of span 12 Ft. as shown in Fig. 5.14.a subjected to two concentrated loads W_1 and W_2. The loading functions $W_1(t)$, $W_2(t)$ are shown in Fig. 5.14.b. Let the beam can be designed by designing three sections1, 2, and 3 marked in Fig. 5.14.a. The serviceability requirement is that the deflection at points 1, 2, 3 on the beam should not exceed 1 inch. The variables can be defined as follows:

F_1, F_2, F_3 = the bending moments in in-tons at Sections 1, 2, 3 respectively (force-state variables).

x_1, x_2, x_3 = the curvature at Sections 1, 2, 3 respectively (deformation state variables).

v_1, v_2, v_3 = the rate of change of bending moment with s.

α_1, α_2, α_3 = the slope of moment-curvature diagram (control variables).

Equilibrium Equations: The moments F_1, F_2, F_3 can be solved in terms of external loads. They are

$$F_1 = 27\ w_1 + 18\ w_2 \quad \text{in tons positive}$$

$$F_2 = 18\ w_1 + 36\ w_2 \quad \text{in tons positive}$$

$$F_3 = 9\ w_1 + 18\ w_2 \quad \text{in tons positive}$$

The bending moment and curvature diagrams are shown in Figs. 5.14.c and d. The functions $F_1(t)$, $F_2(t)$ and $F_3(t)$ are plotted in Fig. 5.14.e. Maximum moments are given by

$$F_{1\ max} = 189 \text{ in tons.}$$

$$F_{2\ max} = 198 \text{ in tons.}$$

$$F_{3\ max} = 99 \text{ in tons.}$$

Boundary $\bar{\bar{B}}$: Let Δ_1, Δ_2, Δ_3 be the deflections at Sections 1, 2 and 3 respectively. The deflections can be expressed in

terms of the curvature as follows (making use of the conjugate beam theory).

$$756\ x_1 + 648\ x_2 + 324\ x_3 = \Delta_1$$

$$648\ x_1 + 1080\ x_2 + 648\ x_3 = \Delta_2$$

$$324\ x_1 + 648\ x_2 + 756\ x_3 = \Delta_3.$$

Equating Δ_1, Δ_2, Δ_3 to 1, the equations to the boundary $\bar{\bar{B}}$ can be obtained. It is the intersection of three surfaces in a three dimensional space.

<u>Differential Constraints</u>:

$$\frac{dx_i}{ds} = \frac{v_i}{\alpha_i} + \frac{1}{10^{10}} |F_i|^3 \ \mathrm{sgn}\ F_i, \quad i = 1, 2, 3.$$

$$\frac{dF_i}{ds} = v_i \ , \quad i = 1, 2, 3.$$

<u>Performance Index</u>:

$$P = \int_0^8 G_0\ ds = \int_0^8 (G_{01} - G_{02})\ ds.$$

<u>Calculation of G_{01}</u> : Assuming that the forces and deformations linearly vary from one section to another.

$$\int_0^{L_j} G_{j,j-1}(\xi)\ d\xi = \frac{F_j\ \dot{x}_j}{3} \Delta L_j + \frac{F_{j-1}\ \dot{x}_{j-1}}{3} \Delta L_j + \frac{F_j\ \dot{x}_{j-1}}{6} \Delta L_j + \frac{F_{j-1}\ \dot{x}_j}{6} \Delta L_j$$

Using the above relation for $j = 1, 2, 3$ and 4 and simplifying

$$G_{01} = 24\,F_1\,\dot{x}_1 + 24\,F_2\,\dot{x}_2 + 24\,F_3\,\dot{x}_3 + 6\,F_1\,\dot{x}_2 + 6\,F_2\,\dot{x}_1 + 6\,F_3\,\dot{x}_2 + 6\,F_2\,\dot{x}_3$$

$$G_{02} = w_1\dot{\Delta}_1 + w_2\,\dot{\Delta}_2 + \dot{w}_1\Delta_1 + \dot{w}_2\Delta_2$$

Using equilibrium equations $G_{01} - G_{02}$ can be simplified. Hence,

$$P = -\int_0^8 \left[\dot{w}_1(756\,x_1 + 648\,x_2 + 324\,x_3) + \dot{w}_2(648\,x_1 + 1080\,x_2 + 648\,x_3)\right] dt$$

The problem formulated above can be solved in a manner similar to that of tension bar. Three intervals 0-2, 2-4, and 4-8 are to be considered using corner conditions at $s = 2, 4$. The problem may be solved by a trial and error procedure. The Table 5.91 shows the computations involved in the problem. The optimal force-deformation relations at Sections 1, 2, 3 are shown in Fig. 5.14.K.

5.7 TIME-DEPENDENT DEFORMATION WITH JUMPS IN STATE VARIABLES

In the preceding paragraphs, loads were assumed as continuous functions of s or t . In many practical cases, it rarely happens that the loading is a continuous function of time. Some times the loads may be applied instantaneously not causing any impact or vibration. In other cases, the rate of application at some interval of loading, may be so fast compared to other intervals of time that the loads may very well be considered as instantaneous. For example, when the time t is taken in terms of years, the application of a concentrated load in a week's time may be as good as an instantaneous application. In civil engineering structures, this type of loading is very common. The problem presented in Section 5.3 is reformulated in this paragraph to allow for such discontinuities in loading.

5.7.1 Reformulation of the Problem:

The effect of instantaneous loading on the structure is to change instantaneously the state vector $\underline{F}$ as well as the $\underline{y}$ component of $\underline{x}$ leading to a jump motion of the state variables with respect to time. In between such jumps, the state variables may remain stationary if the loads remain stationary, and the material of the structure has no time-dependent behaviour. In other cases, state variable moves with time. Piecewise continuity of state variable is not allowed in the formulation presented in Section 5.3. The problem formulated in Section 5.3 may therefore be modified

by a technique similar to that suggested by Vind (137).

(i) Time is not a suitable parameter as stage variable because of the jumps in state variables when expressed in terms of time. Hence time is also taken as the (n+1)th state variable. Let $\underline{X} = \{\underline{x}, t\}$ be the augmented state variable.

(ii) The state vector $\underline{x}$ takes a jump, when at least one component of $\underline{x}$ jumps with respect to time. It is further assumed that r such jumps occur in the loading process with timing $t_1 = 0 < t_2 < t_j < t_r = T$.

(iii) Let j be the total number of jumps occuring before the time t_j (including jump at t_j). An arbitrary stage variable s is chosen as follows:

$$\begin{aligned} s &= t + j, && \text{for } t_j < t < t_{j+1} \\ s_j^- &= t + j-1, && \text{for } t = t_j \\ s_j^+ &= t + j, && \text{for } t = t_j \\ s_1^- &= 0 + 0 && \\ s_r^+ &= S = T + r && \end{aligned}$$

(iv) The loading function $\underline{w} = \underline{w}(t)$ when expressed in terms of s appears as shown in Fig. 5.15.a. In general, the functions $\underline{w}(s)$ between any s_j^- and s_j^+ are nonlinear. However, for all practical purposes, the functions in the said range may be idealized as linear as shown by the firm line in Fig. 5.15.b.

(v) A new variable $w_0 = 0$ or 1 is introduced in order to indicate a jump or smooth motion of state variable with respect to time.

$$w_0 = \begin{cases} 0, & \text{for } s_j^- \leq s \leq s_j^+ \\ 1, & \text{for } s_j^+ \leq s \leq s_{j+1}^- \end{cases} \tag{5.80}$$

(vi) The state Eqs. 5.14 are to be modified as follows: When $t_j < t < t_{j+1}$,

$$\frac{dx_i}{dt} = \frac{v_i(t)}{\alpha_i(t)} + g_i, \quad i = 1, \ldots, n \tag{5.81}$$

When $t = t_j$,

$$\frac{dx_i}{ds} = \frac{dy_i}{ds} = \frac{v_i(s)}{\alpha_i(s)} \equiv f_i', \quad i = 1, \ldots, n \tag{5.82}$$

Combining Eqs. 5.81 and 5.82 and expressing in terms of 's' one gets

$$\frac{dx_i}{ds} = \left[w_0\left\{\frac{v_i(t)}{\alpha_i(t)} + g_i\right\} + (1 - w_0)\frac{v_i(s)}{\alpha_i(s)}\right], \; i = 1, \ldots, n \tag{5.83}$$

Also, $\dfrac{dt}{ds} = w_0$ (5.84)

The initial conditions are to be modified as

$$x_i(0) = x_i(s_1^-) = (x_i)^0 \tag{5.85}$$

$$t(s_1^-) = 0.$$

(vi) Let G_0 in the performance index as given by Eq. 5.27 be

$$G_0 = v_0 \; ; \qquad s_j^- \leq s \leq s_j^+ \tag{5.86}$$
$$\quad = g_0 , \qquad s_j^+ \leq s \leq s_{j+1}^-$$

Performance index can be modified as follows:

$$P = \int_0^S \left\{ w_0 \cdot (g_0) + (1 - w_0) \cdot (v_0) \right\} ds \tag{5.87}$$

5.7.2 Statement of the Reformulated Problem:

Given s = stage variable.

$$s = t + j, \qquad t_j < t < t_{j+1}$$

$$\left. \begin{aligned} s_j^- &= t + j - 1 \\ s_j^+ &= t + j \end{aligned} \right\} t = t_j$$

$$s_1^- = 0 ,$$

$$S = T + r$$

$\underline{X} = \{x_1, \ldots, x_n, t\}$ as state variables (deformations),

$\underline{F} = \{F_1, \ldots, F_n\}$ as state variables (forces),

$\underline{v} = \{v_1, \ldots, v_n\}$ as control variables (force rate), and

$\underline{\alpha} = \{\alpha_1, \ldots, \alpha_n\}$ as control variables (stiffnesses), and

$$\underline{W} = \{w_1, \ldots, w_m\}$$ as the loads to be chosen from.

$$\underline{w}(s) = \{w_1(s), \ldots, w_m(s)\}$$ which is given,

find,

$$\underline{v}^*(s) = \{v_1(s), \ldots, v_n(s)\},$$

$$\underline{\alpha}^*(s) = \{\alpha_1(s), \ldots, \alpha_n(s)\},$$

$$\underline{F}^*(s) = \{F_1(s), \ldots, F_n(s)\},$$ and

$$\underline{X}^*(s) = \{x_1(s), \ldots, x_n(s), t(s)\}$$

such that $P^*(\underline{F}^*, \underline{\alpha}^*, \underline{X}^*) = \underset{\underline{F}, \underline{\alpha}}{\mathrm{Min}}\ P(\underline{F}, \underline{\alpha}, \underline{X})$, where

$$P = \int_0^S \left[w_0 \cdot g_0 + (1 - w_0) \cdot v_0 \right] ds$$

Subject to the following constraints:

(i) $\frac{dx_i}{ds} = \left[w_0 \cdot (f_i + g_i) + (1 - w_0) \cdot f_i \right]$, $i = 1, \ldots, n$

(ii) $\frac{dF}{ds} = \underline{v}$

(iii) $\frac{dt}{ds} = w_0$

iv) $R_i(\underline{v}, \frac{dw}{ds})$, $i = 1, \ldots, P$

(v) $Q(\underline{x}) \leq 0$,

$Q(x') = Q(x'') = 0$

(vi) $K_i(\underline{x}) = 0,$ $\qquad i = 1, \ldots, q$

(vii) $\frac{d\alpha_i}{ds} \leq 0$ or $-|\nu_i(s)v(s)|$ $i = 1, \ldots, n$

(viii) $|x_i^p| \geq d_i$, for $F_i(s)\ v_i(s) \geq 0$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad i = 1, \ldots, n$

$\leq d_i'$, for $F_i(s)\ v_i(s) \leq 0$

and the initial conditions

$$x_i(0) = (x_i)^0 ,$$

$$\qquad i = 1, \ldots, n$$

$$F_i(0) = (F_i)^0 ,$$

$$t(0) = 0$$

5.7.3 Example of Two Bar Truss:

A two bar truss shown in Fig. 5.16. a is loaded by a load w as shown in Fig. 5.16.b. The serviceability requirement is that the maximum deflection of joint C should not exceed 1 inch. The following assumptions are made:

(a) The material is linearly elastic with creep behaviour

$$\underline{\alpha}(s) = \underline{\alpha}$$

(b) The deformation is symmetric.

The problem may be formulated as follows:

$x_1 = x_2 = x =$ the axial strains in the members.

$F_1 = F_2 = F =$ the axial forces in the members.

$w =$ loading function (given)

$v_1 = v_2 = v =$ the control variables

$\alpha_1 = \alpha_2 = \alpha =$ the control variable (stiffness of members)

Reformulation of the Problem:

$w_0 = 0$ for $t = 0$

$w_0 = 1$ for $0 < t \leq 4$

$s = t + j = t + 1$ since number of jump is one.

$S = 5$

$w = 4s, \quad 0 \leq s \leq 1$

$= 4, \quad 5 \geq s > 1.$

The modified load function is plotted in Fig. 5.16.c.

Constraints:

(i) $F = F_1 = F_2 = \dfrac{w}{\sqrt{2}} = 0.707\ w.$

(ii) The boundary $\bar{B}$ is given as in Fig. 5.1.c which is defined by

$x_1 + x_2 \leq 0.01414$, or

$x_2 \leq 0.00707$

(iii) Since the state variable is non-returnable

$(x)^f = 0.00707.$

(iv) State Equation:

$$\frac{dx}{ds} = \frac{v}{\alpha} \quad , \quad 0 < s \leq 1$$

$$\frac{dx}{ds} = \frac{v}{\alpha} + 1x10^{-5}|F|^3 \text{sgn } F \quad , \quad 1 < s \leq 5.$$

$$\frac{dx}{ds} = (1 - w_0)\frac{v}{\alpha} + w_0.(\frac{v}{\alpha} + 1x10^{-5}|F|^3 \text{sgn } F)$$

$$\frac{dt}{ds} = w_0$$

$$\frac{dF}{ds} = w_0(v) + (1 - w_0).v$$

Performance Index is given by

$$v_0 = 2x100.F.\frac{v}{\alpha} - \sqrt{2}.\ w.100.\frac{v}{\alpha} = 100x\sqrt{2}.\frac{dw}{ds}.\ y$$

$$= -200\ vy$$

and

$$g_0 = 0$$

In the modified form

$$P = \int_0^5 \left[(1 - w_0)(-200\ vy) + w_0(0)\right] ds$$

$$= -200 \int_0^5 (1 - w_0)\ y\ v\ ds.$$

The problem may be solved using maximum principle.

$$H = +200\ (1 - w_0)\ y\ v + \lambda_1\ (1 - w_0)\frac{v}{\alpha} +$$
$$w_0(\frac{v}{\alpha} + 1x10^{-5}|F|^3 \text{sgn } F) + \lambda_2.\ v. + \lambda_3.\ w_0$$

$$\frac{d\lambda_1}{ds} = - \frac{\partial H}{\partial y} = - 200(1 - w_0)v , \qquad \lambda_1(5) = 0$$

$$\frac{d\lambda_2}{ds} = - \frac{\partial H}{\partial F} = -\lambda_1 \ w_0 x 1 x 10^{-5} x 3 |F|^2, \ \lambda_2(5) = 0$$

$$\frac{d\lambda_3}{ds} = - \frac{\partial H}{\partial t} = 0, \qquad \lambda_3(5) = 0$$

The variation of v, w_0, λ_1, λ_2 and F with respect to s is shown in Fig. 5.16.d. The values of v, w_0, λ_1, λ_2 and F are substituted in the expression for Hamiltonian. Substituting various values of α in the expression for H, the one giving maximum may be chosen as the optimum. The computation is shown in Table 5.12.

The optimal force-deformation relation is shown in Fig. 5.16.e.

5.8 DISCUSSION

The method of design formulated in the preceding paragraphs is further analysed in the light of structural mechanics to establish its validity. The special features of the proposed method and some interesting conclusions arrived at from the illustrative examples are also presented.

5.8.1 Validity of the Proposed Method:

The proposed design method, to represent the real structural behaviour, must consider to the principles of

continuum mechanics. The basic laws of motion and a constitutive theory are the two expositions that constitute the backbone of continuum mechanics (138). In addition, the stress-strain laws (or constitutive equations) must fulfil a set of conditions or requirements in order to truely represent an elastic-plastic medium (139) which conditions are discussed elsewhere in this section.

(a) Equilibrium Equations and Force Boundary Conditions: Balance of momentum and balance of moment of momentum are two of the laws to be satisfied by any mechanical system. The Eq. 5.10 in the formulation represents the equilibrium equations, and the satisfaction of the equations guarantee that the forces are statically complete and obey the two laws of mechanics given above.

(b) Conservation of Energy: The internal dissipation of energy on account of inelastic and creep behaviour and also due to the propagation of cracks must be taken into account. This would lead to a set of non-conservative force-deformation relations (112), in which the deformations of cross sections are no longer single-valued functions of the forces developed at the sections and vice versa. The concept of minimum potential energy is applicable only for conservative system. In non-conservative systems, energy is expended irreversibly depending on the course of loading. This fact is taken care of in the proposed method in minimizing the performance index. At any stage s, the performance index consists of

energy dissipated in all the prior loading between O-s and the energy potential (in terms of complementary energy) due to loads acting at s. Hence minimization of P is equal to minimization of energy lost + energy potential at s.

(c) Compatibility Conditions and Displacement Boundary Conditions: The deformations of the structure must be compatible among themselves and with the displacement boundary conditions. Eq. 5.26 in the formulation represents these conditions.

(d) Use of Minimum Principles: The aim of the present formulation is to obtain the force-deformation relations of the media to be used so that a specified set of serviceability requirements are satisfied for a given path of loading. In other words, if the force-deformation relations so obtained are used in the usual structural analysis, the true behaviour predicted by this analysis must be within acceptable limits. The displacement must be such that, the deflections and permanent deformation are within the permissible limits at any time t or stage s, so long as the limit value of load is not exceeded. These restrictions on the solution are imposed by the constraint Eqs. 5.23 and 5.25.

The solution of the problem yields the force-deformation relations subject to equilibrium condition, compatibility and boundary conditions, and the serviceability requirements. In order to have unique solution, the solution must satisfy the minimum energy principles. The three

variational principles or minimum principles in elasticity are the minimum potential energy, minimum complementary energy and Reissner's variational theorems (140). Analogous principles are proposed both for the flow theory and deformation theory in plasticity (123). Wang and Prager (141) generalized the minimum principles in plasticity by taking the temperature and creep of material into account. The theorems required in the thesis are stated below.

Consider a region R of the body, bounded by surfaces A_1 and A_2, is subjected to prescribed loads $\underline{w}$ on the surface A_1, and prescribed displacements on surface A_2. Let the internal forces and deformations $\underline{F}$, $\underline{x}$ are in equilibrium with $\underline{w}$ and compatible with boundary discplacement, respectively. Let $\underline{F}$ and $\underline{x}$ be the true forces and true deformations. Now the external loads $\underline{w}$ are increased by $d\underline{w}$, keeping displacements on A_2 constant in an interval ds or dt whichever is applicable depending upon whether time is not important or important. The internal forces $\underline{F}$ change by $d\underline{F}$ and the deformations by $d\underline{x}$ and the displacements in the direction of load $\underline{w}$ by $d\underline{\Delta}$. No body force is considered. For the incremental state of deformation of this type the minimum principles can be stated as follows:

1. <u>Reissener's Variational Theorem</u>: Among all states of stress and displacements which satisfy the boundary conditions of prescribed surface displacements, the actually occurring state of stress and displacement is one determined by the

variational equation

$$\delta \left[\int_{vol} \left\{d\underline{F}.\ d\underline{x} - d\phi\right\} dv - \int_A d\underline{w}.\ d\underline{\Delta}\ dA\right] = 0$$
(5.88)

2. <u>Minimum Principle in the Rate or Flow Theory of Plasticity</u>: Among all rates of forces that are in equilibrium among themselves and with external loads, and among all deformations that are compatible among themselves and with boundary, the true forces and deformations will minimize π given by (123),

$$\pi = \int_v \frac{1}{2}\ d\underline{F}.\ d\underline{x}\ dv - \int_A (d\underline{w}.\ d\underline{\Delta})dA \qquad (5.89)$$

3. <u>Minimum Principle for Inelastic Material with Creep Effects</u>: In this case the above statement holds good with the value of π given by (141).

$$\pi = \int_{vol} (\frac{1}{2}\ dF(dy + dz))dv - \int_A (d\underline{w}.\ d\underline{\Delta})dA \qquad (5.90)$$

<u>To Show That the Minimization of the Performance Indiex P is Equivalent to the Minimization of the Energy Potentials π in Minimum Principle</u>: The performance index is given by Eq. 5.27 as follows:

$$P = \int_0^S G\ ds.$$

Let

$$P^* = \operatorname*{Min}_{\underline{v},\underline{\alpha}} \int_0^S G\ ds = P^*(S) \qquad (5.91)$$

Consider an interval ds just ahead of the stage S.

$$s = S - ds.$$

Now $$\int_0^S G\, ds = \int_0^s G\, ds + G(s)ds$$

$$P^*(S) = P^*(s + ds) = \text{Min}\left[\int_0^s G\, ds + G(s)ds\right]$$

$$P^*(S) = P^*(s + ds) = \text{Min} \int_0^S G\, ds = \text{Min}\left[\int_0^s G\, ds + G(s)ds\right] \qquad (5.92)$$

$$P^*(s + ds) = P^*(s) + \left\{\sum_{i=1}^{n} \frac{\partial P^*(s)}{\partial x_i^*} \frac{dx_i^*}{ds} + \frac{\partial P^*(s)}{\partial t} \frac{dt}{ds}\right\} ds$$
$$+ \text{ higher order terms} \qquad (5.93)$$

Using the principle of optimality in dynamic programming (124), Eq. 5.92 can be written as

$$P^*(S) = \text{Min}\left[P^*(s) + G(s)ds\right] \qquad (5.94)$$

Substituting Eq. 5.93 in Eq. 5.94, one gets

$$\underset{\underline{v},\underline{\alpha}}{\text{Min}}\ G(s) + \frac{\partial P^*}{\partial t}\frac{dt}{ds} + \sum_{i=1}^{n} \frac{\partial P^*}{\partial x_i^*}\frac{dx_i^*}{ds} = 0 \qquad (5.95)$$

This is the Hamilton-Jacobi equation in optimal control problem. It gives that the rate of change of the minimum value of P^* with respect to s is ~~same as~~ the minimum value of G in that interval ds. In the present formulation, G is given by:

$$G = \int_v \left\{\underline{F}(s) \cdot \frac{d\underline{x}(s)}{ds}\right\} dv - \int_s \frac{d}{ds}(\underline{w} \cdot \Delta) ds.$$

Expanding in finite form

$$G = \int_V (\underline{F} + \frac{1}{2}\frac{d\underline{F}}{ds}) \cdot \frac{d\underline{x}}{ds}\, dv - \int_A \Big\{ (\underline{w} + \frac{1}{2} \cdot \frac{d\underline{w}}{ds}) \frac{d\underline{\Delta}}{ds} +$$

$$(\underline{\Delta} + \frac{1}{2}\frac{d\underline{\Delta}}{ds}\frac{d\underline{w}}{ds} \Big\} dA.$$

$$= \int_V \underline{F}\frac{d\underline{x}}{ds}\, dv - \int_A \Big\{ \underline{w} \cdot \frac{d\underline{\Delta}}{ds} + \underline{\Delta} \cdot \frac{d\underline{w}}{ds} \Big\}\, dA +$$

$$\int_A \frac{1}{2}\frac{d\underline{F}}{ds} \cdot \frac{d\underline{x}}{ds}\, dv - \int_A \frac{d\underline{w}}{ds} \cdot \frac{d\underline{\Delta}}{ds}\, dA$$

By principle of virtual work,

$$\int_V \underline{F}\ \frac{d\underline{x}}{ds}\ dv = \int_A \underline{w} \cdot \frac{d\underline{\Delta}}{ds}\ dA.$$

Since $\underline{F}$ is in equilibrium with $\underline{w}$, and $\frac{d\underline{x}}{ds}$ may be considered as the virtual displacements compatible with $\frac{d\underline{\Delta}}{ds}$. Further $\underline{\Delta} \cdot \frac{d\underline{w}}{ds}$ is a stationary quantity for the interval ds, which cannot be minimized. Therefore,

$$\text{Min } G = \int_A (\underline{\Delta} \cdot \frac{d\underline{w}}{ds}) dA + \text{Min} \Big[\int_V \frac{1}{2} (\frac{d\underline{F}}{ds} \cdot \frac{d\underline{x}}{ds}) dv - \int_A (\frac{d\underline{w}}{ds}\frac{d\underline{\Delta}}{ds}\, dA \Big] \qquad (5.96)$$

Hence, minimization of P is done by means of Eq. 5.95 which is equivalent to minimizing the right hand side of Eq. 5.96. It proves the equivalence of the performance index and the rate of potential π.

The solution of optimal control problem is obtained by satisfying the equilibrium, compatibility, and boundary conditions. It is shown that the minimization of P gives a minimum of <u>π at each interval ds</u> from 0 to S. Thus the requirement of minimum principle is satisfied for all states of loading. Since all these conditions are satisfied, the solution corresponds to the actual forces and displacements when the constitutive equations of the type

$$\underline{F}^{*} = \underline{F}^{*}(y)$$

obtained from the optimal control solution are used in an actual analysis.

5.8.2. <u>Physical Significance of the Minimum Performance Index</u>:

Apart from assuring the minimum principle, the minimum value of the performance index P has a physical significance. P^{*} represents the total potential of the system at the end of the period. It consists of the total elastic energy due to the loads acting at the instant s and the entire energy dissipated or that energy which become not recoverable due to residual stress, for the range 0-S. If the elastic energy part remain same, <u>any change in the magnitude of P^{*} may be considered as a measure of damage caused on the structure as the dissipated energy corresponds to the irrecoverable deformations</u>. The purpose of the optimal

control may also be thought of for minimization of the inelastic damage caused on the structure. Later, it is assumed that an imaginary opponent simultaneously tries to maximize this damage, leading to a game situation.

5.8.3. To Show That the Conditions of Plasticity are Satisfied by the solution:

Work hardening type of plasticity is considered in the formulation. Prager (139) states that four conditions are to be satisfied by any stress-strain law of the rate type in plasticity. They are :

(i) Condition of continuity
(ii) Condition of irreversibility
(iii) Condition of uniqueness
(iv) Condition of consistency.

(i) Condition of Continuity: Let the deformation be

$$x_i = y_i^e + y_i^p + z_i, \quad i = 1, \ldots, n \qquad (5.97)$$

where y_i^e = the elastic component of deformation

y_i^p = the plastic component of deformation.

According to Prager (139) the continuity condition is fulfilled if $dy_i^p \to 0$, when dF_i approaches $F_i(s)$. This condition is satisfied in the present formulation by defining the state Equation 5.14 as

$$\frac{dy_i}{ds} = \frac{v_i(s)}{\alpha_i(s)}$$

which implies $\frac{dy_i}{ds} \to 0$ when $v_i(s) \to 0$.

(ii) <u>Condition of Irreversibility</u>: This condition is satisfied if Drucker's postulate which states that the dissipation of energy is positive in any cycle of loading and unloading; is fulfilled. Mathematically the postulate is stated as follows:

$$\underline{F} \cdot d\underline{y}^p > 0 \ , \quad \text{if } d\underline{y}^p \neq 0. \tag{5.98}$$

i.e.

$$\underline{F} \cdot \left(\frac{\underline{v}(s)}{\underline{\alpha}(s+ds)} - \frac{\underline{v}(s)}{\underline{\alpha}(s)}\right) > 0$$

i.e.

$$\underline{\alpha}(s + ds) < \underline{\alpha}(s) \quad , \text{ for } \underline{F}(s)\ v(s) > 0 \text{ (i.e. loading)}$$

$$\frac{d\underline{\alpha}}{ds} < 0$$

which is Eq. 5.11. Hence the irreversibility condition is satisfied as the solution satisfies the Eq. 5.11.

(iii) <u>Condition of Uniqueness</u>: Prager (139) has shown that the condition of uniqueness is satisfied if the stress-strain law satisfies the condition

$$dI = \int_V \left[\delta\ d\ \underline{F} - \delta\ d\ \underline{y}\right] dv \tag{5.99}$$

is positive definite.

sequential decision problem with an integral payoff to the optimization of the Hamiltonian at any one stage s. If Hamiltonian is maximum at any stage s it is maximum everywhere, provided the strategy is admissible.

(ii) The importance of serviceability constraints in getting a solution is brought out by the examples. The boundary surface $\bar{\underline{B}}$ governs the maximum deformation y^c required. Eqs. 5.11 and 5.12 govern the shape of the force-deformation relations. Eq. 5.25 gives the initial value of the stiffness $\alpha = \alpha(0)$ and limits the permanent deformation to the required allowable value. But for these constraints, a solution would not have been obtained because Hamiltonian will be still optimum if a control outside the above-specified limit is taken.

(iii) Constraint Eq. 5.11 is introduced to satisfy the plasticity conditions. If no such restriction is imposed a constant linear path of $\underline{\alpha}$ with $\underline{\alpha} = \underline{\alpha}(0)$ may be obtained as the best control as it gives least value of H. This case corresponds to a linear elastic material, showing that a a <u>linear elastic material is the best structural material from serviceability point of view</u>, as it involves no loss of energy. Also from energy considerations a linear elastic material appears to be superior to nonlinear elastic materials, as the energy required for structural action is less in the case of linear materials.

(iv) <u>Ideal Material</u>: If the plasticity requirement given

by Eq. 5.11 and the upper bound $\alpha_- = \underline{\alpha}(0)$ in Eq. 5.12 can be relaxed, the ideal structural material would be the one with a positive curvature ($\frac{d\underline{\alpha}}{ds} \geq 0$). The force deformation relations corresponding to this case is shown in Fig. 5.17. Such a material requires only least energy to perform the structural action. In the limit a rigid material with force-deformation relations as shown by dotted lines in Fig. 5.17 is the best. In this case, if the material takes a path of unloading as shown by the dotted lines in Fig. 5.17, it is a still better control. However, it represents that energy is to be stored in the material in a structural action instead of dissipation which may not be possible. Hence, such force-deformation relations have to be necessarily elastic. The materials with such force-deformation relations are called <u>ideal locking materials</u> (143). The muscular cells in living organisms are said to have such stress-strain curves.

(v) The example of three bar system shows that the force-deformation relations are not always unique in redundant systems especially single load condition. The same serviceability conditions can be obtained by many combinations of members. However, when the cost-effectiveness condition is imposed as shown in next chapter, the choice becomes unique.

5.8.6 Special Features and Potentialities of the Method:

In spite of the complex mathematical structure of

the formulation, the method is distinct and has the following special features that distinguishes the design method from conventional design processes.

(i) It has a direct design concept in which the rĕquired structural behaviour (expressed in terms of force deformation relations) is obtained as an output of the design process as task curves.

(ii) It gives a design by which the deflections and permissible deformations are kept within the allowable limits. An assurance of safety and serviceability for normal load conditions may give enough courage to the designer to attempt a trade off between safety under abnormal load conditions and cost of structure.

(iii) The elimination of material considerations from design computation offers greater flexibility to designer to choose the members from a wider class of cross sections with different geometry and material that have a behaviour similar to the idealization used in design. This will promote the concept of design of materials for specific use rather than the conventional method of designing for specific materials.

(iv) It gives a method of solution applicable to elastic and inelastic structures with or without creep effects.

(v) The method is essentially intended to give a basis

for the further extensions into the cases of design under risk or uncertainty. However, the method itself can serve as an independent design process for deterministic conditions. When a final choice of cross section is to be made, the required safety factors may be incorporated. The use of such a safety factor is dispensed with by the design process described in Chapters 7-10.

(vi) The force-deformation relations obtained for serviceability requirement offers a basis for the further design for cost-effectiveness. This aspect is studied in Chapter 6.

5.8.7 Further Extensions of the Problem:

The method may be extended further to the following cases without much difficulty:

(i) Design of structures with distributed parameter systems (beams, plates, shells etc.). Guidance may be taken for this case from the optimal control theory of distributed parameter systems.

(ii) Design of structures subjected to moving loads. A moving load may be simulated as a collection of loads applied one after another. This method is further described in Chapter 8.

(iii) Design of structures subjected to alternate load conditions. Two alternate loads on the structure may

be simulated as two loads acting on the structure one applied after another. The force-deformation relation obtained will be for both the loads.

(iv) Design of structures for environmental conditions like temperature, settlement etc. The functions of these phenomena with respect to time may also be taken along with the loading functions in the design.

(v) For repeated loading, cyclic loading and other complex forms of loading shown in Fig. 5.18.

(vi) Inertial elasticity, damped inertial elasticity, viscoelasticity etc. involving ordinary differential equations of higher order as the governing equations may also be solved by this procedure by suitably reformulating the problem, with more state variables. For example, when equation in the inertial (delayed) elasticity of the form (144)

$$m \frac{d^2x}{dt^2} + G x = F \tag{5.100}$$

are involved, the problem may be reformulated as follows:

$$\frac{dx^1}{dt} = x^2$$

$$\frac{dx^2}{dt} = \frac{d^2x^1}{dt^2} = \frac{F}{m} - \frac{Gx^1}{m} .$$

In place of a single variable x, two state variables x^1, x^2 are defined and the problem can be reduced to an optimal control problem. Similar modification can also be

made when the order of differential equation is more than two.

(vii) Incorporation of a minimum weight criterion in the performance index that can simultaneously lead to a minimum weight design along with serviceability control.

(viii) Development of an efficient algorithm exclusively for structural optimal control problems. Advantage may be taken from the existing computational techniques in optimal control theory.

(ix) The uniqueness of the solution may also be studied further.

(x) The feasibility of stochastic control formulation for the application of problems involving random loading like earthquake may be studied.

5.9 SUMMARY

The inelastic design method proposed in this chapter can be summarily stated as follows. It is required to find out the geometry and material of the cross sections of members of a structure with given configuration and connections for a given loading condition with given load paths. The design problem is considered as an optimal control problem in which its structural deformation is assumed to be controlled by the external loads and internal force-deformation relations of the cross sections. For the given external control, namely load, the optimal force-deformation relations

that can control the serviceability requirement within the allowable limits is obtained as a solution of the problem. Knowing these relations called task curves, the cross section can be chosen such that the section has identical force-deformation relation.

Though the concepts and method are more rational and basically sound, the method in this form is not enough for practical designs, as the various uncertainties in the actual design situation are not considered in this proposal. Very often, the loading function may not be known. This method is intended to form a framework of design for serviceability that can be further extended to take care of the complexities of a practical situation. The subsequent chapters deal with the uncertainties in design.

CHAPTER SIX

INELASTIC STRUCTURAL DESIGN UNDER RISK FOR COST-EFFECTIVENESS

6.1 INTRODUCTION

The method formulated in Capter 5 considers the serviceability and safety under normal 'deterministic' load conditions only. The magnitudes of the loads are not to exceed arbitrarily specified limit The loads are, in general, random and it is hardly possible in many cases, to limit the magnitudes to such arbitrary levels. The consequences of loads exceeding to abnormal magnitudes are to be investigated in order to safeguard the structure from undue damages. Also, the economic involvement in the design and construction together with the costs associated with failure have to be considered. In the present chapter, loads are assumed to be of statistical nature with known probabilities, and a method of inelastic design is formulated. The design is a decision process under risk. A cost-effectiveness model developed in Chapter 4, is made use of as the criterion of design by which a trade-off between the cost of the structure and the safety and ductility requirements is achieved. The present method is supplementary to what has been proposed in Chapter 5. The task curves obtained in that chapter, are extended in such a way that the cost-effectiveness given by Eq. 4.14 is optimum. The two methods together give the behavioural requirements of

a structure that has to be safe, serviceable and economical. The ductility requirements are also fulfilled in this formulation.

The statistical character of the material behaviour is not taken into account, as the method is intended to find out what the task curves would be if the structure is rational in its behaviour. Also the uncertainties of the type discussed in Chapter 4 are not considered in this formulation. They are taken up in the subsequent chapters.

The design problem is stated in Section 6.2 and a brief outline of the procedure is given in Section 6.3. The decision process using cost-effectiveness criterion is first illustrated through an example of a three bar system in Section 6.4. The general concepts of the formulation are explained in Section 6.5. A method of design applicable for statically determinate systems is formulated in Section 6.6 and that for statically indeterminate systems is given in Section 6.7. Section 6.8 contains a brief discussion on the method.

6.2 STATEMENT OF THE PROBLEM

The following information is given:

(i) The complete layout of the structural members, and the types of connections that are proposed to use.

(ii) The loads acting on the structure and their

probability density functions. The distributions are assumed to be normal.

(iii) The portion of force-deformation relations obtained for serviceability requirements under normal load condition (called task curves).

(iv) Other relevant information to find the cost of failure and other design data.

It is required to extend the task curves that are already given, to such levels that the cost-effectiveness criterion is optimum. Knowing the task curves, the cross sections can be chosen by the methods described in Chapter 10.

6.3 OUTLINE OF THE METHOD

6.3.1 Assumptions:

The following assumptions are made in the formulation presented in this chapter.

(i) Deformations are small.

(ii) Loading is quasistatic. No dynamic or fatigue effect is considered.

(iii) Failure does not occur as a result of time-dependent deformations.

(iv) Failure due to elastic instability does not occur.

(v) Abnormal magnitudes of the loads are attained only once in the life time of the structure.

Repeated loads are not considered in this case.

(vi) Loads (External loads) are assumed to be statistically independent.

(vii) No change in the position or direction of loads is considered.

6.3.2 <u>Outline of the Method:</u>

In agreement with the concept of design presented in Chapter 3, the structure is assigned the role of a decision maker. The structure is supposed to decide its <u>course of action</u> or behaviour optimally, when nature applies a set of loads with a random selection of their magnitudes. Though the true state of loading on the structure is not known, the probability associated with the states of loading, is known, which information leads to a decision under risk. Depending upon the loads chosen by nature, each alternative choice of the structure would lead to any one of a set of failure modes with some probability of occurrence. Since the structural behaviour is assumed deterministic, the probability of occurrence of the failure mode is the probability of the state of loading that causes this failure. The cost-effectiveness criterion in this formulation given in Chapter 4 may be taken as the basis of design. Eq. 4.14 represents the mathematical statement of the criterion. The alternative that has the acceptable least value of K_T (subject to the constraints) is chosen as the optimal decision (The absolute

minimum of K_T is 1.0 which corresponds to a case of no failure. This model differs from the expected worth model of Haider (67) and the expected loss criterion of Sawyer in two respects.

(i) In the present work, the nonlinear force-deformation relations are the real design variables. The set of force-deformation relations corresponding to the optimum cost-effectiveness is the final decision.

(ii) The probability of state of loading is not the true probability of failure as used by Haider or Sawyer as, in the present method, strength is deterministic.

6.4 DESIGN OF A THREE BAR SYSTEM FOR COST-EFFECTIVENESS

A design for cost-effectiveness is illustrated through an example of a three bar system. A general method of design is presented in the subsequent sections. The example is intended to bring out the special features of the cost-effectiveness approach and the advantage of direct design.

Design of Three Bar System:

Consider the design of the bars 1, 2 and 3 of the 3-parallel bar system shown in Fig. 6.1. The axial force-axial strain relations of the bars are to be found out. Knowing the force-deformation relations, the material and cross section can be chosen.

The following requirements are to be met with:

(i) The system is to be serviceable for load w between 0 and 40 Tons (The design is carried out in Chapter 5. The axial force-axial strain relations are shown in Fig. 5.12.c.).

(ii) The load w is random in magnitude. The probability distribution is considered discrete as shown in Fig. 6.1.b.

(iii) The cost-effectivenss must be optimum.

Assumptions:

(i) Force-deformation relations are deterministic

(ii) Material is assumed elasto-plastic

(iii) The possible failure modes and the associated cost of failure are given below.

Mode of failure	Cost of failure	Amount in Rs.
1. Unserviceability	C_{sc}	100/-
2. One member fails (by yielding)	C_p^1	$10(F)_{max}$
3. Two members fail (by yielding)	C_p^2	$10(F_1 + F_2)_{max}$
4. Three members fail with warning	C_w	$10(F_1+F_2+F_3)_{max}$
5. Three members fail without warning (one or two members have brittle failure or all the three yield simultaneously).	C_c	$10(F_1+F_2+F_3)_{max}$ + 5000

$$\text{Cost of structure} = C_s = 10(F_1+F_2+F_3)_{max}$$

F_1, F_2, F_3 are the forces in the members. The cost of the structure is assumed to be proportional to the maximum force developed in each bar. This is only an approximate way of finding the cost of the system. The cost of failure without warning is taken as Rs.5000/-. This high value is purposefully taken to illustrate the importance of a warning before failure and to show how the cost-effectiveness model takes this necessity of giving prior warning into account. Cost-effectiveness factor is given by

$$K_{Ti} = 1 + \sum_j p_j K_{ij} \tag{6.1}$$

where

K_{Ti} is the cost-effectiveness of ith alternative structure represented by a set of force-deformation relations

p_j is the probability of occurrence of jth state of nature (load).

$$K_{ij} = \frac{\text{Cost of failure at jth state of nature}}{\text{Cost of ith alternative chosen}}$$

Since serviceability design is done for w equal to or less than 40 tons, serviceability failure is assumed to occur when the load exceeds 40 tons. A member is said to have failed when it yields or fractures. If the yielding of

the three bars occur in a staggered manner the failure is assumed to occur with warning. However, if only one or two bars are provided, the failure is assumed to be sudden.

Equilibrium Equation:

Let, F_1, F_2, F_3 be the forces in the bars 1, 2, 3 respectively. Equilibrium equation is given by

$$F_1 + F_2 + F_3 = W \tag{6.2}$$

Compatibility condition is given by

$$y_1 = y_2 = y_3 = \frac{\Delta}{100} \tag{6.3}$$

where y_1, y_2, y_3 are the axial strains in bars 1, 2 and 3 respectively and Δ is the total deflection.

Method of Solution:

The method of solution is illustrated in Table 6.1. Altogether 12 trials are made, three of them with only a single bar, five of them with 2 bars and four with 3 bars. The alternative sets of force-deformation relations in each trial are shown in Fig. 6.1.c. The entries belonging to each alternative action are made along a row in Table 6.1. The states of loading ranges from 40 to 48 tons and are entered along the columns. The probability of each state of nature is entered in each column. Each action-state combination is analyzed, the possible modes of failure and the associated costs are determined. A sample calculation

can be shown as follows. Consider the alternative no. 6. It consists of two bars with maximum forces 22 tons and 24 tons respectively. Both members are ductile.

Cost C_s of the system $= (22 + 24)10 =$ Rs. 460/-

At load $w = 40$ tons, Cost of failure $C(40) =$ Rs. 100/-

$$K(40) = 0.217$$

Similarly at $w = 44$ tons, member one fails. Therefore

$$C(44) = 220$$

$$K(44) = \frac{220}{460} = 0.478.$$

At $w = 46$, both members fail,

$$K(46) = \frac{460 + 5000}{460} = 11.85$$

$$\therefore K_T = 1 + \Sigma\, p(w)\, K(w) = 1 + 0.217(0.035 + 0.025 + 0.017 + 0.010)$$

$$+ 0.478(0.005 + 0.003) + 11.85(0.002 + 0.001 + 0.002)$$

$$= 1.082$$

The cost of failure K_{ij} are entered in the table at ith action and jth state of nature. The cost effectiveness factor K_T is entered in the last column. The corresponding force-deformation relations are plotted in Fig, 6.1d. Certain advantages of a design of this type are discussed in Section 6.8.

6.5 A GENERAL FORMULATION OF THE PROBLEM

The example given above illustrates the basic approach to the decision under risk. However, the method of design

presented is not feasible for all types of structures and loads. In most practical problems, several loads with different probability density functions may act on the structure. Further a listing of the alternatives as given in the problem is not possible on account of the numerous possibilities. The compatibility equations and equilibrium equations are more involved, which may bring added difficulties. A formulation of the problem in a general form is presented in this section.

6.5.1 Alternative Actions or Strategy of Structure:

Let us assume that N cross sections of the structure are to be chosen for complete design of the system. Let there be a total of n active force-deformation relations at these sections under the action of external loads. The choice of N sections may therefore be considered as the choice of the force-deformation relations which are called the task curves, as they represent the task to which the sections are put in. A set of n force-deformation relations is called an alternative action or strategy of structure. It may also be called a policy. It represents the course of action or behaviour the structure puts in to carry the loads. Each alternative action represents an alternative structure. As a first attempt it is assumed that no interaction of force-deformation relations at a section occurs. For example, yielding of a section with respect to moment may not be affected by the shear force, twisting moment or

axial forces. Thus,

$$\underline{F} = \underline{F}(y) = \left\{F_1(y_1) \ldots, F_n(y_n)\right\} \qquad (6.4)$$

are n such force-deformation relations representing an alternative action. The stiffness α at any y' is given by

$$\alpha_i(y') = \frac{dF_i(y\ )}{dy_i}\Big|_{y_i = y_i'}, \quad i = 1, \ldots, n \qquad (6.5)$$

Any change in the shape of force-deformation relation may be made by suitably changing the slope $\alpha_i(y)$ and the total deformation y_i^d. It is assumed that infinitely many alternative actions can be formulated by altering the shape of functions $\underline{F} = \underline{F}(y)$.

6.5.2 Force-Space and Deformation-Space:

A concept of a force-space and that of a deformation space is introduced here. At any state of loading, the sections will have a set of forces and the structure will go to a deformed state. Let $\underline{F} = \left\{F_1, \ldots, F_n\right\}$ be the vector that represents the forces in the structure. It can be considered as a point in an n-dimensional Euclidean space $\underline{\bar{F}}$ called force-space. As considered in Chapter 5, $\underline{Y} = \left\{y_1, \ldots, y_n\right\}$ is a deformation vector in the Euclidean n space $\underline{\bar{Y}}$, called deformation space.

Functions given by Eq. 6.4, map a point in $\underline{\bar{Y}}$ space onto a point in $\underline{\bar{F}}$ space, and vice versa.

6.5.3 Loading Space and State of Nature:

Let m independent loads be acting on the structure. Because of the randomness of the magnitude of loading, any combination of magnitudes may act on the structure. Let $\underline{w} = \{w_1 \; ... \; w_m\}$ be the magnitudes of the m loads at a state of loading. It can be considered as a point in an m-dimensional Euclidean space $\underline{\overline{W}}$, called load space. Since no change in direction of the loads is considered, all loads may be taken as positive loads, and hence only the positive region of the $\underline{\overline{W}}$ space need be considered.

Each point in the $\underline{\overline{W}}$ space represents a state of nature. The state that actually occurs is called the true state of nature which is unknown.

6.5.4 Failure Modes and Outcome Sets:

Each action-state combination leads to any one of the failure states or to a state of no failure. Failure, if occurs, may happen in different modes causing different degrees of damage. Let us assume that the failure modes are the following. The cost of failure is indicated against each of them. The mathematical definition and interpretation will be given later.

1.	Unserviceability	C_{sc}
2.	Repairable damage	C_d
3.	Partial collapse	C_p
4.	Total collapse with warning	C_w
5.	Total collapse without warning	C_c

The details of the failure modes are given in Sections 4.3.1 and 4.3.2. The losses due to failure are impact functions of time. As a conservative (or pessimistic) approach, it is assumed that the loads causing a particular type of failure act at a time when the cost of failure is at its peak value. Thus C_{sc}, C_d, C_p, C_w and C_c are the maximum of the functions $C_{sc}(t)$, $C_d(t)$, $C_p(t)$, $C_w(t)$ and $C_c(t)$ respectively within the interval of 0 to T.

If the ith action (structure) and the load $\underline{w}$ act together, the cost of failure is $C_i(\underline{w})$ which may be 0 or any one of C_{sc}, C_d, C_p, C_w, and C_c. The factor $K_i(\underline{w})$ is given by

$$K_i(\underline{w}) = \frac{C_i(\underline{w})}{C_{si}} \tag{6.6}$$

where C_{si} is the cost of structure which is not yet known. It may vary from action to action. The relative values of C_{si} among the actions may be estimated with reasonable degree of accuracy from the nature of force-deformation relations.

6.5.5 Probabilities of Various States of Nature:

The loads are assumed to be of random magnitude, the probability density functions of which are known. The various states of nature $\underline{w}$ may have their own probabilities of occurrence. The probability of occurrence of any particular state can be determined as follows:

The probability density functions of the loads may

be normal, lognormal or any other distribution obtained. Let all the loads are normally distributed with distributions $N(\mu_1, \theta_1)$, $N(\mu_2, \theta_2)$ ..., $N(\mu_m, \theta_m)$ for $w_1, w_2, \ldots, w_m$ respectively. μ represents the mean and θ the standard deviation. The probability density function of the random variable w_i is given by

$$f(w_i) = \frac{1}{\theta_i \sqrt{2\pi}} e^{-\frac{1}{2} \frac{(w_i - \mu_i)^2}{\theta_i}}, \quad i = 1, \ldots, m \tag{6.7}$$

The probability

$$P\left[w_i' \leq w_i \leq w_i' + \Delta w_i\right] = \int_{w_i'}^{w_i' + \Delta w_i} f(w_i)\, dw_i \tag{6.8}$$

Let $F_p(w_i')$ be the cumulative distribution, given by

$$F_p(w_i') = \int_{-\infty}^{w_i'} f(w_i)\, dw_i \tag{6.9}$$

where $f(w_i)$ is a continuous function. Probability that w_i lies in a small interval Δw_i at w_i' is approximately given by $f(w_i')\,\Delta w_i$

Now, the <u>multivariate distribution</u> (145) for the m-dimensional random variable can be considered. Let $f(w_1, \ldots, w_m)$ be the density function and $F_p(w_1, \ldots, w_m)$ the cumulative distribution function.

$$F_p(\underline{w}') = P\left[\underline{w} \leq \underline{w}'\right]$$

$$= \int_{-\infty}^{w_m'} \cdots \int_{-\infty}^{w_2'} \cdots \int_{-\infty}^{w_1'} f(\underline{w})\, dw_1\, dw_2 \ldots, dw_m \tag{6.10}$$

$$P(\underline{w}' \leq \underline{w} \leq \underline{w}'+\Delta\underline{w}') = \int_{w_m'}^{w_m'+\Delta w_m} \cdots \int_{w_1'}^{w_1'+\Delta w_1} f(\underline{w})\, dw_1\, dw_2 \ldots, dw_m \tag{6.11}$$

$$\approx f(\underline{w}')\Delta w_1 \Delta w_2 \ldots, \Delta w_m$$

probability of $\underline{w}$ lying in a region R is given by

$$P\left[\underline{w} \in R\right] = \int_R \cdots \int f(\underline{w})\, dw_1\, dw_2 \ldots, dw_m \tag{6.12}$$

Let $w_i = w_i'$, $w_j \leq \infty$, $j = 1, \ldots, m$, $j \neq i$ define a plane $\underline{M}_i$ in the m-dimensional space. The marginal distribution is given by

$$P\left[w_i \leq w_i',\ w_j \leq \infty \begin{matrix} j = 1, \ldots m \\ j \neq i \end{matrix}\right] = \int_{-\infty}^{w_i'} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\underline{w})\, dw_m, \ldots, dw_1\, dw_i \qquad i = 1, \ldots, m. \tag{6.13}$$

Now let us consider the assumption of statistical independence of the loads. In this particular case, the equations given above can be simplified considerably. Thus Eq. 6.10 reduces to

$$F_p(\underline{w}') = F_p(w_1')\, F_p(w_2') \ldots F_p(w_m') \tag{6.14}$$

Eq. 6.11 reduces to

$$P(\underline{w}' \leq \underline{w} \leq \underline{w}' + \Delta\underline{w}) = P\Big[(w_1' \leq w_1 \leq w_1' + \Delta w_1') \cap (w_2' \leq w_2 \leq w_2' + \Delta w_2) \ldots \cap (w_m' \leq w_m \leq w_m' + \Delta w_m)\Big]$$

$$= P(w_1' \leq w_1 \leq w_1' + \Delta w_1') \times$$

$$P(w_2' \leq w_2 \leq w_2' + \Delta w_2) \times$$

$$\cdots$$

$$P(w_m' \leq w_m \leq w_m' + \Delta w_m)$$

$$\approx f(w_1')\, f(w_2') \ldots f(w_m') \Delta w_1 \Delta w_2 \ldots, \Delta w_m \quad (6.15)$$

Eq. 6.13 can be written as

$$P\left[w_i \leq w_i',\ \underset{\substack{j=1,\ldots m\\ j\neq i}}{w_j \leq \infty}\right] = \int_{-\infty}^{w_i'} f(w_i)\, dw_i \prod_{\substack{j=1,\ldots m\\ j\neq i}}^{m} \int_{-\infty}^{\infty} f(w_j)\, dw_j$$

$$= F_p(w_i') \quad (6.16)$$

Now consider the manifold $\underline{\overline{M}}$ obtained by the intersection of the hyper planes $\underline{\overline{M}}_i$. The probability that $\underline{w}$ lies in the region R enclosed by the coordinate planes and surface $\underline{\overline{M}}$ is given by

$$P(\underline{w} \in R \leq \underline{\overline{M}}) = F_p(w_1')\, F_p(w_2') \ldots, F_p(w_m')$$

$$= F_p(\underline{w}') \quad (6.17)$$

Also

$$P(\underline{w} \notin R \leq \underline{\overline{M}}) = 1 - F(\underline{w}') \quad (6.18)$$

Now consider that the Manifold $\underline{\overline{M}}$ is shifted to $\underline{\overline{M}} + \Delta\underline{\overline{M}}$ such that $\underline{w} = \underline{w}' + \Delta\underline{w}$. Then

$$P(\underline{w} \in R \leq \underline{\bar{M}} + \Delta \underline{\bar{M}}) = F_p(\underline{w}' + \Delta \underline{w}) \tag{6.19}$$

The probability that $\underline{w}$ lies in the region enclosed by $\underline{\bar{M}}$ and $\underline{\bar{M}} + \Delta \underline{\bar{M}}$ is given by

$$P(\underline{w} \in R,\ \underline{\bar{M}} \leq R \leq \underline{\bar{M}} + \Delta \underline{\bar{M}}) = F_p(\underline{w}' + \Delta \underline{w}) - F_p(\underline{w}') \tag{6.20}$$

Multivariate Normal Distribution: Let $\{F_1, \ldots, F_n\}$ be the random variables. Let $\{m_1, \ldots, m_n\}$ be the mean and $\{d_1, \ldots, d_n\}$ the standard deviation. The multivariate distribution of $\underline{F} = \{F_1, \ldots, F_n\}$ is normal and is given by:

$$f(\underline{F}) = \frac{\sqrt{|\underline{A}|}}{(2\pi)^{\frac{1}{2}n}} e^{-\frac{1}{2}(\underline{F} - \underline{m})^T \underline{A} (\underline{F} - \underline{m})} \tag{6.21}$$

where

$$\underline{F} = \{F_1, \ldots, F_n\}$$

$$\underline{m} = \{m_1, \ldots, m_n\}$$

A^{-1} is the covariance matrix which is positive definite. The covariance matrix is given by

$$A^{-1} = E\left\{(\underline{F} - \underline{m})(\underline{F} - \underline{m})^T\right\}$$

where $E(\cdot)$ represents the expected value of the event. A typical element d_{ij} of the matrix A^{-1} is given by

$$d_{ij} = E\left\{(F_i - m_i)(F_j - m_j)\right\}$$

Linear Addition of Normal Distribution: Let $w_1, \ldots, w_m$ be a set of statistically independent random variables with normal distributions having $\mu_1, \mu_2, \ldots, \mu_m$ as means of variables, and $\theta_1^2, \ldots, \theta_m^2$ as the variance respectively. The probability distribution of $b_j w_j$, where b_j is a constant, is also normal with $b_j \mu_j$ as mean and $b_j^2 \theta_j^2$ as variance. Let $F_1, \ldots, F_n$ be a set of random variables such that

$$[F] = [b_{ij}] [w] \qquad (6.22)$$

Let any component F_i be

$$F_i = \sum_j b_{ij} w_j \qquad (6.23)$$

The probability density function of F_i is the distribution of $\sum b_{ij} w_j$

$$m_i = b_{i1}\mu_1 + b_{i2}\mu_2 + \ldots b_{ij}\mu_n \quad \text{and} \qquad (6.24)$$

$$d_i = \sqrt{b_{i1}^2\theta_1^2 + b_{i2}^2\theta_2^2 + \ldots + b_{in}^2\theta_n^2} \qquad (6.25)$$

where m_i is the mean and d_i is the standard deviation.

In the formulation, $\underline{w}$ denote the peak value of load reached during the life time, and it is implied that a value $\underline{w}$ occurs only once during the life period. However, loading is a continuous process and the same combination $\underline{w}$ may repeat several times. This general phenomenon is not considered in this formulation.

6.5.6 Expected Cost of Failure and Cost-Effectiveness Criterion:

The expected cost of failure of each alternative action can be calculated as follows. Let $P(\underline{w})$ is the probability of the state of load $\underline{w}$ and $C_i(\underline{w})$ is the cost of failure of ith alternative at state $\underline{w}$. The expected cost of failure is given by

$$E_i(C_f) = \sum_{\underline{w} \in \overline{\underline{W}}} P(\underline{w})\ C_i(\underline{w}), \quad i = 1, \ldots, k \quad (6.26)$$

Adding to Eq. 6.8, the cost of ith alternative structure,

$$C_{Ti} = C_{si} + \sum_{\underline{w} \in \overline{\underline{W}}} P(\underline{w})\ C_i(\underline{w}), \quad i = 1, \ldots, k \quad (6.27)$$

Dividing with C_{si}

$$K_{Ti} = 1 + \sum_{\underline{w} \in \overline{\underline{W}}} P(\underline{w})\ K_i(\underline{w}), \quad i = 1, \ldots, k \quad (6.28)$$

Instead of summing the product for each $\underline{w}$, a regionwise summation is proposed in the later section.

6.5.7 Optimal Action:

The optimal action for the structure is that alternative which has the acceptable least value K_T. The absolute minimum of K_T is 1.0 corresponding to a case with no failure of any type, which is never sought for. The cost of structure C_s for such a case is ∞

6.5.8 Constraints:

The constraints that restrict the choice of the alternatives are the following:

(i) Constraints on the Force-Deformation Relations to be Chosen:

As in Chapter 5, the behaviour of the sections may be idealized to elastic (linear or nonlinear), elasto-plastic, or strain hardening. The type of idealization is reflected on the shape of the curve.

Some of the constraints on the shape due to the assumed idealizations are given below:

1. Elastic

$$\alpha_i(y) = \alpha_i = \text{constant}, \quad i = 1, \ldots, n \qquad (6.29)$$

2. Elasto-plastic

$$\left.\begin{aligned} \alpha_i(y) &= \alpha_i \quad y \leq y_{yield} \\ &= 0 \qquad y_{yield} \leq y \end{aligned}\right\} \quad i = 1, \ldots, n \qquad (6.30)$$

3. Nonlinearly inelastic

$$0 \leq \alpha_i(y) \leq \alpha(y - dy) \qquad (6.31)$$

and $\alpha_i(y) = \alpha_i\big|_{y=0}$ for $F_i \cdot dF_i \leq 0, \; i = 1, \ldots, n$

In order to have the serviceability requirements for normal load conditions, the force-deformation relations chosen must contain the task curves obtained in Chapter 5 as a part

of them. Thus

$$F_i = F_i^*(y_i^*) \quad \text{for } y_i \leq y_i^c,$$
$$\qquad\qquad\qquad i = 1, \ldots, n \quad (6.32)$$
$$= F_i(y_i) \quad \text{for } y_i \geq y_i^c,$$

where y_i^c is the maximum value reached by the ith deformation in the serviceability design.

This condition will give a basis for choosing the alternative actions, or else, a search in a vast ocean of alternative actions would have been necessary. The decision carried out for serviceability requirements limit the choice from a vast set to a few alternatives. Further the constraints on shape offers to restrict the alternatives.

(ii) Equilibrium Equations: As given in Chapter 5, the equilibrium equations can be written as

$$R_i(\underline{F}, \underline{w}) = 0, \quad i = 1, \ldots, p \qquad (6.33)$$

There will be p independent equations connecting n forces and m loads.

For a statically determinate system, n is equal to p. Hence for any given $\underline{w}$, a force vector $\underline{F}$ can be found out. Thus for any $\underline{w}$ in $\underline{W}$ space, there is a unique $\underline{F}$ in $\underline{F}$ space. For a region in $\underline{W}$ space, there is a corresponding region in $\underline{F}$ space.

In the case of statically indeterminate system, $n > p$. For any $\underline{w}$ in $\underline{W}$ space there can be many $\underline{F}$ in $\underline{F}$.

However compatibility constraints offer a surface $\underline{\bar{C}}$ in $\underline{\bar{Y}}$ space corresponding to which there will be a surface mapped in the $\underline{\bar{F}}$ space by force-deformation relations. The $\underline{F}$'s chosen in the $\underline{\bar{F}}$ space must belong to the surface $\underline{\bar{C}}$.

(iii) Compatibility Constraints: The compatibility equations and displacement boundary conditions together can be represented by the equations

$$K_i(\underline{Y}) = 0, \quad i = 1, \ldots, q \tag{6.34}$$

These equations define a manifold of dimensions less than n in the Euclidean n-space $\underline{\bar{Y}}$. It restricts that the deformation vector $\underline{Y}$ must lie on the surface $\underline{\bar{C}}$.

If one or more sections, yield or break, the form of the compatibility equations changes. Correspondingly, the surface defined will also change. It is required to define the new surfaces created by such yielding or fracture of sections.

6.5.9 Significance of Failure Modes:

The significance of the various failure modes can be explained for our formulation as follows:

(a) Unserviceability: As described in Chapter 5, the serviceability requirements on deflections and rotations, define a manifold $\underline{\bar{B}}$

$$Q(\underline{Y}) = 0 \tag{6.35}$$

in the $\underline{Y}$ space. <u>If the point y that represents the deformed state at any stage of loading falls outside this manifold, serviceability failure is assumed to occur.</u> When the manifold $\underline{C}$ due to compatibility constraints is also present, the intersection of $\bar{\underline{C}}$ and $\bar{\underline{B}}$ define a boundary in $\bar{\underline{C}}$ beyond which serviceability failure is indicated.

(b) <u>Structural Damage or Failure</u>: The various failure modes of structure are presented in Section 4.3. The significance of these modes in the present formulation of the problem in the three spaces can be explained as follows.

The brittle fracture of a cross section at any force level indicates the termination of the force-deformation relations at that force level. With this the corresponding forces and deformations cease to exist.

If a cross section yields with respect to any force-deformation relation, the mapping of the deformation in the $\bar{\underline{Y}}$ space corresponding to force F_{yield} is not unique. The same F_{yield} may be mapped on to more than one y. The fracture or yielding can be conveniently be represented by hyper planes in the force space. Consider that the ith force-deformation relation indicates yielding or fracture of the section when the force reaches a value F_i^*. In other words, it is the maximum capacity of the section with respect to that force-deformation relation. It shows that hyper plane, keeping $F_i = F_i^*$, may be defined in the force-space which

divides the space into two regions. The plane is normal to the F_i axis. No force $\underline{F}$ lying on the side of the plane not containing origin (called outside) can occur on the structure, In statically determinate system if $\underline{F}$ is outside the hyper plane, failure is certain. This concept is made use of in Section 6.6.2 to define region of collapse in statically determinate system.

In statically indeterminate system, as soon as the force vector reaches the hyper plane, the force-deformation relation $F_i = F_i(y)$ ceases to exist when the section fractures. It denotes a reduction in the dimension of both force and deformation spaces by one and the problem is to be redefined in the lower dimension space. When the section yields, the force vector starts moving along the hyper plane in which case the surface is a barrier to the force-vector $\underline{F}$.

In the deformation space also corresponding changes occur. As in the force space, a fracture causes a reduction in the dimension of deformation space. In the case of yielding the component y_i become undefined. The deformation vector moves parallel to the y_i axis. In statically determinate system, as the deformation becomes uncontrolled, the motion of $\underline{y}$ denotes the collapse of the structure. Statically indeterminate structures respond to the yielding or fracture of a member in a different way. As in force space, whenever a section fails by brittle fracture, a reduction

in dimension of deformation space occurs. In the lower dimension space the deformations may be constrained by a new manifold $\underline{\bar{C}}^*$ due to the compatibility condition. With yielding, there is no reduction in the dimension of space. But, the deformation vector causes to move along $\underline{\bar{C}}$. Instead a new manifold $\underline{\bar{C}}^*$ of lower dimension is formulated and the force-vector is constrained to move along the surface $\underline{\bar{C}}^*$. Since the vector is still controlled on the surface, failure may not occur at this stage.

Now the collapse of statically indeterminate systems can be explained as follows. If the structures fail by successive brittle fracture of members, the problem is to be redefined in lower dimension at every time a failure occurs. This reduction in dimension continues until the dimension becomes n-r when the structure is equivalent to a statically determinate system. Any further reduction indicates that failure is certain. Corresponding to the reduction in dimension, a reduction in the manifold $\underline{\bar{C}}$ also occurs.

In the case of yielding, the reduction in the dimension of manifold continues as the members yield one by one until the dimension becomes zero. Further yielding of any section causes collapse.

If the reduction in dimension of either the space or the manifold is not one after another and occurs simultaneously, failure is sudden without any warning. Also in certain indeterminate systems, failure of one or two members may result

in complete collapse. It denotes that the yielded component y_i cannot be controlled by forcing the vector on to the manifold $\bar{\underline{C}}^*$ of reduced dimension. Consequently the vector jumps out of the manifold $\bar{\underline{C}}^*$ causing uncontrolled motion of $\underline{y}$ resulting in a failure.

The above concepts may be made use of in formulating a cost-effectiveness model for statically indeterminate systems.

The degree of warning or the duration of period between first yielding and complete collapse depends not only on the structural behaviour, but also on the duration and manner of loading. This latter aspect of the problem is not considered in this formulation. A structure is said to fail with warning if the members fail gradually one after another with visual signs of yielding, cracking etc.

6.6 DESIGN OF STATICALLY DETERMINATE STRUCTURES

For purpose of design, the problem can be divided into two types:

(i) Problems in which no compatibility constraint is present (Problems related to statically determinate structures).

(ii) Problems in which compatibility constraints are present (Problems related to statically indeterminate structures).

In this section, the solution of the first type of problem is dealt with. An iterative solution, herein called

policy iteration procedure, is formulated and applied to a two bar truss. The formulation is general and applicable to any statically determinate system with discrete variables. A set of force-deformation relations belonging to the selected cross sections form a policy of the system. The policy is optimal if the cost-effectiveness criterion for that policy is acceptable optimum.

6.6.1 Iteration Process:

The iteration process consists of the following steps. Consider that a set of force-deformation relations are initially chosen. Each relation, must contain as a part of it, the force-deformation relations obtained by the serviceability design.

(a) Policy Evaluation Operation: For any set of force-deformation relations, whether assumed initially or obtained after a policy improvement routine (as given below), the cost-effectiveness factor K_T is evaluated. It gives the merit of the policy.

(b) Policy Comparison Operation: The policies evaluated in two successive trials are compared. The one with minimum value is retained, and fed into the policy improvement routine for further improvement.

(c) Policy Improvement Routine: The policy is improved in this routine by suitably changing the shape of force-deformation relations (This operation is done by adjusting $\underline{\alpha}(y)$ and $\underline{y}^d$).

The operations are schematically shown in Fig. 6.2. The three operations are cyclically carried out, until an acceptable minimum value of K_T is obtained.

6.6.2 Policy Evaluation Operation:

It is the most complex operation that is to be carried out. It consists of the assessment of failure modes, if the particular set of force-deformation relations are chosen for design, the assessment of the cost of failure associated with each mode of failure, the identification of the regions in the load, force or deformation space which represent the various failure modes and the probability of load $\underline{w}$ occurring in these regions. It consists of the following computations and operations.

(i) Cost Evaluation: For the policy or strategy under consideration, the approximate initial cost C_s is found out. As in the example described in Section 6.4, the cost may be assumed proportional to the maximum forces in the members or so.

(ii) Listing the failure modes: The failure modes are listed for the policy under consideration. For a statically determinate structure, three regions or states of the structure may be identified.

(a) Serviceable State: It is the region denoted by the constraint Eq. 6.34.

(b) Unserviceable State: If the constraint Eq. 6.34 is

violated, the structure is said to be in unserviceable state.

(c) Collapse Region: It is the collection of that states of loads $\underline{w}$ for which at least one member will fail. A manifold in force space that separates the region of collapse from the region of survival (serviceable and unserviceable states) is derived in this section.

The three regions are mutually exclusive. Any single load vector $\underline{w}$ cannot cause more than one state. This mutual exclusiveness is true for only static and quasistatic loads. In the case of impact, repeated or dynamic loads the three states may overlap. This case is not considered in this study.

(iii) Evaluation of the Cost of Failure: The costs of failure may be evaluated by methods described in Chapter 4. Let the costs be as follows:

1. Serviceability $C_1 = 0$
2. Unserviceability $C_2 = C_{sc}$
3. Collapse without warning $C_3 = C_c$

C_{sc} and C_c are described in Sections 4.3 and 4.4

(iv) Evaluation of Probability of States: The probability of the three states of nature are evaluated. Let p_1, p_2, p_3 be the probabilities of serviceable, unserviceable and collapse states. To find the probabilities, the regions in the load space that cause each state of the structure may be identified.

Then p_1, p_2 and p_3 are the probabilities of load $\underline{w}$ occurring in regions that cause serviceability, unserviceability and collapse respectively.

(a) Probability p_1 of Serviceable State of Structure:

The state of serviceability is assumed to be assured by the design. By game theory approach to design described in Chapter 7, it is possible to design a structure that is serviceable for any combination of load $\underline{w}$ so long as the load lies within the normal load condition defined by $\underline{W}$ as given in Eq. 5.6. Let $\underline{M}_s{}^W$ be the manifold defined by the upper limits to normal load condition

$$\underline{M}_s{}^W = \bigcap_{i=1}^{n} \underline{\bar{M}} \left\{ \begin{array}{ll} w_i = W_i, & w_j \leq \infty \\ & j = 1, \ldots, m \\ & j \neq i \end{array} \right\} \tag{6.36}$$

p_1 is the probability that $\underline{w}$ is in the region that contains the origin separated by $\underline{\bar{M}}_s{}^W$. From Eq. 6.17,

$$p_1 = P(\underline{w} \in R \leq \underline{\bar{M}}_s{}^W) = F_p(\underline{W}) \tag{6.37}$$

(b) Probability p_3 of Collapse State: The collapse state may be identified by defining a manifold $\underline{M}_c{}^F$ in force-space as follows.

Consider the set of force-deformation relations $\underline{F} = \underline{F}(y)$ that is taken for evaluation of the cost-effectiveness.

Probability density function of F_i is the linear additive distribution of $\underline{w}$ obtained using Eq. 6.36. Since normal distributions are assumed for load $\underline{w}$, the distribution of $\underline{F}$ will also be normal. The method of addition of distributions is given in Section 6.5.5.

Since normal distribution is assumed for the random load vector $\underline{w}$, the forces $F_i(i = 1,\ldots,n)$ calculated from them are also normally distributed. However, the components of the force vector $\underline{F}$ are not statistically independent and a componentwise integration as that for load $\underline{w}$ is not possible. Therefore the multivariate distribution of $\underline{F}$ is to be found out. Let $f(F_1, \ldots, F_n)$ be the multivariate density function of $\underline{F}$. Since the components $F_i(i = 1,\ldots,n)$ are normally distributed the function $f(\underline{F})$ is also normally distributed (146). The multivariate distribution of n variables $\underline{F}$ is given by Eq. 6.21. The marginal distribution is

$$P\left[F_i \leq F_i^*,\ \underset{\substack{j=1,\ldots,n\\ j\neq i}}{F_j \leq \infty}\right] = \int_{-\infty}^{F_i^*}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} f(\underline{F})\ dF_1,\ldots dF_{i-1},\ dF_{i+1},\ldots dF_n,\ dF_i \tag{6.42}$$

$$P\left[\underline{F} \in R \leq \underline{M}_c^F\right] = P\bigcap_{i=1}^{n}\left(F_i \leq F_i^*,\ \underset{\substack{j=1,\ldots,n\\ j\neq i}}{F_j \leq \infty}\right) \tag{6.43}$$

$$p_3 = P\left[\underline{F} \notin R \leq \underline{M}_c^F\right] = 1 - P\left[\underline{F} \in R \leq \underline{M}_c^F\right] \tag{6.44}$$

The evaluation of the probability p_3 can be done, in general,

by means of numerical integration.

(c) Probability p_2 of Unserviceability:

$$p_2 = 1 - (p_1 + p_3) \tag{6.45}$$

(v) Evaluation of Cost-Effectiveness: The cost-effectiveness factor K_T is given by

$$\begin{aligned} K_T &= 1 + p_1 \frac{C_1}{C_s} + p_2 \frac{C_2}{C_s} + p_3 \frac{C_3}{C_s} \\ &= 1 + p_1 K_1 + p_2 K_2 + p_3 K_3 \end{aligned} \tag{6.46}$$

6.6.3 Policy Improvement Routine:

After comparison of two successive policies, the one with less value of K_T is improved further by changing the shape of force-deformation relations. A systematic procedure would help to have quicker convergence. It is easier to consider a case in which the factor K_T is maximum and then to improve successively the curves so that the factor K_T gets reduced. In statically determinate systems, it is immaterial whether the section yields or fractures. Further the deformations beyond normal load state are not of importance. The

force level in each relation is the important consideration for collapse. However, for consistency and to maintain the similarity with the force-deformation relations that exist in practice, the constraints given by Eqs. 6.29-6.31 must be taken into consideration. Further in the policy improvement routine, the portion of curves obtained for the normal load conditions should not in any case be changed, as it would violate the serviceability requirement.

6.6.4 Example of Two Bar Truss:

The two bar truss designed in Chapter 5 for serviceability is further designed in this section for cost-effectiveness by the policy iteration procedure discussed in the previous sections. In order to illustrate the importance of cost-effectiveness model in fixing the safety level, two separate cases are studied. In one case the cost of collapse is considered as the cost of structure only. In the second case a higher cost of collapse (Rs. 5,000/- in addition to the cost of structure) is taken.

The truss is as shown in Fig. 5.11.a. It is subjected to two loads w_1 and w_2 as shown in the figure. The following data are given:

(i) The truss is designed for serviceability requirements when the load is less than or equal to 5 tons and 4 tons

$$0 \leq w_1 \leq 5\ \mathrm{T}$$
$$0 \leq w_2 \leq 4\ \mathrm{T}$$

The force-deformation relations for the two members under this condition are obtained as in Fig. 5.11.g and h.

(ii) The loads w_1 and w_2 are random. The probability density functions are given as follow:

For w_1, the distribution is $N(4, 0.78)$

For w_2, the distribution is $N(3, 0.78)$

The limit values 5 tons and 4 tons correspond to 90 % probability.

(iii) Cost of unserviceability

C_{sc} = loss in the input cost + loss in the benefit

= 0 + Rs. 500

= Rs. 500/-

(iv) The cost of collapse

Two cases are considered

<u>Case 1</u>: $C_c = (F_1 + F_2)10$ where F_1 and F_2 are the forces in the members.

<u>Case 2</u>: $C_c = (F_1 + F_2)10 + \text{Rs.}5000$

Let F_1 and F_2 be the forces. From equilibrium equations

$$F_1 = 0.707\ w_1 + 0.707\ w_2$$

$$F_2 = 0.707\ w_1 - 0.707\ w_2$$

<u>Calculation of Probabilities:</u>

$$P\left\{(w_1 \leq 5T,\ w_2 \leq \infty) \cap (w_1 \leq \infty,\ w_2 \leq 4T)\right\} = F_p(5)\ F_p(4)$$

$$= 0.81$$

The probability distributions of F_1 and F_2 may be obtained from that of w_1 and w_2 as follows:

Let $N(m_1, d_1)$ and $N(m_2, d_2)$ be the distributions of the F_1 and F_2 respectively.

$$m_1 = 0.707\ (4 + 3) = 4.949$$

$$m_2 = 0.707\ (4 - 3) = 0.707$$

$$d_1 = d_2 = \sqrt{\tfrac{1}{2} \times 0.78^2 + \tfrac{1}{2} \times 0.78^2} = 0.78$$

The distributions of F_1 is $N(4.949, 0.78)$ and that of F_2 is $N(0.707, 0.78)$.

The bivariate distribution function can be given as follows (145):

$$f(F_1, F_2) = \frac{1}{2\pi\ d_1\ d_2 \sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\frac{(F_1-m_1)^2}{d_1^2} - 2\rho\frac{(F_1-m_1)(F_2-m_2)}{d_1\ d_2} + \frac{(F_2-m_2)^2}{d_2^2}\right]\right\}$$

where ρ is the correlation coefficient between F_1 and F_2 given by

$$= \rho_{12} = \frac{d_{12}}{d_1\ d_2}$$

Evaluation of : $d_{12} = E(F_1 \; F_2) - E(F_1) \; E(F_2)$

$$= E\left\{\frac{w_1^2 - w_2^2}{2}\right\} - E\left\{\frac{w_1 + w_2}{\sqrt{2}}\right\} E\left\{\frac{w_1 - w_2}{\sqrt{2}}\right\}$$

$$= \frac{(\theta_1^2 + \mu_1^2) - (\theta_2^2 + \mu_2^2) - (\mu_1^2 - \mu_2^2)}{2} = \frac{\theta_1^2 - \theta_2^2}{2}$$

$$d_1 \; d_2 = \sqrt{\text{var } F_1} \; \sqrt{\text{var } F_2} = \sqrt{\text{var } \left(\frac{w_1 + w_2}{\sqrt{2}}\right)} \; \sqrt{\text{var}\left(\frac{w_1 - w_2}{2}\right)}$$

$$= \sqrt{\frac{\theta_1^2 + \theta_2^2}{2}} \; \sqrt{\frac{\theta_1^2 + \theta_2^2}{2}} = \frac{\theta_1^2 + \theta_1^2}{2}$$

$$\rho = \frac{\theta_1^2 - \theta_2^2}{\theta_1^2 + \theta_2^2}$$

Knowing ρ, the values related to the bivariate distribution can be obtained from tables given in Ref. 146. In the problem under consideration $\rho = 0$ since $\theta_1 = \theta_2$. Hence F_1 and F_2 are statistically independent. The probabilities are taken from the tables of normal distribution. Fig. 6.3 illustrates the planes separately the various regions of serviceability, collapse etc.

p_3 , is calculated for various trials and is entered in Table 6.2. A sample calculation is shown below.

$p_3 = 0.0059059$, $p_1 = 0.800041$, $p_2 = 1 - p_1 - p_3 = 0.1940531$

$K_T = p_2 \times K_2 + p_3 \times K_3 = 2.11728$ for Case 1.

From Table 6.2, it is seen that a section with higher strength (8.0) than in Case 1 has to be resorted to in Case 2 in order to get the same difference of K_T between two successive trials.

6.7 POSSIBLE EXTENSION TO THE DESIGN OF STATICALLY INDETERMINATE STRUCTURES

Certain steps regarding the possible extension of the procedure given in Section 6.6 to statically indeterminate structures are considered in this section. No detailed analysis is carried out. The procedure of design in this case is more involved due to the presence of compatibility constraints. A simultaneous consideration of all the three spaces, (load space, force space and deformation space) is necessary. Further, the identification of regions of failure are more difficult than in a statically determinate system.

6.7.1 Iteration Process:

The iteration process is similar to that for statically determinate structures described in Section 6.6.1. However, the policy evaluation operation is more complicated and greater care is needed in the policy improvement routine.

6.7.2 Policy Evaluation Operation:

In this section, only those aspects that deserve special considerations are discussed while other operations remain same as that for statically determinate structures.

(i) Failure Modes, Costs of Failure and Probabilities: The failure modes and the associated probabilities and costs of failure are given below. Two separate cases are to be considered;

(a) a gradual collapse with warning

(b) a sudden collapse without warning.

Case 1: Collapse With Warning:

No.	State of Structure	Cost of Failure	Probability of Failure
1.	Serviceable	$C_1 = 0$	p_1
2.	Unserviceable	$C_2 = C_{sc}$	p_2
3.	Damage of ith type	$C_3^{\,i} = C_d^{\,i}$	$p_3^{\,i}$
4.	Collapse	$C_4 = C_w$	p_4

Case 2: Collapse Without Warning:

1.	Serviceable	$C_1 = 0$	p_1
2.	Unserviceable	$C_2 = C_{sc}$	p_2
3.	Collapse	$C_3 = C_c$	p_3

(ii) Evaluation of Probabilities: The evaluation of probabilities is too cumbersome, though not impossible. A systematic procedure may be developed using the concepts of the three spaces, the manifolds and the reduction of dimension of spaces and manifolds. The possibilities of such an approach are worth studying. However, it is not attempted in this investigation to study the problem in detail.

6.7.3 Policy Improvement Routine:

The process of improvement of the policy is

similar to that for statically determinate systems. However, two important aspects must be taken note of. Firstly, the number of alternatives to be searched in this case is enormous and unless a well developed procedure is adopted to improve the policy, there are chances of missing some optimal policies. Secondly, the deformations outside the serviceable state are also important due to redundancy and by proper choice of the stiffnesses $\underset{\sim}{\alpha}$ better economy may be obtained.

6.8 DISCUSSION

The direction design method for cost-effectivenss, proposed in this chapter, has certain special features as discussed below.

(i) The method is essentially a decision approach in which the structure is assumed to decide its course of action (represented by force-deformation relations) under risk. The risk is due to the random nature of loading on the structure which is in excess of the 'normal design' load.

(ii) The design trades off the cost of structure to the expected cost of failure under abnormal loads which is a measure of the structural effectiveness as shown in Chapter 4. Instead of arbitrarily specifying a safety level, the required safety of the system is assessed considering the associated consequences of failure and the cost of the structure. However, the safety and serviceability under normal load conditions are not subjected to any trade off and the trade off is effected

only for abnormal loads.

(iii) Inelastic reserve strength is partly made use of to resist the abnormal loads especially when warning is required before failure.

(iv) The example of the three bar system shows that ductility in terms of the deformation capacity of a section is also obtained as an ouput. by this method. Thus safety, serviceability, ductility and economy are incorporated in the cost-effectiveness criterion of design.

(v) Example of three bar truss shows the superiority of a cost-effectiveness criterion over minimum weight design. A Mitchell structure, which is the ideal minimum weight design for a scheme of loading as given is a single bar system. The cost-effectiveness criterion shows that a three bar system is best suited as it can give enough warning before failure. To illustrate this phenomenon, a very high cost of failure is taken in this problem. It points to the fact that in civil engineering structures where weight is not an important factor, cost and cost-effectiveness must be the design criterion. However. inaccurate the estimation of the cost may be, the design may give a better attention to the requirements of design. If weight is also an important factor, a trade off between cost-effectiveness and weight may be resorted to. Such a procedure would lead to Pugsley's concept of a well-balanced structure discussed in Chapter 1.

(vi) The probability of states considered here is different from the concept of probability of failure. In this case the strength and behaviour of the structure is considered deterministic and the load probabilistic. If structural behaviour is also considered random, the problem may not be as simple as the way considered in the procedure discussed here due to the nonlinearity of response. The nonlinear structural behaviour is shown in Chapter 10 as stochastic process. The randomness in the structural behaviour is considered in the subsequent decision of the choice of structural members described in Chapter 10. In the proposed method, a numerical value of the probability of failure is not sought for.

(vii) No allotment of probabilities of failure to various members is needed.

(viii) The distribution of force vector $\underline{F}$ is obtained as the resultant distributions of loads $\underline{w}$ evaluated through equilibrium equations. Any error in the estimation of the density functions of load $\underline{w}$ may partly be compensated by the addition of distributions (59). However, the error in the estimation of the density function is considered as an uncertainty problem and is compensated for through a statistical game formulation in Chapter 9.

(ix) The performance of a design for serviceability under normal load conditions helps the cost-effectiveness design in several ways. It allows to separate the state of normal

load condition for which no trade off is permitted, as they are considered at 'abnormal' levels of load. The evaluation of the probability of serviceable state becomes easy as seen from Eq. 6.37. Further, the initial portion of curves obtained gives a basis to start the search for an optimal set of force-deformation relations.

(x) The concepts of three spaces (force, load and deformation) manifolds (representing compatibility conditions), reduction of dimension of space and manifold etc. give an insight into the process of failure. Also it offers some basis by which the regions can be divided for various states, the different types of failure can be interpreted and the probabilities can be evaluated.

(xi) The example of three bar system illustrates further that a design of statically indeterminate system becomes unique under a cost-effectiveness criterion. It further shows that a design for safety of statically indeterminate system under single load condition is not always unique and for unique solution additional conditions like minimum weight or cost-effectiveness are to be imposed.

(xii) The example of two bar truss illustrates how the behavioural requirement of structures changes with the change in the consequences of failure. For a higher cost of failure, a set of force deformation relations showing greater capacity is needed.

6.9 SUMMARY

This chapter presents a direct method of design for cost-effectiveness under risk. The complete range of force-deformation relations can be obtained by this method following a policy iteration procedure.

This method forms a basis for direct design when the probability density functions are exactly known. Further it forms a basis for the extension to take the uncertainty in the probability density functions into account.

The next chapter presents the outline of the direct inelastic design under uncertainty.

CHAPTER SEVEN

OPTIMUM INELASTIC DESIGN UNDER UNCERTAINTY

7.1 INTRODUCTION

The methods of design developed in Chapters 5 and 6 are intended to provide a basis for design for serviceability under normal load conditions, and for safety and economy (in the form of cost-effectiveness) under abnormal load conditions. In both the cases, no uncertainty in the information of design data was considered. In the present chapter and the following three chapters design is considered as a design under uncertainty, and the outline of a methodology for optimum proportioning for structures of inelastic materials is presented. The concepts and methods developed in the previous chapters offer the basic framework of design.

The complete outline of a method of design is presented in this chapter, and the details are worked out in the subsequent three chapters. In this decision process, the structure is assigned the status of a decision maker, operating under the conditions of uncertainty. Structural action is assumed to be a game played by the structure against nature, and a competitive situation is created. The uncertainties of various origin are taken up in stages. In the proposed game, structure is assumed to select its own strategies namely the cross sectional properties, so that serviceability is achieved under normal conditions of loading, and optimum cost-effective-

ness under abnormal loading conditions, which trades off the cost of structure and the safety level required.

7.2 STATEMENT OF THE DESIGN PROBLEM

The designer is provided with the configuration of a structural system, including the arrangement of members, and types of connections. It is required to choose the material, and geometry of the member cross sections. The following assumptions are made in the proposed design methodology:

(i) The limit values of loads which separates the normal and abnormal load conditions are known. The structure is to be serviceable within these limit values.

(ii) When the loads are within the limit values, the scheme of loading is assumed to be uncertain. These are the strategic uncertainties discussed in Chapter 4 (sequence, timing, direction and position of loads).

(iii) Loads are assumed to be of random magnitudes, the abnormal values of which are acting in random combinations. The probability density functions of these random phenomena are in general not exactly known. However, it is presumed that the bounds within which the density functions lie are known. Within these bounds the occurrence of all functions are equally likely.

(iv) The structural behaviour of the cross sections represented by the force-deformation relations is also random

due to the randomness of the material behaviour and cross sectional dimensions.

(v) A good deal of non-measurable uncertainty exists due to imperfect knowledge, design idealizations and assumptions and inaccuracies in the computation, and due to constructional and operational errors.

(vi) The material of the structure may be nonlinearly inelastic with or without creep effects.

(vii) It is assumed that environmental conditions other than loads remain unchanged, and do not affect the structural performance.

(viii) The loads are of the quasistatic type, and no fatigue and dynamic effects are considered.

(ix) Deformations are small.

(x) Connections are assumed to be so strong that no failure occurs due to the failure of connections.

The design requirements are the following:

(i) The structure is to be serviceable within the normal load conditions. No damage of any type is allowed, and no compromise on serviceability requirements are permitted within the normal load conditions.

(ii) Once the limit values that bounds the normal load conditions are exceeded, the emphasis is more on safety and overall economy than on serviceability. Safety and economy

are linked together by a cost-effectiveness criterion as developed in Chapter 4.

(iii) Failure of the structure is not to be defined in terms of a single mode like 'weakest link' or 'fail safe'. All types of possible failure modes with the loss associated with each failure, are to be considered.

When all the three requirements given above are satisfied, the resulting design is considered optimal with regard to safety, serviceability, ductility and economy.

7.3 A STAGE BY STAGE DECISION PROCESS

The problem as stated above is too complex to be dealt with in a single stage of decision. Hence a stage by stage decision process is proposed in which the uncertainties of different origin are dealt with in stages.

7.3.1 Present and Proposed Method; A Comparison:

This process differs from the current practice of dealing with uncertainty. At present, uncertainties are accounted by any one of the following ways:

(i) By means of factor of safety or load factor in the deterministic approach to design.

(ii) By means of partial safety factors in the CEB approach (26).

(iii) By judgemental probabilities in the extended

reliability approach (27).

(iv) By probabilistically evaluated safety factors (32).

Turkstra (22), in 1962, treated for the first time structural design as a decision under uncertainty, and a Bayesian approach was proposed. Similar studies were made by Benjamin (23), and Sexmith (25). In this work, structural design is considered as a decision under uncertainty. Unlike the decision approaches of Turkstra, Benjamin and Sexsmith, the structure itself is simulated to be a decision maker participating in a game. The structure reacts to the loads acting on it, the magnitudes and the loading process of which are uncertain. Further, the behaviour of the structure itself is uncertain.

7.3.2 Proposed Direct Design Method:

As in Chapters 5 and 6, the force-deformation relations of the cross sections are taken as the central decision parameters, based on which the stage by stage operations are carried out as follows:

Stage 1: Design for serviceability under strategic uncertainty of loading: Design for serviceability is carried out in this stage. Consider that the force-deformation relations are deterministic, and the loads are within the limit values. The uncertainties are of the strategic type discussed in Chapter 4. Structure decides its optimal set of force-deformation relations under the uncertain behaviour of loading

such that serviceability requirements are satisfied. This is the first step shown in the flow diagram in Fig. 7.1. The portion of force-deformation relation constructed in Stage 1 is shown in Fig. 7.2.

<u>Stage 2</u>: <u>Design for cost-effectiveness under statistical uncertainty</u>: Design for optimum cost-effectiveness is carried out in this stage. As in Stage 1, force-deformation relations are assumed to be deterministic, but the loading is assumed to be probabilistic. The uncertainties are of the statistical type pertaining to the loads. In this stage, structure completes the force-deformation relations already obtained in Stage 1, extending them for the abnormal load conditions. The choice corresponds to the minimum cost-effectiveness of the system given by:

$$K_T = 1 + \sum_i p_i K_i \qquad (7.1)$$

where K_i is the cost of the ith mode of failure expressed as a fraction of the initial cost of the structure. This stage is schematically shown in the second block in Fig. 7.1. The curves obtained as the output are called the task curves and are shown in Fig. 7.2.

<u>Stage 3</u>: <u>Design for non-measurable uncertainties and shifting of task curves</u>: Three marginal factors r_s, r_c and r_d are decided by a decision process based on preferences or otherwise. The factors are assumed to represent the non-

measurable uncertainties that may arise in the design process. The task curves are shifted to conservative positions using the marginal factors as shown in Fig. 7.3. The third block in Fig. 7.1, and the thick curve in Fig. 7.2 indicates the operations and the results involved in this stage.

Stage 4: Selection of material and cross section for cost-effectiveness: The material and geometry of the cross sections are decided at this stage. The material behaviour is assumed to be a stationary Markov process with unknown probabilities, as suggested in Chapter 4. The other requirements are:

(i) The cost effectiveness criterion

$$C_T = C_s \, (1 + \sum_i p_i K_i) \qquad (7.2)$$

is a minimum with respect to C_s, keeping p_i constant at the values obtained in Stage 2.

(ii) The force-deformation relations of sections corresponding to the minimum C_T must coincide with the shifted task curves obtained in Stage 3. This stage is shown in fourth block of Fig. 7.1.

The method is called a direct design method as the decisions are direct, directly leading to the final choice of sections.

A design is complete only when all the four stages given above are complete. The design method in each stage is

7.4 STRUCTURAL ACTION GAME

The decisions in stages one and two, are based on the assumption of a structural action game. According to this assumption, structural action is simulated as a game played by structure against nature. This game differs from the structural design game of Turkstra (22), in which design is assumed to be a game played by designer against nature. It is a 'pessimistic' approach and the safety levels are fixed considering the above.

7.4.1 Description of the Game:

The game assumed here is a two player game. Player 1 is the nature, which causes various loads, externally applied displacements and other environmental effects to act on the structure. Player 2 is the structure, which reacts to the aggressive moves of player 1, by deforming and being stressed. The game is assumed to be zero-sum, that is, the loss of one player is the gain of the other. The payoff that measures the loss or gain will be different in Stage 1 and Stage 2, which will be discussed later. The strategy of the structure is the selection of the cross sections in terms of the force-deformation relations they possess. The strategies of nature are the uncertainties in the loading process; statistical or strategic. Though nature is capable of acting in an unlimited way, it is assumed that bounds within which the nature acts are known

through experimental study. It is further assumed that both structure and nature act rationally to better their gains within the bounds specified. In the case of structure such an assumption is valid as no uncertainty or randomness is considered. Because of the assumption, a situation in which two parties react with conflicting interest, is created; a situation which can be studied by Von Neumann's theory of games (85). Since the loading is a continuous process, the game considered is a multi-stage game extending over the entire service life of the system. The sequence of moves made by both parties, during the life of the structure is called a <u>play</u> of the game. The period, during which the play takes place, is called the <u>period</u> of the play. At each instant of the play, the two parties, have to choose their action out of the various alternatives available to them. The complete set of moves that each player chooses in the course of a play is called the <u>strategy</u> of the player. In a game, the players can plan their strategies in advance. The forces and stiffnesses in each of the force-deformation relations together constitute the strategy of the structure. Structural design is the selection of these strategies to better its own benefit when an equally interested antagonist is trying to minimize the benefit. This would, as seen later, lead to a conservative design within the bounds in which the nature's action is uncertain.

During service life, only a single play takes place. But in the design of structures there are two requirements to

be met with. They are

(i) The serviceability is to be guaranteed for the normal load conditions.

(ii) Optimum cost-effectiveness is to be achieved.

The real single play of the game is, therefore, replaced by two imaginary plays which are to be played one after another with two different payoffs. The plays are designated as 'normal play', and 'survival play' respectively.

7.4.2 Normal Play:

This play constitutes the decision process in Stage 1 given in Section 7.3. The purpose of the play is to guarantee the serviceability requirements. The magnitudes of the loads acting on the structure do not exceed the limit values. This load condition is called the normal load condition as defined in Chapter 4. These limits are deterministic quantities generally specified by codes, or may be the characteristic values of loads defined by CEB, which are the loads with a specified probability of exceedence. In this play, the strategic variations of loads constitute the strategy of nature. Any variation in the course of loading is assumed to be equally likely, as no compromise on the safety and serviceability is allowed during normal load conditions. The optimal solution of the game gives the force-deformation relations (task curves) for the normal load conditions. The formulation of the game made as a differential game problem is given in Chapter 8.

7.4.3 Survival Play:

This is to be played on completion of the normal play, and it constitutes the decision model for Stage 2. The purpose of this play is to assure safety with least expected cost of failure, when loads of abnormal magnitudes act. The force-deformation relations obtained in the normal play are suitably extended to meet this requirement. The emphasis is on the economy of the system and its failure. Loads are considered to be random with unknown probabilities. The unknown probability density functions are assumed to be between two bounds obtained through experiments. Within the bounds, the occurrence of any function is assumed to be equally likely. The functions that lie between the bounds form the strategy of nature. The play is called survival play as the payoff is the penalty for the survival of the structure. The payoff is the cost-effectiveness factor given by Eq. 7.1. The detailed formulation is presented in Chapter 9.

7.5 EVALUATION OF MARGINAL UNCERTAINTY FACTORS

The decision in Stage 3 consists of the evaluation of the marginal uncertainty factors that takes into account the non-measurable uncertainties.

7.5.1 Non Measurable Uncertainties:

Game theory is a versatile tool to arrive at pessimistic decisions when uncertainty exists. But theory of games,

or in general, all mathematical models for decision making, presumes that all the possibilities or alternatives are known to the decision makers (players) before hand. Hence a game formulation of the problem allows for the uncertainty only within this assumption. Any uncertainty, due to the omission of a load, or an error in the estimation of its position cannot be taken care of in a game. The non-measurable uncertainties that may arise in a structural design process can be listed as follows:

1. Uncertainty due to the omission of one or more design data like loads.
2. Uncertainty due to insufficient scientific knowledge or technological skill.
3. Uncertainties of design origin arising from idealizations, assumptions, inaccurate modelling etc.
4. Uncertainties of computational origin, computational inaccuracies.
5. Uncertainties of constructional origin, faulty workmanship.
6. Little skilled inspection and lack of repairs.
7. Uncertainties in the estimation of needs of design; assessment of the serviceability needs, cost of construction, cost of failure of different modes, and the assessment of the cost-effectiveness model.

These uncertainties are mostly of the non-random non-measurable type. By assuming that the effect of these uncertainties are to cause a wrong assessment of the task curves (force-deformation relations required), and the real force-deformation relations of the cross sections, the non-measurable uncertainties can be compensated by shifting the task curves to the conservative side by introducing marginal uncertainty factors as given below.

7.5.2 Marginal Uncertainty Factors:

The effects of the uncertainties given above on the curves may be any one or more of the following.

(i) The estimated strength, and actual strength may be different.

(ii) The estimated stiffness, and actual stiffness may be different.

(iii) Ductility may be different from that is actually needed.

Hence strength, stiffness and ductility are corrected to the conservative side by three factors r_s, r_c and r_d respectively. The method of correction is shown in Fig. 7.3. The same factors are assumed to compensate the errors in both the curves-task curve and the force-deformation relation of the cross section. The following points may be noted in assessing the factors.

(i) The marginal factors that are needed for the uncertainties discussed are much smaller than the conventional factor of safety, or the partial safety factors of CEB, because many of the severe uncertainties are accommodated within the decision Stages 1, 2 and 4.

(ii) Joints and connections very often are the weakest points of the structure especially of steel and prefabricated concrete structures. The strength of a connection is not improved by the introduction of the marginal factors as discussed above, and any uncertainty in the behaviour of connections is not compensated for, and must be dealt with separately.

(iii) A greater margin may be provided for those cross sections, the failure of which have a pronounced effect on the overall safety of the system. Thus a bottom column in a multistoreyed building may require a greater margin than a beam in the top floor.

The marginal uncertainty factors can be defined as follows:

(a) Marginal factor r_s for strength

Let F_L and F_R be the maximum forces in the new shifted task curve and the task curve respectively for any deformation. Then the marginal factor r_s is defined as:

$$r_s = \frac{F_L - F_R}{F_R} \tag{7.3}$$

Knowing a preassigned value of r_s, and F_R, F_L can be computed. The factors r_s can be split as

$$r_s = r_s' + r_s'' \qquad (7.4)$$

where r_s' = the uncertainty factor for strength

r_s'' = the factor that raises the capacity of critical sections, the failure of which have a greater effect on the overall safety of the structure. It is analogous to the capacity reduction factor for columns given in ACI Codes.

(b) Marginal factor r_c for stiffness.

Let Y_L and Y_R be the maximum <u>deformations at the normal load conditions</u> in the shifted task curve and task curve respectively. Then

$$r_c = \frac{Y_R - Y_L}{Y_R} \qquad (7.5)$$

(c) Marginal factor r_d for ductility.

Ductility of a cross section is defined here as the capacity to undergo deformation without fracture or separation. It is measured by the total deformation of the section. A marginal factor r_d is introduced to improve the ductility that compensates for any uncertainties. Thus

$$r_d = \frac{Y_L - Y_R}{Y_R} \qquad (7.6)$$

Here Y_L' and Y_R' are the total deformations of the section. r_s, r_c and r_d are positive numbers with numerical values lying between 0 and 1. The use of factors r_s, r_c and r_d for shifting the curve is shown in Fig. 7.3.

7.5.3 Evaluation of the Factors:

The uncertainties for which the factors chosen are of the non-measurable type, and hence a rigorous mathematical procedure is not possible to find the factors. The extent to which the uncertainties exist can only be assessed in qualitative terms. The choice of the factors therefore depends more on the personal judgement of the decision maker. However, the use of a suitable decision technique would lead to a systematic process. A decision approach based on preferences is proposed for the evaluation of the factors.

7.5.4 Optimal Decisions Based on Preferences:

The theory and solution method of an optimal decision problem based on preferences is presented in this section. The formulation is due to Radner (148). The decision problems of this type are of very practical use when preferences and decisions are to be made by using personal judgement.

Basic Principles: The basic principles can be summarized as follows. Individuals, when faced with two alternative actions, will in general prefer one to another or he will be indifferent between them. If such preference or indifference can be

expressed with every pair of alternatives, all the alternatives can be ordered based on the preferences. Numerical utilities can be assigned to the alternatives in such a way that they represent the individuals' preferences, if such utilities exist. It is to be noted here that preferences among alternatives come prior to the numerical characterization. Thus an alternative A is preferred to B not because A has a higher utility. On the other hand, a higher utility is assigned to A, as A is preferred to B.

Statement of the Problem: The choice of suitable marginal uncertainty factors is considered as the decision problem. The alternatives are the members that are admissible. Mathematically the problem can be stated as follows:

Given

(i) A set A of alternative actions $A_1, A_2, \ldots, A_n$. (It is assumed that the alternatives are finite.)

(ii) A set B of alternative states $B_1, \ldots, B_\ell$ of the environment. Each element of the alternative states is called an event. (Events are assumed to be finite in number).

(iii) A set R of alternative outcomes ρ or results of his action. ρ is associated with each action-event pair (A, B). Thus $\rho = \rho(A, B)$.

The decision maker has to choose an action A_1 that is optimal in some sense.

<u>Preference Order</u>: A partial preference order on the alternatives is a relation $\succsim$ called preference and indifference. Thus $A_1 \succ A_2$ indicates A_1 is strictly preferred to A_2. Similarly $A_1 \sim A_2$ indicates A_1 is indifferent to A_2. $A_1 \succsim A_2$ indicates A_1 is preferred or indifferent to A_2.

If $A_1 \not\succsim A_2$, and $A_2 \not\succsim A_1$, then A_1 and A_2 are incomparable.

<u>Assumptions</u>: In order to satisfy some consistency requirements, certain assumptions are made as follows:

1. <u>Transitivity</u>: $A_i \succsim A_j$ and $A_j \succsim A_k$ imply $A_i \succsim A_k$.
2. <u>Reflexivity</u>: For every A_i in A, $A_i \succsim A_i$.
3. <u>Completeness</u>: For every A_i and A_j in A, $A_i \succsim A_j$ and/or $A_j \succsim A_i$.

It is shown that if the set A is finite, it is always possible to have such preferences among all its alternatives.

<u>Utility</u>: A preference ordering can usually be represented by a numerical function. Thus $\Omega(A_i)$ is a numerical measure representing the ordering, if for all A_i and A_j in A, $A_i \succsim A_j$ is equivalent to $\Omega(A_i) \geq \Omega(A_j)$. If Ω represents ordering, any strictly increasing function of Ω also represent the ordering.

Let u be the numerical function that represents

the preference ordering of the alternatives A_i with respect to any one event B_i. u is called a <u>utility function</u>. With the utility u and outcome function ρ, a <u>payoff</u> function ω can be obtained as

$$\omega(A, B) = u[\rho(A, B)] \tag{7.6}$$

<u>Probabilities of the Events</u>: The relative likelihood of various events, or states of the environment can be represented by a probability ϕ (B) distribution among the events. The probabilities may be obtained objectively or may be assigned subjectively by the decision maker using his personal judgement. When the probabilities are of the latter type, they are called personal probabilities. They possess the following properties.

$$\phi_j \geq 0, \quad j = i, \ldots, \ell \tag{7.7}$$

$$\sum_{j=1}^{p} \phi_j = 1 \tag{7.8}$$

<u>Expected Pay-off</u>: With the allotted probabilities on B, environment is assumed to select a mixed strategy of all events. Knowing ω (A, B) and ϕ_j, the quantity Ω(A) can be found as

$$\Omega(A_i) = \sum_{j=1}^{l} \omega_{kj} \phi_j, \quad i = 1, \ldots, n \tag{7.9}$$

The quantity $\Omega(A_i)$ is called the expected pay-off associated with the action A_i.

Optimal Choice: With the expected pay-offs that show the ordering of all the actions, the decision maker can choose the optimal action as the one having the optimum value of expected pay-off.

7.5.5 Application to the Evaluation of Marginal Uncertainty Factors:

The decision problem discussed above is applied to the evaluation of marginal factors r_s, r_c and r_d. The method is particularly suited to this choice, as the factors are chosen using the judgement of the decision maker.

It is to be noted that the factors r_s, r_c and r_d are to be evaluated for each of the n task curves leading to the evaluation of 3n factors. In order to avoid the tremendous work involved, it is assumed that the factors r_s' , r_c and r_d are the same for the task curves irrespective of the members to which they belong. This assumption can be justified due to the fact that the factors are chosen to compensate for the human errors, which may not have any dependence on the type of the member. The factor r_s'' is chosen for each of the task curves separately depending upon the importance of the members to which the curves belong, on the overall safety of the structure.

States of Environment: The first step in the evaluation of the factors r_s' , r_c and r_d is to list out the uncertainties that may arise. The uncertainties form the events B_i.

Care should be taken not to leave any event arbitrarily, however, uninfluential it may be. Tables 7.1, 7.2, and 7.3 show the states of environment that affects r_s', r_c and r_d respectively. They include nonmeasurable uncertainties arising from the human interference which may be in design, construction and/or operation of the system.

Alternative Actions: The purpose of the decision process is to choose numerical values for r_s', r_c and r_d. Generally, the range within which these quantities lie can be assessed before hand. The factors can take any value within the bounds. For convenience, it may be assumed that the numerical values of r_s', r_c and r_d belong to a set of finite numbers (say 0.0, 0.1, 0.2, 0.3, ..., 1.0). These numbers form the alternative actions in the decision process.

Rating the Events: Each of the events or the uncertainties listed in Tables 7.1, 7.2 and 7.3 is rated by the designer to show to what extent each of them may influence the system. As no numerical measure is available, the rating is done using the personal judgement of the decision maker. The ratings are also listed in the Tables against each of the event. They are expressed qualitatively, as a qualitative judgement is easier than a quantitative one. Also, it is required to write the rating list in the order in which they may occur in the system. For example, the ratings for workmanship are very good, good, fair and poor. In the actual system it may be

judged in the order good, very good, fair, poor. It indicates that there is every chance that workmanship is good, with less chance it may be very good, or fair, and the chance that it becomes poor is least. Table.7.4 shows the rating made.

Outcome Set, Utility and Payoff: In the decision of marginal safety factors the utility function $u(\rho)$ is taken as

$$u(\rho) = \rho \tag{7.10}$$

Hence the payoff function is given by

$$\omega(A, B) = \rho(A, B) \tag{7.11}$$

The outcome set $[\rho]$ is a matrix whose elements represent the numerical representation of the effect of the choice of the alternative actions namely the numerical value of r_s', r_c and r_d on the particular event. The qualitative rating of the events given above and the degree of safety required in serviceability, safety and ductility form the basis of preference. The preferences are expressed in the form of numbers. Because of Eqs. 7.10 and 7.11, the outcome set is equal to the payoff.

Probabilities: In the usual decision problems only one of the alternative events may be actually occurring. The assigned probabilities indicate with what likelihood a particular event can occur. In the structural system, on the other hand, more than one events may occur. The probability allotted to the

events may be viewed as the probability with which nature mixs the strategies. The probabilities are judged by the decision maker and subjectively assigned to each event. If two events are given, the decision maker may be able to judge with what probability the two will be acting. From such pairwise judgement a global allocation of probabilities may be made.

Expected Payoff: Knowing the payoff of all the events and the probabilities, the expected payoff of each action can be calculated by means of Eq. 7.9.

7.5.6 Illustrative Example:

The method is illustrated for the example of a 3-bar system. The evaluation of r_s is shown and that of others can similarly be done.

Evaluation of r_s': Let the numerical value is to be chosen from 0.1, 0.2, 0.3 and 0.4. The outcome set R is given in Table 7.4 as a 4 x 12 matrix which is the payoff matrix due to Eqs. 7.10 and 7.11. The rating for the events are also shown in Table 7.4. The probability allotted to each event is also shown in Table 7.4. The expected payoff is calculated and tabulated from which the value of r_s' is chosen.

$$r_s' = 0.2$$

Evaluation of r_s': Since the failure of all the members are equally serious, the value of r_s'' is same for all the members.

Let it be 0.1.

$$r_s = r_s' + r_s'' = 0.2 + 0.1 = 0.3$$

7.5.7 Discussion:

The decision theory approach presented in this section gives a systematic procedure for the evaluation of the marginal factors to represent a set of nonmeasurable uncertainties. The method recognizes the subjective nature of the problem and the dependence on the judgement of the decision maker. The expected utility criterion is a convenient tool that characterises the preference ordering of a complex problem in a simple manner. The decision maker is required to express preferences among actions for relatively simple events. However, the vagueness of judgement and the arbitrariness of numerical characterization will have pronounced effect on the choice. But, considering the subjectivity and vagueness of the problem itself, this is negligible.

The proposed factor r_s has great similarity with the factor of uncertainty used in the extended reliability method (27), as both are intended to cover the same sort of uncertainties. However r_s does not represent the uncertainty due to unknown probabilities which is taken care of by statistical game. Ang (29) proposes to relate the uncertainty factor to judgemental probabilities. Any such relation which does not consider the utilities involved, will be arbitrary. The

method proposed in this chapter is more rational, as the judgement is done for each of the uncertainties and the decision is made based on judgement and preferences.

The Tabulation method of the Institution of Engineers, London (34) to evaluate the load factors recognizes the existence of the various uncertainties and relies on the judgement as in the present method. However, the choice of factors is not based on any ordering of the alternative actions as done in the present work. Instead, the factors are chosen from a table depending upon the judgement of the situation made by the decision maker. This empirical approach makes the tabulation method simpler but not so rational.

7.6 FINAL CHOICE OF MEMBER CROSS SECTIONS

The three stages of decisions discussed so far are intended to decide what type of behaviour is needed so that the structure is safe, serviceable and efficient under an uncertain loading environment. It remains to select the cross sections that can afford the required behaviour economically. The decisions in Stage 4 involve the selection of the material, shape and dimensions of the cross sections. The criterion of choice is the cost-effectiveness criterion given by Eq. 7.2. The method of choice is presented in detail in Chapter 10.

7.7 ARRANGEMENT OF MEMBERS

In the method proposed in the previous sections, it

is assumed that the member arrangement is known. However, member arrangement may also influence the optimality of design. Hence the arrangement of members is also to be taken as a decision problem.

The method of linking this decision problems to what is described in this chapter is proposed in Chapter 3. If such a cycle of design is also carried out, the arrangement of members may be treated as an additional set of strategy which jointly minimizes the cost-effectiveness along with other strategies described in this chapter. This formulation is not studied here in detail.

7.8 SUMMARY

The various stages involved in a direct design method for design under uncertainty are presented. The design is divided into four stages. In the first two stages, a structural action game is assumed and the force-deformation relations of various sections required for serviceability and optimum cost-effectiveness are found out by the two plays of the game. The third stage is to compensate for the non-measurable uncertainties involved in a design process. A decision procedure based on references is presented to evaluate the marginal factors. The fourth stage of decision is to select the cross section of members and is dealt with in detail in Chapter 10.

The next chapter deals with the normal play of the structural action game.

CHAPTER EIGHT

NORMAL PLAY OF STRUCTURAL ACTION GAME

8.1 INTRODUCTION

The normal play of the structural action game described in Para 7.4 is formulated as a full-fledged design method in this chapter. As proposed earlier, this play of the game is intended to design the force-deformation relations (task curves) required to satisfy the serviceability conditions. The magnitudes of the loads acting on the structure are assumed to be within the bounds specified, for which condition the structure is to be serviceable. For this load condition, (called the normal load condition), the loading process or path of loading (sequence, timing and manner of loading) is assumed to be uncertain. The inelastic design process formulated in Chapter 5 treating design as an optimal control problem is extended as a game problem to accommodate the said uncertainties in loading process. The design problem thus become a differential game problem.

The detailed formulation is presented in Section 8.4, while Section 8.5 contains the illustrative examples. The problem is reformulated in Section 8.6 to allow for the jumps in state variables and in Section 8.7 to consider the change in position. The validity of the formulation is discussed in Section 8.8.

8.2 DIFFERENTIAL GAME

Differential game ''...... is a mathematical entity which fuses game theory, the calculus of variation, and control theory ''(149). It makes the theory of games applicable to problems, when the dynamics of the situation must be taken into account. Thus, like optimal control theory, differential game is a multistage decision model. In 1954, Isaacs intiated studies in this branch of mathematics Later Berkovitz (150-152), Pontryagin (153) and others (154) enriched the theory by supplying necessary theorems. Ragade, and Sarma (155) applied the differential game to solve optimal control problems in the face of uncertainty. Differential game is still in its infancy. A good account of the theory can be had from references (149-157).

8.3 STATEMENT OF THE PROBLEM

The following information is given:

(i) The structural type is chosen, and the arrangement of members and the types of connections are also selected.

(ii) The loads acting on the structure are also known. The magnitudes of the loads, called the normal values of the loads within which the structure is to be serviceable are also specified.

(iii) The loading process is uncertain. The sequence, and timing and direction of the loads are not known and may

follow any path. In case a change in position of the loads occurs, that may also be specified in which case they also are to be considered uncertain. In the present formulation no such change in position of loads is considered.

(iv) The service life of the structure is specified as T.

It is required to find out the force deformation relations (called the task curves) of the cross sections for the range of loading specified.

The criterion of decision is that the structure is to be safe and serviceable, i.e. the deflections, rotations and permanent deformations must be within the allowable limits. No compromise on the safety and serviceability is allowable in the normal load conditions. Hence all loading paths are assumed to be equally likely for serviceability design.

8.4 FORMULATION OF THE PROBLEM:

The inelastic design of the structure under uncertainty is formulated in this section as a differential game problem. The basic assumption is the structural action game, according to which structural action is assumed to be a game played by structure against nature. The assumptions made in the formulation as optimal control problem hold good in the present formulation as well.

8.4.1 Brief Outline of the Formulation:

The formulation is basically analogous to the

formulation as optimal control problem. But, unlike in the latter method, two parties namely structure and nature, having conflicting aims, control the deformation state of the structure. In optimal control formulation, nature's control namely the loading path has been assumed known. In differential game, the path of load is uncertain. It is assumed that nature may choose a loading path that maximizes the damage. As before, structure selects its control (force-deformation relation) such that the damage is minimum. But the minimization in this case is to be done when an equally interested antagonist is trying to maximize the payoff.

The controls are the strategies of the two players. Because of the simultaneous minization and maximization, the optimal strategies of the players correspond to a saddle point in the payoff surface in the space $(\underline{v}, \underline{\alpha}) \times \underline{u}$. The equilibrium and compatibility conditions, and the serviceability requirements enter in the formulation as constraints.

Most of the terminology defined in Chapter 5 holds good in this case also. Hence the definitions of terminology, and the descriptions presented in Chapter 5 are not repeated here. The correspondence in terminology in structural design, optimal control and differential game problems is presented in Table 8.1. which would clarify the various terminologies and their equivalence.

8.4.2 Stage Variables:

s and t are the stage variables chosen as in Chapter 5.

8.4.3 State Variables:

(a) Deformation Space $\bar{Y}$: $\underline{x} = \left\{x_1, \ldots, x_n\right\} \in E^n$ is the state vector that represent the deformed state of the system. The path $\underline{x}(s) = \left\{x_1(s), \ldots, x_n(s)\right\}$ traced by the vector $\underline{x}$ is called the path of state vector. Initial value of state vector is at $s = 0$

$$\underline{x}(0) = (\underline{x})^0 \equiv \left\{x_1{}^0, \ldots, x_n{}^0\right\} \tag{8.1}$$

Similarly at $s = S$

$$\underline{x}(S) = (\underline{x})^f \equiv \left\{x_1{}^f, \ldots, x_n{}^f\right\} \tag{8.2}$$

At any s,

$$x_i = y_i + z_i, \quad i = 1, \ldots, n \tag{8.3}$$

When t is taken as an additional state variable, $\underline{X} = \left\{\underline{x}, t\right\} \in E^{n+1}$ is taken as the augmented state vector.

(b) Force Space $\bar{F}$: As in optimal control formulation the forces $\underline{F} = (F_1, \ldots, F_n)$ are state variables in the force space $\bar{F}$. The vector $\underline{F}$ represents the state of the forces in the space $\bar{F}$. The path $\underline{F}(s) = \left\{F_1(s), \ldots, F_n(s)\right\}$ represents the path of force vector in the space $\bar{F}$. At $s = 0$

$$\underline{F}(0) = \left\{F_1{}^0, \ldots, F_n{}^0\right\} \tag{8.4}$$

(c) Load Space $\underline{\underline{W}}$: Let n loads are acting on the structure. At any stage, let $\underline{w} = \{w_1, \ldots, w_n\}$ be the magnitudes of the loads. The vector $\underline{w}$ represents a point in the n-dimensional load space $\underline{\underline{W}}$. The vector w is called a state vector in the load space $\underline{\underline{W}}$. The path of load $\underline{w}$ is given by $\underline{w}(s) = \{w_1(s), \ldots, w_m(s)\}$. At $s = 0$

$$\underline{w}(0) = \{0, \ldots, 0\} \tag{8.5}$$

8.4.4 Players:

Structure and Nature are the players taking part in the game. Nature is the maximizing player and is called player 1. Structure is called player 2, and is the minimizing player.

Game theory assumes that the players are rational players who always try to better their gains. The validity of this assumption is discussed in Section 8.8.

8.4.5 Strategies:

The proposed game consists of a sequence of moves starting from $s = 0$ to $s = S$. At each instant of the play, both players make their moves out of the total moves available to them. The totality of the moves constitute a strategy of the player. The strategies are chosen by both players from respective strategy functions. Strategy and strategy functions are the counterparts of control and control law in optimal control problem.

Strategies of Player 1: Player 1 is provided with control variables or strategies namely the uncertainties associated with loads. The m loads acting on the structure are the strategies in the custody of nature. However, the assumption that the controls are 'inertialess'(capable of taking jumps-piecewise continuous) does not allow loads $\underline{w}$ to be directly taken as controls. Hence, the loads $\underline{w}$ are taken as state variables as given in Section 8.4.3, and their rates are taken as controls or strategies. Thus, $\underline{u} = \left\{ u_1, \ldots, u_m \right\}$ representing the rate of loading, are the controls or strategies in the custody of nature. From practical considerations the rates are kept below certain limits in order to avoid impact, vibration etc. and to keep the increment of loads always small. The limits are

$$-U_i^L \leq u_i \leq U_i^u , \qquad i = 1, \ldots, m \qquad (8.6)$$

The path $\underline{u}(s) = \left\{ u_1(s), \ldots, u_m(s) \right\}$ is called the strategy function. In optimal control problem, the loading function $\underline{w}(s)$ and the rates $\underline{u}(s)$ given by Eq. 5.6 are known a priori, and the solution holds good only for these known functions. But, in practice, loading function may not be known beforehand. Hence Eqs. 5.6 are also unknown functions. According to the game concept, player 1 is assumed to choose that function which maximizes the payoff to its benefit.

Strategies of Player 2: As in optimal control problem, two sets of strategies are possessed by player 2.

(i) $\underline{v} = \{v_1, \ldots, v_n\}$ the force rates associated with the state variables chosen from control laws

$$\underline{v}(s) = \{v_1(s), \ldots, v_n(s)\} \tag{8.7}$$

In statically determinate system, the controls $\underline{v}$ are uniquely determined in terms of $\underline{u}$ and hence $\underline{v}$ may not be treated as unknown strategies.

(ii) $\underline{\alpha} = \{\alpha_1, \ldots, \alpha_n\}$ the stiffness of the n force-deformation relations chosen from strategy functions

$$\underline{\alpha}(s) = \{\alpha_1(s), \ldots, \alpha_n(s)\} \tag{8.8}$$

8.4.6 Order of Moves and Information Pattern:

At any instant s, the play can be continued by player 1 initiating the next move. The response of the structure for any external load is so quick that it may be assumed that player 2 makes the move simultaneously with player 1.

It is also assumed that at any s, the states $\underline{x}$, $\underline{F}$ and $\underline{w}$, and all the prior positions occupied by the state variables together with the moves made by both parties are known to both of them. By definition, such a game is called a game of perfect information.

8.4.7 Constraint:

The constraint equalities and inequalities are the same as that in the optimal control problem formulated in Chapter 5. They are the following:

(i) $0 \leq \underline{w} \leq \underline{W}$, (8.9)

(ii) $R_i(\underline{F}, \underline{w}) = 0, \quad i = 1, \ldots, p$ (8.10)

(iii) $K_i(\underline{x}) = 0, \quad i = 1, \ldots, q$ (8.11)

(iv) State equations

(a) $\frac{dx_i}{ds} = f_i(\underline{x}(s), \underline{F}(s)\ \underline{v}(s), \underline{\alpha}(s)), i = 1,\ldots,n$ (8.12)

with initial conditions given by Eq. 8.1. Eqs. 5.15 to 5.20 that show the various idealizations hold good for this case also.

(b) $\frac{dF_i}{ds} = v_i \quad , \quad i = 1,\ldots, n$ (8.13)

with initial condition given by Eq. 8.4

(c) $\frac{dw_i}{ds} = u_i \ , \quad i = 1, \ldots, m$ (8.14)

with initial condition given by Eq. 8.5

(v) Serviceability constraints

(a) $Q(\underline{x}) \leq 0$

$Q(\underline{\bar{x}}') = Q(\underline{\bar{x}}'') = 0$ (8.15)

(b) $|x_i^p| \leq d_i$, for $F_i(s)\ v_i(s) \geq 0$

$\geq d_i'$, for $F_i(s)\ v_i(s) \leq 0$ (8.16)

(vi) Plasticity condition

(a) $\frac{d\alpha_i}{ds} \leq 0$ or $-|\mathcal{V}_i(s)v_i|$ $i = 1,,\ldots,n$ (8.17)

(b) $0 < \alpha_i \leq \alpha_i(0)$, $i = 1, \ldots, n$ (8.18)

8.4.8 Payoff or Performance Index:

The numerical quantity which the players strive to minimize in game is called the payoff. It is the reward gained by the maximizing player, or the loss occurred to the minimizing player. They are the same due to the assumption of zero-sum by which the loss occurred to one is the gain of the other.

The payoff is given by

$$P(\underline{x}, s, \underline{F}, \underline{v}, \underline{u}, \underline{\alpha}, \underline{w}) = \int_0^S G_0(\underline{x}(s), s, \underline{F}(s), \underline{v}(s), \underline{u}(s), \underline{\alpha}(s), \underline{w}(s))ds \qquad (8.19)$$

The strategies $\underline{v}$ and $\underline{\alpha}$ minimizes P, and $\underline{u}$ maximizes it. G_0 is the same as the quantity defined in Chapter 5. Eqs. 5.28 to 5.33 also hold good in the present formulation.

8.4.9 Saddle Point and Value of the Game:

Consider that there exists strategies $\underline{u}^*$, $\underline{v}^*$, $\underline{\alpha}^*$ such that

$$P^*(\underline{v}^*, \underline{\alpha}^*, \underline{u}^*) = \max_{\underline{u}} \min_{\underline{v},\underline{\alpha}} P = \min_{\underline{v},\underline{\alpha}} \max_{\underline{u}} P \qquad (8.20)$$

and

$$P(\underline{v}^*, \underline{\alpha}^*, \underline{u}) \leq P^*(\underline{v}^*, \underline{\alpha}^*, \underline{u}^*) \leq P(\underline{v}, \underline{\alpha}, \underline{u}^*), \qquad (8.21)$$

The quantity $P^*(\underline{v}^*, \underline{\alpha}^*, \underline{u}^*)$ is called the value of the game

and $\underline{v}^*$, $\underline{\alpha}^*$, $\underline{u}^*$, define a saddle point in the space ($\underline{v}$, $\underline{\alpha}$) x $\underline{u}$. Eqs. 8.20 and 8.21 constitue the necessary and sufficient condition for the existence of a saddle point (152). The value P^* must be single-valued, though the other two pairs $P(\underline{v}^*, \underline{\alpha}^*, \underline{u})$ and $P(\underline{v}, \underline{\alpha}, \underline{u}^*)$ may be multivalued (152). The existence of saddle point is a must for pure strategy solutions. The existence of saddle point and value for the structural action game are further discussed in Section 8.8.

8.4.10 Optimal Path and Optimal Strategies:

The path $\underline{x}^*(s)$, $\underline{F}^*$, $\underline{w}^*$ corresponding to the value of the game is called the optimal paths of state variables. The requirement that P^* must be single valued does not preclude the possibility of there being more than one trajectory $\underline{x}^*(s)$ (152). However, P^* takes same value along all such paths. The strategies $\underline{v}^*(s)$, $\underline{\alpha}^*(s)$ and $\underline{u}^*(s)$ associated with the value of the game are called, optimal strategies.

8.4.11 Force-Deformation Relations from Optimal Strategies:

By the procedure described in 5.3.11 the force-deformation relations can be obtained from $\underline{F}^*$ and $\underline{\alpha}^*$ as

$$F_i^* = F_i^*(y_i^*) \quad , \quad i = 1, \ldots, n \qquad (8.22)$$

These are the task curves that are needed to satisfy the serviceability requirements and laws of motion. In an optimal control problem, these curves correspond to the minimum P^* whereas in differential game they correspond to the saddle point.

It shows that <u>the task curves are obtained for the worst course of loading</u>. It is shown later in Section 8.8 that the <u>structure if designed with these task curves will be serviceable for any other course of loading</u>.

8.4.12 <u>Statement of the Problem</u>:

Now, the design problem can be formally stated as a differential game problem as follows:

Given

s = stage variable,

$\underline{w} = \{w_1, \dots, w_n\}$ (loads)

$\underline{F} = \{F_1, \dots, F_n\}$ (forces)

$\underline{x} = \{x_1, \dots, x_n\}$ (deformations) as state variables and

$\underline{u} = \{u_1, \dots, u_m\}$ the strategies in the hands of player 1

$\underline{v} = \{v_1, \dots, v_n\}$ and

$\underline{\alpha} = \{\alpha_1, \dots, \alpha_n\}$ the strategies in the hands of player 2

determine

$$u_i^* = u_i^*(s)\ , \qquad i = 1, \dots, m$$

$$v_i^* = v_i^*(s)\ , \qquad i = 1, \dots, n$$

$$\alpha_i^* = \alpha_i^*(s)\ , \qquad i = 1, \dots, n$$

$$w_i^* = w_i^*(s) , \qquad i = 1, \ldots, m$$

$$F_i^* = F_i^*(s) , \qquad i = 1, \ldots, n \quad \text{and}$$

$$x_i^* = x_i^*(s) , \qquad i = 1, \ldots, n$$

such that

$$P^*(\underline{v}^*, \underline{\alpha}^*, \underline{u}^*) = \min_{\underline{v},\underline{\alpha}} \max_{\underline{u}} \int_0^S G_0(\underline{x}, s, \underline{F}, \underline{v}, \underline{\alpha}, \underline{w}, \underline{u})ds$$

$$= \max_{\underline{u}} \min_{\underline{v},\underline{\alpha}} \int_0^S G_0(\underline{x}, s, \underline{v}, \underline{F}, \underline{\alpha}, \underline{u}, \underline{w})ds \tag{8.19}$$

subject to the constraining conditions given by Eqs. 8.6 and 8.9 to 8.18. and the initial condition given by Eqs. 8.1, 8.4 and 8.5.

8.4.13 Solution of the Problem:

A solution to the differential game problem must yield the following

1. The value of the game $P^*(s)$
2. Optimal paths $x_i^*(s)$, $F_i^*(s)$, $\quad i = 1, \ldots, n$

   and $w_i^*(s)$, $\quad i = 1, \ldots, m$
3. Optimal strategies $u_i^*(s)$, $\quad i = 1, \ldots, m$

   $v_i^*(s)$, $\quad i = 1, \ldots, n$

   and $\alpha_i^*(s)$, $\quad i = 1, \ldots, n$

The method of solution of the differential game is

somewhat similar to that of optimal control problem. But it involves certain added difficulties such as the existence and uniqueness of solution and occurrence of singular surfaces and conjugate points. Sarma and Ragade (156) have discussed some of the difficulties encountered in solving differential game problem. The problem, in its most general form, can be solved only with the help of an algorithm and by resorting to numerical methods. An analytical solution becomes unwieldy when the number of variables increases.

The necessary conditions for the existence of solution offer an analytical basis for solving simple problems. The necessary conditions for the existence of solution to the differential game problem have been studied by three different approaches as given below.

(i) Using the principle of optimality

(ii) Variational approach

(iii) Geometric approach.

Isaacs (149) made use of the principle of optimality and obtained a partial differential equation. Berkovitz (150) also obtained by Hamilton - Jacobi theory of calculus of variation a similar condition. Also, by using Weistrass condition, Berkovitz (150) has obtained another set of conditions presented in Section 5.4. Leitman and Mon (166) by a geometric approach obtained a set of necessary conditions, which is an extension of the geometric theory

of optimal control. The geometric approach has the appeal of Isaacs approach based on physical consideration and the equations obtained are similar to those obtained by Berkovitz (151). A differential game problem may be stated as follows (155):

Determine a $\underline{u}^* \in U$, and $\underline{\alpha}^* \in \mathcal{A}$ such that,

$$P^*(\underline{u}^*, \underline{\alpha}^*) = \min_{\underline{\alpha}^* \in \mathcal{A}} \max_{\underline{u}^* \in U} P(\underline{u}, \underline{\alpha}) \tag{8.23}$$

where

$$P(\underline{u}, \underline{\alpha}) = h(x(T), T) + \int_0^T G_0(\underline{x}, \underline{u}, \underline{\alpha}, t)\, dt$$

(h is called terminal payoff.) subject to the conditions

$$\frac{dx_i}{ds} = f_i(\underline{x}, \underline{u}, \underline{\alpha}, t), \quad x_i(0) = x_i^0, \quad i = 1, 2 \ldots, n. \tag{8.24}$$

$$\theta_j(\underline{x}, \underline{\alpha}, t) \geq 0, \quad j = 1, \ldots \ell \tag{8.25}$$

$$\theta_j'(\underline{x}, \underline{u}, t) \geq 0, \quad j = 1, \ldots \ell' \tag{8.26}$$

$$Q(x(T), T) = 0 \tag{8.27}$$

Isaacs (149) divides the solution to the problem into two divisions.

(i) Solution in the small: The state space is divided into a number of subregions separated by surfaces called singular surfaces. In each part the solution is smooth. The smooth part of solution found between singular surfaces is termed solution in the small. The solution technique in the small is that of differential equation. The value of the game P^* and the optimal strategies $\underline{u}^*$ and $\underline{\alpha}^*$ are continuous and are of class C' in this case.

(ii) Solution in the large: On the singular surfaces, a variety of special behaviour occurs. Solution in the large consists of the identification of the singular surfaces and the assembly of smooth parts into total solution.

Solution in the small: The equations for solution in the small are the following (151, 155). The problem stated earlier may be treated as a statement of the problem in th division. Let

$$H = G_0 + \underline{\lambda}\ \underline{f} \tag{8.28}$$

Let $\underline{u}^*$, $\underline{\alpha}^*$ be the optimal strategies and $\underline{x}^*(t)$ the optimal path respectively. Then

$$\text{(i)}\quad \frac{dx_i}{dt} = f_i = \frac{\partial H}{\partial t}(t,\ \underline{x},\ \underline{u}^*(t,\ \underline{x}),\ \alpha^*(t,\ \underline{x}), \underline{\lambda}),\ i=1,\ldots,\ n \tag{8.29}$$

$$\text{(ii)}\quad \frac{d\lambda_i}{dt} = -\frac{\partial H}{\partial x_i}(t,\ \underline{x},\ \underline{u}^*(t,\ \underline{x}),\ \alpha^*(t,\ \underline{x}), \underline{\lambda}),\ i = 1,\ldots,\ n \tag{8.30}$$

where

$$\frac{\partial H}{\partial x_i} = \frac{\partial H}{\partial x_i} + \frac{\partial H}{\partial \underline{u}} \cdot \frac{\partial u}{\partial x_i} + \frac{\partial H}{\partial \underline{\alpha}} \cdot \frac{\partial \alpha}{\partial x_i}$$

(iii) $\frac{\partial H}{\partial u_i} + \underline{\mu}' \cdot \frac{\partial \underline{\theta}'}{\partial u_i} = 0, \quad i = 1, \ldots, m$ (8.31)

(iv) $\frac{\partial H}{\partial \alpha_i} + \underline{\mu} \cdot \frac{\partial \underline{\theta}}{\partial \alpha_i} = 0, \quad i = 1, \ldots, n$ (8.32)

(v) $\underline{\mu} \cdot \underline{\theta} = 0$ (8.33)

(vi) $\underline{\mu}' \cdot \underline{\theta}' = 0$ (8.34)

(vii) $\underline{\mu}' \geq 0, \quad \underline{\mu} \leq 0$ (8.35)

(viii) At each $t \in [0, T]$, H must be such that

$$\begin{aligned} & H^*(t, \underline{x}^*, \underline{u}^*(t), \underline{\alpha}^*(t) \ (t)) \\ &= \underset{\underline{u}}{\text{Max}} \ \underset{\underline{\alpha}}{\text{Min}} \ H(t, \underline{x}^*, \underline{u}(t), \underline{\alpha}(t), \underline{\lambda}(t)) \\ &= \underset{\underline{\alpha}}{\text{Min}} \ \underset{\underline{u}}{\text{Max}} \ H(t, \underline{x}^*, \underline{u}(t), \underline{\alpha}(t), \underline{\lambda}(t)) = -\frac{\partial P^*}{\partial t} \end{aligned} \tag{8.36}$$

where

$$\underline{u}^*(t) = \underline{u}(t, x^*)$$

$$\underline{\alpha}^*(t) = \underline{\alpha}(t, x^*)$$

(ix) $\underline{\lambda}$ is continuous in the domain. At the point of discontinuity $\underline{\lambda}$ possess a left hand limit λ_k^- and a right

hand limit λ_k^+ .

(x) If $\underline{x} = \underline{x}^*(t)$, then

$$\underline{\lambda} = \frac{\partial P^*}{\partial \underline{x}} \tag{8.37}$$

(xi) P^* satisfies the Hamilton-Jacobi equation. Hence value of the game P^* may be obtained from this equation. The equation is

$$\underset{\underline{\alpha}}{\text{Min}} \; \underset{\underline{u}}{\text{Max}} \; G_0(t, \underline{x}, \underline{u}, \underline{\alpha}) + \underline{\lambda} \cdot \underline{f}(t, \underline{x}, \underline{u}, \underline{\alpha}) + \frac{\partial P^*}{\partial t} = 0 \tag{8.38}$$

or

$$G_0(\underline{x}, \underline{u}^*, \underline{\alpha}^*) + \frac{\partial P^*}{\partial \underline{x}} \cdot \underline{f}(t, \underline{x}, \underline{u}^*, \underline{\alpha}^*) + \frac{\partial P^*}{\partial t} = 0 \tag{8.39}$$

(xii) <u>Tranversality Condition</u>: Let the equations $Q(x(T), T) = 0$ represent the terminal surface. Let is be rewritten as

$$\underline{x} = \underline{x}(\underline{\sigma}) \; , \quad T = T(\underline{\sigma}) \tag{8.40}$$

where $\underline{\sigma}$ is an n-vector of free variables. The n-relations required at the terminal time T to give the transversality conditions are

$$\underline{\lambda} \cdot [\underline{M}] = - \left(G_0 + \frac{\partial h}{\partial T}\right) \cdot \frac{\partial T}{\partial \underline{\sigma}} - \frac{\partial h}{\partial \underline{x}} \cdot \frac{\partial \underline{x}}{\partial \underline{\sigma}} \tag{8.41}$$

An element in the ith row and jth column of $[M]$ is given by

$$f_i \cdot \frac{\partial t}{\partial \sigma_j} - \frac{\partial x_i}{\partial \sigma_j} \tag{8.42}$$

(xiii) $$H^*(\underline{x}^*, t, \underline{u}^*, \underline{\alpha}^*, \underline{\lambda}) = -\frac{\partial \psi}{\partial t}\Big|_{t=T} \tag{8.43}$$

where

$$\psi(\underline{x}, \underline{u}, \underline{\alpha}, t) = h(\underline{x}(t), t) + \eta\, Q(\underline{x}(t), t), \eta \neq 0$$

<u>Solution in the large</u>: Isaacs (149) has discussed the properties of several types of singular surfaces like, transition surface, semi-permeable surface, barrier etc. Berkovitz (151) has given the following continuity properties on the switching surfaces (singular surfaces that separate two regions in which the optimal path may be continuous).

(i) An optimal trajectory must not be tangent to the switching surface, when it crosses the surface.

(ii) Let $(\tau, \underline{x}')$ be a point on the switching surface where the trajectory crosses. If this point is a point of discontinuity of $\underline{u}$ or $\underline{\alpha}$ or both, there may be more than one solution of the state equation satisfying (τ, x'). Also, a solution unique at (τ, x') may bifurcate at some point when $\underline{u}$, $\underline{\alpha}$ or both discontinuous. If $\underline{u}$, $\underline{\alpha}$ are both continuous at (τ, x') there will be a unique solution for the state equation.

(iii) If on the manifold $\underline{\bar{M}}$ which the trajectory crosses at $(\tau, \underline{x}')$, only one of $\underline{u}^*$, or $\underline{\alpha}^*$ become discontinuous, then $\underline{\lambda}$ is continuous at $t = \tau$.

$$\underline{\lambda}^-(\tau) = \underline{\lambda}^+(\tau) \tag{8.44}$$

otherwise

$$H(M_\tau^-)\frac{\partial \tau}{\partial \underline{x}''} - \underline{\lambda}^-(\tau)\frac{\partial \underline{x}'(\tau)}{\partial \underline{x}''} = H(M_\tau^+)\frac{\partial \tau}{\partial \underline{x}''} - \underline{\lambda}^+(\tau)\frac{\partial \underline{x}'(\tau)}{\partial \underline{x}''} \tag{8.45}$$

where $\underline{x}''$ is the value of $\underline{x}$ on the adjacent manifold from where the trajectory emerged. $H(M_\tau)$ represents the value of H on the manifold M_τ.

8.5 APPLICATION TO THE DESIGN OF STRUCTURES

In this section, the formulation of structural design as differential game and the solution of the problem are presented. The application is intended to illustrate the technique involved so as to bring out the characteristics of the problem. Computational aspects of the problem are not discussed here.

8.5.1 Characteristics of the Solution of Structural Action Game:

In structural action game, the trajectory of $\underline{x}$ is such that its smooth portions are separated by switching

surfaces. When $\underline{u}$, $\underline{v}$ and $\underline{\alpha}$ are all continuous and smooth, the trajectory $\underline{x}$ is also smooth. Discontinuities in the strategies occur as described below.

(i) $\underline{u}$ changes abruptly with or without a change in sign. A jump in magnitude of $\underline{u}$ causes a corresponding jump in the magnitude of $\underline{v}$. A change in sign of $\underline{u}$ causes a change in sign of $\underline{v}$ and a jump in $\underline{\alpha}$.

(ii) $\underline{u}$ may be continuous but some component of $\underline{v}$ may become zero (denotes yielding).

The boundary $\underline{\bar{B}}$ that represents the limit to serviceability is a switching surface or terminal surface depending upon whether $\underline{x}'$ of $\underline{x}$ that belongs to $\underline{\bar{B}}$ is an intermediate point in the trajectory or the final value $\underline{x}^f$.

8.5.2 <u>Application to Statically Determinate Systems</u>:

As discussed in Chapter 5, the design of statically determinate systems is simpler due to the simplifications described in Section 5.5.1. The state variables $\underline{F}$ and strategies $\underline{v}$ may be eliminated by means of equilibrium equations. No equality state variable constraint due to combatibility conditions need be considered.

8.5.3 <u>Example of Tension Bar</u>:

Consider a tension bar of 100 inches length as shown in Fig. 5.8.a. It is subjected to an axial load w

which is bounded as

$$0 \leq w \leq 4 \text{ tons}$$

It is required to find out the force-deformation relations of the bar in order to satisfy the following.

1. The bar must be safe for the range of load specified.
2. The elongation should not exceed 1 inch.
3. The permanent deformation should not be more than 0.001 in/in.
4. $-1 \text{ ton} \leq u \leq 1 \text{ ton}$.

The variables may be defined as in the example of tension bar presented in Section 5.5.2.

$$F,\ w,\ y = \text{state variables}$$

$$v,\ \alpha = \text{strategies of player 2}$$

$$u = \text{strategy of player 1.}$$

Let $S = 8$. The equations may be written as follows.

$$P = -100 \int_0^8 y \frac{dw}{ds}\, ds$$

<u>Constraints</u>

$$0 \leq w \leq 4$$

$$-1 \leq u \leq 1$$

$$u = v,\ F = w$$

$$(\alpha(0)-\alpha) \geq 0$$

$$\alpha - \alpha(0)\left[1 - \left|\frac{F}{50}\right|\right] \geq 0 \qquad \text{for} \quad u \geq 0$$

$$\alpha - \alpha(0)\left[1 - \left|\frac{F-F^{e}}{50}\right|\right] \geq 0 \qquad \text{for} \quad u < 0$$

$$y \leq 0.01$$

$$\left|y - \frac{F}{\alpha(0)}\right| \leq 0.004, \quad u \geq 0$$
$$\geq 0.002, \quad u < 0$$

The equations for solution of the problem can be obtained as follows. Total duration of play S is 8. Also, since $u = v$ and $F = w$, F and v are eliminated by substituting u for v and w for F .

$$H = -100\, y\, u + \lambda_1 \frac{u}{\alpha} + \lambda_2 u$$

$$\frac{dy}{ds} = \frac{u}{\alpha}, \quad \text{with } y(0) = 0$$

$$\frac{dw}{ds} = u, \quad \text{with } w(0) = 0$$

$$\frac{d\lambda_1}{ds} = 100\, u$$

From the solution of case three of example of Tension Bar in Chapter 5,

$$\frac{d\lambda_2}{ds} = -\frac{\mu_1\, v(\alpha(0) - \alpha)}{\alpha^2\, F} \qquad \text{for } u \geq 0$$

$$\frac{d\lambda_2}{ds} = -\frac{\mu_1 \, v\,(\alpha(0) - \alpha)}{\alpha^2 |F - F^e|} \operatorname{sgn} \left|\frac{F - F^e}{10}\right|, \text{ with } \lambda(8) = 0$$

optimal strategy $u^* = -1 \operatorname{sgn} \lambda_2$.

It is required to find a u^* and α^* such that

$$P^*(u^*, \alpha^*) = \underset{\alpha}{\text{Min}} \; \underset{u}{\text{Max}} \; H$$

$$0 \leq w \leq 4.$$

In the example of tension bar in Chapter 5, the type of optimal strategy α^* is shown for different types of loading. Hence minimization with respect to α is not presented in detail. Only the maximization with respect to u is presented.

Table 8.2 contains the value of $H(u, \alpha^*)$ different values of u. Fig. 8.1.a illustrates the loading strategies considered for solution. It can be seen that Trial 4 in which the section is loaded to 4 tons and fully unloaded has the maximum value of P^*. This example shows that loading and subsequent unloading of forces will give a positive payoff which is maximum when the section is loaded to the maximum. Hence the optimal strategy of player 1 is to load each section to the maximum force level possible and then unload it fully or partially. This load strategy is called the worst course of loading. The optimal strategy α^* corresponds to the worst course of loading. The optimal force deformation

relation is the same as that shown in Fig. 5.9.e.

8.5.4 Example of Two Bar Truss:

The two bar truss shown in Fig. 5.11.a is subjected to two loads whose normal load conditions are defined as follows.

$$0 \leq w_1 \leq 5 \text{ tons}$$

$$0 \leq w_2 \leq 4 \text{ tons}$$

It is required to find out the task curve satisfying the following conditions.

(i) The joint c should not deflect more than 1" horizontally and 1" vertically,

(ii) The permanent deformations of the members should not be more than 0.002 in/in.

Formulation of the Problem: Variables are defined as in the example of two bar truss in Section 5.5.3. The duration of loading S is taken as 12. The equations may be formulated as follows.

$$P = \int_0^{12} - \left\{ 100 \frac{(y_1 + y_2)}{\sqrt{2}} u_1 + 100 \frac{(y_1 - y_2)}{\sqrt{2}} u_2 \right\} ds$$

$$\frac{dy_1}{ds} = \frac{u_1 + u_2}{\alpha_1 x \sqrt{2}}, \quad \frac{dy_2}{ds} = \frac{u_1 - u_2}{\sqrt{2}\, \alpha_2}$$

$$\frac{dw_i}{ds} = u_i \quad , \quad i = 1, 2$$

$$\frac{d\lambda_1}{ds} = \frac{100}{\sqrt{2}} u_1 + 100 \frac{u_2}{\sqrt{2}} \quad , \qquad {}_1 (12) = 0$$

$$\frac{d\lambda_2}{ds} = 100 \frac{u_1}{\sqrt{2}} - 100 \frac{u_2}{\sqrt{2}} \quad , \qquad {}_2 (12) = 0$$

$$H = -100\left\{ \frac{(y_1+y_2)}{\sqrt{2}} u_1 + \frac{(y_1-y_2)}{\sqrt{2}} u_2 \right\} + \lambda_1 \frac{u_1+u_2}{\sqrt{2}\,\alpha_1}$$
$$+ \lambda_2 \frac{(u_1-u_2)}{\sqrt{2}\,\alpha_2} + \lambda_3 \frac{(u_1+u_2)}{\Sigma\sqrt{2}} + \lambda_4 \frac{(u_1-u_2)}{\sqrt{2}}$$

The constraints are

$\alpha_i \leq \alpha_i(0)$ for loading of the section

$\alpha_i = \alpha_i(0)$ for unloading and reloading

$\alpha_i = \alpha_i(0) \left[1 - \left|\frac{F_i}{20}\right|\right] \geq 0 \quad i = 1, 2$

$0 \leq w_1 \leq 5$ tons

$0 \leq w_2 \leq 4$ tons $\qquad y_1 + y_2 \leq \pm\, 0.01414$

$0 \leq |u_1| \leq 1.25 \qquad y_1 - y_2 \leq \pm\, 0.01414$

$0 \leq |u_2| \leq 1$

The method of maximization of Hamiltonian H with respect to u is illustrated in Table 8.3. Two trials with same value of α and different values of u are shown for this purpose. Several trials are needed to arrive at an optimum solution. The force-deformation relation is shown in Fig. 8.2.

8.5.5 Application to Statically Indeterminate System:

The application of the differential game formulation of structural design to statically indeterminate systems is straightforward. Unlike in statically determinate system, the strategies $\underline{v}$ are also to be treated as unknown strategies. Also, the state variable equality constraints due to compatibility conditions bring in additional difficulties.

The problem in its general form can be solved only with the aid of an efficient algorithm.

8.5.6 Inelastic Material With Creep Effects:

When the structure is to be made of materials with time-dependent behaviour, the formulation presented in Section 8.4 can very well be applied provided the loads are continuous functions of time. With the following changes, the problem can be formulated to take care of creep effects.

(a) Time t is taken as the stage variable in place of S.

(b) The state equations contain terms that account for the creep effects.

(c) The payoff function should be modified taking the additional energy due to creep deformations.

When steady state creep is considered, the state equation may be written in the form,

$$\frac{dx_i}{ds} = \frac{v_i}{\alpha_i} + a_i |F_i|^{\gamma} \operatorname{sgn} F_i, \quad i = 1, \ldots, n$$

8.6 TIME DEPENDENT DEFORMATION WITH JUMPS IN STATE VARIABLES

The formulation presented in Section 8.4, holds good when the state variables are continuous functions of θ or t. As discussed in Chapter 5, it may happen that loads, when expressed as functions of time, may show instantaneous changes with respect to t. The effect of such sudden loading is to cause a jump in state variables in an otherwise continuous motion with respect to t. Ordinarily, optimal control theory and differential game are not applicable for such situations. However, the problem may be reformulated in a manner similar to the one discussed in Chapter 5, so that it reduces to a differential game problem. The solution aspects are not discussed as the problem is too complex, and requires further study and deeper understanding.

Reformulation of the Problem:

The reformulation is to be done exactly in the same manner as described in Section 5.7.1. However, the following differences may be noted.

(i) In optimal control problem with given loading path, the number of jumps r in a sequence is known. In differential game, as the loading path is also unknown, the number of jumps that totally occurs are not known. For convenience, it may be assumed that altogether only r jumps occur in the service period.

(ii) The loading path, being known a priori in optimal

control formulation the time t_j at which the jth jump occurs is also known. Hence the selection of the value of the parameter w_0, as 0, or 1 is not difficult. In differential game, the time of choice of w_0 as 0 or 1 is totally unknown as the functions $\underline{w}(s)$ themselves are unknown. <u>Hence w_0 is assigned as an additional strategy in the hands of player 1, who chooses the value 0 or 1 at appropriate time such that</u> the payoff is maximized.

The problem may be mathematically stated in a manner similar to that given in Section 5.7.2 with the introduction of $\{w_0, u_1, \ldots, u_m\}$ as maximizing strategies of player 1 and $(w_1, \ldots, w_m)$ as the state variable in the load space.

8.7 PROBLEMS WITH CHANGE IN POSITION OF LOADS

In the problems formulated hitherto, no change in the position of loads is considered. The uncertainty arising from the change in position can also be easily taken into account in the above formulation. Change in position of loads is very common in bridges, buildings etc. where moving loads, and movable live loads are acting. If all the possible positions occupied by the loads can be located beforehand, the uncertainty can be accounted as follows:

Consider, for example, that at any stage s, a load w_1 occupy any of the 4 positions L_1, L_2, L_3 and L_4 on a structure. The exact position out of the four given above,

occupied by w_1 are not known. The following change in the problem formulation can be made to compensate for uncertain position of loads.

Instead of a single load w_1, assume that 4 loads w_{11}, w_{12}, w_{13}, w_{14} are acting at locations L_1, L_2, L_3 and L_4 respectively such that only one of them may occur at any time t. Mathematically the above-mentioned requirement may be written as

$$w_1(s) = w_{11}(s) \cup w_{12}(s) \cup w_{13}(s) \cup w_{14}(s)$$

and

$$w_{11}(s) \cap w_{12}(s) \cap w_{13}(s) \cap w_{14}(s) = \phi$$

The introduction of the concept of sequential application of loads helps the above mentioned adjustments.

8.8 DISCUSSION

The validity of the method as a conservative approach to design under uncertainty, and some of the special features of the formulation are discussed in this section.

8.8.1 Comparison With Existing Methods of Design:

The proposed method, while satisfy the basic laws of motion that are the fundamental axioms of any structural mechanics problem differs from the existing methods of both utilitarian and non-utilitarian design processes. This

difference is only in the procedure adopted, and not in the basic requirements. The existing methods of design make use of force-deformation relations as known parameters that help to relate stresses, strains, displacements etc. in terms of the design variables namely cross sectional dimensions. In the proposed method, the force-deformation relations, that are required for the serviceability requirements consistent with the basic axioms of structural mechanics, are constructed and their cross sections that have similar force-deformation relations, are chosen. The force-deformation relations therefore serve as an intermediate step of decision to separate the choice of cross section from the analysis of structure for design requirements. This method has specific advantages in the design and optimization process, Some of the advantages are discussed in Chapter 10.

8.8.2 Comparison with the Optimal Control Formulation of Structural Design:

The differential game formulation is a direct extension of the optimal control theory approach given in Chapter 5. The essential characteristics of the optimal control formulation are retained in the extension also. The discussions that had in Chapter 5 on the validity of optimal control formulation as a design tool holds for differential game model as well. The solutions obtained by the two methods differ as described below.

The control problem assumes that the path of external load is known as a function of s or t . The optimal solution of the problem, therefore, is valid only for the given path of loading. In differential game, the loading path is considered uncertain. The solution process therefore, checks for all possible paths of loading and the force-deformation relation that minimizes the payoff for the worst path of loading is picked out. As any other path of loading can lead only to a less payoff, the solution obtained is conservative for all courses of loading other than optimal one. Thus differential game gives a conservative or pessimistic solution for an uncertain situation of loading.

8.8.3 Justification of the Rule of Saddle Value:

The acceptance rule in game theory (see Table 2.6) that picks out the optimum strategies of both players is the saddle value theorem (or minimax theorem). According to this rule, the optimal strategy or payoff corresponds to the maximum of the minimum gain the player 1 can have. Similarly player 2 has the optimal strategy corresponding to the minimum of the maximum gain his opponent can have or loss he himself suffers. If a saddle point exists,

$$\min \max P = \max \min P = P^*$$

The quantity P^* is called the value of the game. If player 2 sticks on to its optimal strategy namely force-deformation relations, it can be sure of a payoff equal to the value of

the game, even if the opponent also sticks on to its optimal strategy. Any deviation from the optimal strategy by nature causes only a reduction in the payoff. Thus the optimal strategy of structure is obtained by assuming that nature will also choose the optimal strategy. In other words, nature is assumed to be a rational player, who always strives to maximize the benefit.

Serious objections may be raised on this assumption of rationality when it is known that nature does not care to have the optimum outcome. It may be feared that such a design will lead to very uneconomical structure. It can be seen from the discussion below that such a fear is not with any basis.

Two assumptions are involved in the formulation of the game problem related to this aspect. They are: (i) the nature is a rational player who strives to maximize the payoff, namely the damage measured in terms of the energy by choosing an optimal path of loading and (ii) all paths of loading are equally likely. It is to be noted that the magnitudes of the loads are kept below the limit values within which the structure is to be serviceable. Within this normal load condition no compromise on serviceability and safety is permitted and hence the solution is obtained for the optimal path of loading. Thus the worst case attitude is taken with regards to path of loading, and not with respect to magnitude. A design for a worst magnitude of load not considering the risk involved may lead to highly uneconomical designs, whereas

a worst case attitude about the path of loading may not lead to such uneconomic designs because it is the magnitude of the loads that governs the dimensions and cost of structure more than the path of loading.

Thus the pessimistic approach to design is with regard to serviceability for a limited magnitude of load. At such a normal condition, the assumption that all allowable paths are equally likely may also be justified.

8.8.4 Existence of Saddle Point:

As mentioned before the existence of a saddle point is one of the prerequisites for pure strategy solutions. However, in structural design problems, the non-existence of saddle point is not a calamity. If no saddle point exists, the problem may still be treated as a game problem and a minimax solution may be sought for instead of a saddle point solution. The necessary and sufficient conditions for existence of a saddle point are given by Eqs. 8.20 and 8.21. By definition, the proposed game is a game of perfect information in which the state variables $\underline{x}$ at any s are known to both players. According to game theory, saddle point does exist for games with finite strategies that are of perfect information. However, the existence of saddle point in the structural action game is shown below from physical considerations.

<u>To show that for the game stated in Section 8.4.12, at least one saddle point does exist, and if more than one saddle point exists, all of them will have same value</u>

It is required to show that

(i) $\min\max \underline{P} = \max\min P = P^*$

(ii) $P^*(\underline{v}^*, \underline{\alpha}^*, \underline{u}) \leq P^*(\underline{v}^*, \underline{\alpha}^*, \underline{u}^*) \leq P(\underline{v}, \underline{\alpha}, \underline{u}^*)$ and

(iii) P^* is the same for all optimal pair $(\underline{v}^*, \underline{\alpha}^*)$, $\underline{u}$

(i) Assume that: $\min_{\underline{v},\underline{\alpha}} \max_{\underline{u}} P \neq \max_{\underline{u}} \min_{\underline{v},\underline{\alpha}} P$.

Then from game theory it is known that

$\max\min P < \min\max P$.

Consider now a discrete version of the differential game with finite number of strategies. Let player 1 has k strategies or k sets of the loading path, and player 2 has ℓ sets of force-deformation relations leading to ℓ structures. The payoff matrix (P) for this discretised problem is a k x ℓ matrix, each row of which represents a path of loading and each column a structure. The payoff matrix is shown in Table 8.4. The element P_{ij} is the payoff of jth structure for ith path of loading. The physical significance of the payoff is already described in Chapter 5 as the energy which is a sum of the energy dissipated during the period 0-S of loading and the recoverable energy at the final stage S. The minimum of any

the energy payoff is greater than the value. Thus

$$P(\underline{v}^*, \underline{\alpha}^*, u) \leq P^*(\underline{v}^*, \underline{\alpha}^*, \underline{u}^*) \leq P(\underline{v}, \underline{\alpha}, \underline{u}^*)$$

(iii) According to game theory if more than one saddle point exists, all of them will have same value (152). From structural point of view, it shows the existence of more than one structure with same amount of damage.

Note: In cases where the various paths of loading have no effect on energy just as in elastic systems, a valley with constant minimum energy value may occur instead of a saddle point.

8.8.5 <u>Safety and Serviceability:</u>

The optimal strategies $\underline{v}^*$, $\underline{\alpha}^*$ corresponds to the worst path of loading. If the structure is designed with $\underline{v}^*$, $\underline{\alpha}^*$, then it will have less damage at any loading. Hence serviceability is guaranteed.

<u>To show that the structure with $\underline{F}^* = \underline{F}(\underline{v}^*)$ is safe for all paths of loading provided the limit values are not exceeded.</u>

In order to have safety, the force-deformation relations obtained must be completely obtained for all combinations of loading within the limit values. In the proposed game, player 1 strives to maximize the energy dissipated and the energy conserved in the system. The

dissipation is maximum when energy is dissipated to the fullest extent at all sections. In inelastic material, the dissipation is maximum when each section is loaded to the maximum possible force level by taking suitable combinations of loading and then unloading them. This process of loading to the fullest extent is done for all sections, and the force-deformation relations obtained by the optimal solution will have the maximum force and maximum deformation possible at each section. Hence if the structure is designed with these curves as the force-deformation relations, it will be safe for all paths of loading provided the limit values are not exceeded.

8.8.6 Stability:

Stability is another requirement in a design for normal load conditions. It may include (i) Elastic stability (ii) stability against creep buckling and (iii) inelastic stability. A structure is said to be unstable, when atleast one of the deformations become very large as the external load is infinitesimally increased. Hoff (158) and Ziegler (159) advocate that such a dynamic criterion is the realistic one not only in usual dynamic and static situation but also under non-conservative loads. This criterion is implicitly satisfied when $\underline{\alpha}(s)$ is taken as greater than zero, which limits the increase in the deformations to finite values. This condition is satisfied for the whole service life from O-S. Hence, it may be noted that instability of the second and third types may be absent in the structure so long as the limit values of

the loads are not exceeded. Elastic instability or buckling requires additional considerations. However, the problem of stability requires further examination and no conclusions are arrived at in this investigation on the stability of the system.

8.8.7 Special Features of the Method:

Differential game formulation has all the special features of the optimal control formulation discussed in Chapter 5. In addition, the following points may be noted.

(i) The uncertainty in the sequence and timing of the load are automatically taken care of by the game formulation. A design with minimum damage for the worst course of loading is obtained. Uncertainty in the direction and position of load may also be taken care of by proper modifications suggested in Section 8.7. In the existing method also, the aspect of the problem is taken care of by placing the load in such a way that worst effect is produced on the structure. Therefore, the pessimistic solution due to game approach no way leads to a more costly design than that is obtained by the present design process. A proper algorithm will automatically take care of the uncertainty, while in the present design practice, any omission on the part of designer to correctly locate the worst positions may lead to unsafe designs.

(ii) The maximization of payoff is done by player 1 by

stressing the cross-sections to the maximum extent possible, and by subsequent unloading. The presence of any external load at the final stage would give a negative value of payoff (since the net value of payoff is negative) which will not be a best policy for player 1 as the payoff is less. Hence, a best strategy for player 1 is to unload the structure fully at the final time S so that the payoff is the loss of energy from O - S which is a positive quantity.

(iii) When creep deformation is present, the best strategy for player 1 would be to apply all the loads at the earliest instant possible and to allow the worst combination to continue until the final time is reached. At the final time the loads may instantaneously be removed.

(iv) For player 2, the best strategy $\underline{\alpha}$ would be the one set that allows the sections or joints below load to deflect more than other points on the structure so that the sum of external work and complementary work is increased due to the increase of the deflection thereby leading to a less value of payoff.

8.8.8 Extensions of the Work:

The method proposed in this section has been studied in detail only for a very special case of statically determinate system. It can be extended to more complex cases. All extensions proposed for optimal control formulation may be

attempted for differential game as well.

8.9 SUMMARY

The normal play of the structural action game presented in Chapter 7 is studied in detail, and a design method is formulated. The problem is formulated as a differential game problem which is an extension of the optimal control formulation presented in Chapter 5. The loads are assumed to be within specified limit values, and the loading process is considered uncertain. The solution of the problem gives the task curves required for serviceability under uncertain loading process. Extensions to cases where time-dependent behaviour is present and to a case where loads are instantaneous applied in a long process causing jumps in state variables are discussed. The method is illustrated through examples of tension bar, and two bar truss. The validity of the method is discussed in Section 8.8, showing that the structure if designed with the optimal force-deformation relations is safe and serviceable for any path of loading provided the magnitude lie within normal load range.

CHAPTER NINE

SURVIVAL PLAY OF THE STRUCTURAL ACTION GAME

9.1 INTRODUCTION

The normal play of the structural action game has been formulated in the preceding chapter. This play of the game chooses the optimal force-deformation relations for safety and serviceability requirements under normal load condition. The subsequent survival play of the game for cost-effectiveness criterion is formulated in this chapter. The strategy of structure is the same force-deformation relations, chosen in Chapter 8, while that of nature is the uncertainty in the information of probability density functions and the timing of the loads. Wald's minimax game (88) is applied to find the optimal strategy of structure. The method proposed in this chapter is an extension of the cost-effectiveness design presented in Chapter 6 considering the uncertainty in the probability density functions.

Statistical game and minimax rule are briefly presented in Section 9.2. The problem is stated in Section 9.3 and a brief outline is given in Section 9.4. The detailed formulation and the solution process are given in Section 9.5. The method is illustrated by means of a simple example. Section 9.6 is a brief discussion on the formulation.

9.2 STATISTICAL GAME AND MINIMAX RULE

9.2.1 Bayesian and Minimax Models in Structural Design:

The formulation of structural design as statistical decision game is not new. Turkstra (22) seems to be the first to attempt a Bayesian approach to the solution of structural design problem. This attempt is followed by Benjamain's (23) extensive form of Bayesian decision for earthquake loads and Sexsmith's (25) approach to the design of beams.

A Bayesian decision model is extremely useful in situations where the decision maker can improve the statistical information through experiments. When additional information is available, the a priori distribution (assumed or objectively measured) can be modified in the light of the additional data obtained from experiments. In a structural design problem, the designer may have neither the resources nor the time to conduct an experiment for improving his knowledge of the probability distribution of loads. Hence, the direct use of Bayesian decision model may not be helpful as a decision tool for uncertainty in structural design. However, a Bayesian model can be used effectively for the improvement of codes from time to time, as and when additional data are obtained. For example, consider the case of a probability density function for wind loads. Every year additional data may be obtained by conducting observations. This additional information may be used to improve the existing distribution through a Bayesian decision model. This process of refinement may be continued

making the function closer and closer to reality.

Simultaneously with this refinement, the designer must be able to work with the existing information giving due considerations to the inaccuracies and to arrive at reasonably accurate designs. Two considerations are necessary in this process. In one, the inaccuracy in the data or more precisely the degree of confidence with which the data can be used must be transmitted to the designer. This can be specified by specifying the bounds to the real unknown functions, which bounds may be reduced gradually by the Bayesian experimental process. The second consideration is a decision model for using the data with limited uncertainty to arrive at reasonably acceptable decisions. A minimax game of Wald's type is proposed in this chapter for the latter situation. This model takes a conservative decision within the bounds specifying the uncertainty.

9.2.2 Statistical Decision Game:

Statistical decision game is assumed to be a game played by the decision maker against nature. Nature is designated as player 1 who selects a probability distribution, of its choice. Player 2 is the decision maker who makes a decision under the uncertain information of the probability density function. Such models are extensively used in the decisions applied to statistical problems. It is based on the principle that a statistical procedure should be evaluated by its

consequences at the particular circumstance. In 1939, Wald adopted this concept for statistical problems. Later in 1950, the minimax game is proposed by him.

Wald's model of decision theory has the formal structure of game theory and is a special case of the latter. Game theory considers an intelligent opponent while in statistical game, nature is a neutral player who may not care for the outcome. In spite of this fact, Nature is assumed to be a rational player in the minimax game problem, leading to a pessimistic solution as described below.

9.2.3 Minimax Rule:

Let the decision maker (player 2) has a set of strategies $\underline{\alpha}$ of which he has to choose one that gives optimal outcome under the situation. Player 1, (Nature) has a set of strategies $\underline{\theta}$ of which it is assumed to choose an optimal one. For each combination of strategies α_i belonging to $\underline{\alpha}$ and θ_j belonging to $\underline{\theta}$, there is an outcome $u(\alpha_i, \theta_j)$ which is a measure of the loss suffered by the decision maker. With the knowledge of $u(\underline{\alpha}, \underline{\theta})$ the decision maker can select an optimal strategy $\underline{\alpha}$ as follows.

$$u(\underline{\alpha}) = \min_{\underline{\alpha}} \max_{\underline{\theta}} u(\underline{\alpha}, \underline{\theta}) \tag{9.1}$$

The rule thus states that the strategy that gives the minimum of all maximum losses is the acceptable one. If the decision maker sticks on to the optimal strategy any deviation by

nature from its own optimal strategy results in a less loss to the former. On the other hand, if the nature sticks on to its own optimal strategy, decision maker may or may not have another strategy that has a greater gain. Thus the minimax rule is a pessimistic one that assures the least loss a decision maker can afford. Unlike in game theory, a search for saddle point is not necessarily made in minimax game. One such saddle point may or may not exist. Since no saddle point condition is searched, the minimization and maximization can be done independently from one another.

9.3 STATMENT OF THE PROBLEM

(i) Assumptions: All assumptions made in Chapter 6 hold good in the present formulation.

(ii) The Data Given: The following informations are given

(a) The force-deformation relations required for safety and serviceability under normal load conditions are known by a design described either in Chapter 5 or in Chapter 8.

(b) The external loads $\underline{w}$ acting on the structure are random. The variables $\underline{w}$ are assumed to be normally distributed. However, either mean or standard deviation is assumed to be unknown within two bounds as described in Chapter 4. Within the bounds, all probability density functions are equally likely.

(iii) It is required to carry out the cost-effectiveness design considering the uncertainty in the probability density functions.

9.4 BRIEF OUTLINE OF THE METHOD

The present formulation constitutes the survival play of the game described in Chapter 7. The strategies of the structure are the force-deformation relations, of which the initial portions are already known from serviceability considerations. It is required to select an optimum set of force-deformation relations (task curves). The strategies of nature are the probability density functions the range of which is given. Nature may choose any combination of these functions. Also, the timing of the application of the loads on the structure is also considered as a strategy belonging to nature. By minimax rule, the structure chooses a set of force-deformation relations as its optimal strategy corresponding to the worst course of action of nature (worst combination of distribution within the bounds) so that the loss it suffers in the minimum of the maximum loss among all alternatives. The cost-effectiveness factor K_T given by Eq. 7.1 is the pay-off. Nature can maximize the pay-off by maximizing the probabilities p_i by choosing proper density function and the costs of failure C_i by choosing appropriate time at which the cost of failure is high.

9.5 FORMULATION OF THE PROBLEM

The formulation for cost-effectiveness design under known probability holds good in this case also with the following changes.

(i) In the cost-effectiveness design under risk situation,

the probabilities are known quantities for any specified state of structure. But, it is not fully known, in the game formulation because of the unknown density functions associated with the loads.

(ii) The time t of each mode of failure is such that the cost of the particular mode of failure is high.

(iii) The optimum policy of structure corresponds to the min max K_T instead of min K_T taken in cost-effectiveness design.

9.5.1 Strategy of Nature:

The uncertainty in the information of the exact probability density functions is assigned to nature as its strategy. Nature is assumed to choose a combination of such functions so that it receives maximum benefit under the competitive moves.

One way of mathematically stating the uncertainties is described in Chapter 4, where the standard deviations $\underline{\theta}$ are treated as unknown parameters. This method of definition of the uncertainties is further discussed here.

The type of distribution, the mean and standard deviation are the usual parameters needed to define a density function. Any one or all of them may generally be considered unknown. As a first attempt, let us assume that the distribution belongs to a class, say normal distribution. Thus let $N(\mu_1, \theta_1), \ldots, N(\mu_m, \theta_m)$ be the normal distributions for m

loads acting on the structure.

In each of the distributions, either the mean or deviation may be treated as unknown and other parameter as known.

Case 1. Mean μ is unknown deviation θ is known: This uncertainty is not with regard to the shape but with regard to the position of the function. When it is difficult to exactly locate the mean, this sort of uncertainty may be assumed. Let $\_$ be such that

$$\mu_i^L \leq \mu_i \leq \mu_i^u , \quad i = 1, \ldots, m \tag{9.2}$$

In this case the choice of a proper μ is the strategy of nature. This strategy is called μ strategy. μ strategy is shown in Fig. 9.1.a.

Case 2. Deviation θ is unknown, mean μ is known: This uncertainty refers to the shape of the normal distribution. Very often it is difficult to correctly estimate the shape experimentally. In such cases, θ may be treated as an unknown parameter such that

$$\theta_i^L \leq \theta_i \leq \theta_i^u , \quad i = 1, \ldots, m \qquad (.3)$$

The choice of a proper θ is the strategy of nature. This strategy is called θ - strategy to distinguish it from μ-strategy. The θ - strategy is shown in Fig. 9.1.b.

Timing of the Application of Load: The cost-effectiveness factor K_T given by Eq. 7.1 is a function of time. Hence, if the timing of the application of load is properly chosen by nature, the value of K_T can be maximized. This is possible by choosing the appropriate time of application of load so that the cost of failure $C_i(t)$ is maximum. Hence, timing of load is also considered as a strategy of nature.

9.5.2 Minimax Solution:

Let K_T represents the cost-effectiveness factor given by Eq. 6.26 as

$$K_T = 1 + \Sigma\, p(\underline{w})\; K(\underline{w}) \tag{9.4}$$

This factor is taken as the pay-off which is a loss to the player 2 (structure). In the method proposed in Chapter 6, $p(\underline{w})$ for any $\underline{w}$ is taken as a known quantity. In the minimax game, $p(\underline{w})$ is unknown since $\underline{\mu}$ or $\underline{\theta}$ is unknown. The maximizing player chooses a $\underline{\mu}$ or $\underline{\theta}$ and the time t such that K_T is maximum. Simultaneously, the minimizing player chooses the force-deformation relations such that K_T is minimum. Let the strategy of player 2 be denoted by (F-y). The problem can be stated as follows:

Find an $(F\text{-}y)^*$ such that the pay-off

$$K_T^*((F\text{-}y)^*) = \underset{(F\text{-}y)}{\text{Min}}\ \underset{\underline{\mu}\text{ or }\underline{\theta},t}{\text{Max}} \left[1 + \Sigma\, p(\underline{w})\; K(\underline{w})\right] \tag{9.5}$$

Eq. 9.5 is called the minimax rule of the survival play of structural action game. The maximization and minimization may be carried out separately for any load $\underline{w}$ chosen.

9.5.3 <u>Maximization of Pay-off</u>:

The terms $p(\underline{w})\ K(\underline{w})$ in the Eq. 9.4. are all positive quantities. Hence, maximization of each term would lead to a total maximum. For any $\underline{w}$, nature must find a suitable combination of $\underline{\mu}$ or $\underline{\theta}$ such that $p(\underline{w})$ is the maximum. Also, the time t is chosen such that $C(\underline{w})$ is maximum. This freedom of choice is allowed to player 1 within the bound given by Eq. 9.2 or 9.3. If $\underline{\theta}$ happens to be the strategy, the upper bound would very often give the maximum p when load is at the tails of the distribution. However, if $\underline{\mu}$ is the strategy, careful consideration is necessary in the choice of the value $\underline{\mu}$. For any load w_i, the distribution function may be placed in such a way that the mean μ_i is within the limits and the probability $p(w)$ is maximum.

In the practical solution of problems, the probability of regions with same cost of failure is added up. Hence, the regionwise maximization with respect to $\underline{\mu}$ or $\underline{\theta}$ strategies would become necessary. This can be done by finding the maximum probability p_i of the region by choosing appropriate probability density functions within the bounds.

9.5.4 Illustrative Example:

The maximization of pay-off K_T with respect to μ-strategies or θ-strategies may be illustrated through a simple example of three parallel bar system.

Consider a three bar system as shown in Fig. 9.2.a. It is subjected to a load w which is random. The probability density function of w is considered normal with either μ or θ known. Consider that the structure has force-deformation relations as shown in Fig. 9.2.b. The cost-effectiveness is given by

$$K_T = 1 + p_1 . K_1 + p_2 . K_2 + p_3 K_3$$

where

$$K_1 = \frac{C_1}{C_s}$$

$$K_2 = \frac{C_2}{C_s}$$

$$K_3 = \frac{C_3}{C_s}$$

and p_1, p_2, p_3 are the probabilities of the three states of serviceability, damage and collapse respectively. For purpose of comparison, the distribution is assumed as N(40, 5) and the cost-effectiveness factor K_T is calculated as 0.3469312.

Case 1. Nature with μ strategy: In this case, is assumed to be unknown such that

$$36 \text{ tons} \leq \mu \leq 40 \text{ tons}.$$

and $\theta = 5$.

The maximum values of p_1, p_2, p_3 in this case are found out. The max p_1 corresponds to the lower limit and the other two correspond to upper limit.

Case 2. Nature with θ-strategy: In this case, θ is assumed to be unknown such that

$$5 \leq \theta \leq 10$$

$$\mu = 40 \text{ tons}$$

The maximum p_1 p_2 corresponds to lower bound and p_3 corresponds to upper limit.

The example illustrates how the value of K_T changes due to maximization. It may be noted that the probabilities in each row for μ-strategy or θ-strategy will not get added up to unity.

9.5.5 Minimization of Pay-off:

Minimization of pay-off can be done exactly in the same manner as proposed in Chapter 6. A policy iteration procedure may be followed. In the policy iteration procedure a maximizing operation may also be added in the policy evaluation operation. At this operation nature is assumed to select the μ-strategies or θ-strategies and time in such a way that K_T is maximum. The maximum value of K_T can be obtained by a manner similar to the one explained in example given in the above section. When probability distributions of the internal forces are found

out from that of external loads, by means of equilibrium equations. The two limits may be estimated in such a way that the limits in the case of forces are the possible extremes knowing the bounds to the distribution of $\underline{F}$ the maximization can be carried out. The flow chart for the new policy iteration scheme is shown in Fig. 9.3.

9.6 DISCUSSION

The uncertainty in the information of probability distributions is considered as a strategy belonging to nature and a game is formulated. The minimax solution gives the optimal set of force-deformation relations that minimize the cost-effectiveness for the worst combination of probability density functions (that maximizes the pay-off) within the specified bounds. So long as a distribution is unknown, such a conservativ estimate taking a worst case attitude is admissible. The error due to the worst case approach depends on the bounds specified rather than the adoption of minimax rule. Hence, the bounds must be obtained in a more accurate way.

The process of maximization becomes too complex due to the complexity of normal function. It is worthwhile to study the use of algebraic approximations of normal function or some distributions that can be expressed by algebraic functions so that the differentiation with respect to μ can be easily done. Such approximations in the distribution is permissible on account of the worst case attitude taken in this formulation.

Also, the expansion of the probability density function f(w) into a series of the form known as Gram-Charlier Series of Type A (160) given by

$$f(w) = \sum_{j=0}^{\infty} C_j \; Z^{(j)}(w)$$

where $C_0 = 1$, $C_1 = 0$, $C_2 = \frac{1}{2}(\theta^2-1)$, $C_3 = -\frac{1}{6}\mu^3$ etc. may be attempted to use. The series may be truncated at convenient level of accuracy. $Z(w)$, $Z^{(1)}(w)$, $Z^{(2)}(w)$, $Z^{(3)}(w)$ for a normal distribution are tabulated by Owen (146). μ^3 is the third moment about the mean of w. The computation of multivariate distribution for cost-effectiveness design described in Chapter 6 and for the maximization of K_T with respect to $\underline{\mu}$ or $\underline{\theta}$ would be easily done using such algebraic functions. However, this extension requires considerable study before any concrete proposal can be made about the applicability of the method.

9.. SUMMARY

The survival play of the structural action game is presented in this chapter. The cost-effectiveness design is reformulated taking the maximization of the factor K_T by nature choosing appropriate probability density functions. Two types of uncertainty have been discussed; one is the uncertainty in the mean and the other the uncertainty in the deviation.

By this design, the force-deformation relations are obtained taking the uncertainty in the statistical property also into account.

CHAPTER TEN

CHOICE OF STRUCTURAL MATERIALS AND MEMBERS

10.1 INTRODUCTION

In Chapters 5 to 9, several decision techniques for obtaining the task curves for various conditions of loading have been presented. The curves are obtained as the optimal course of action or behaviour required to be followed by the structure under appropriate conditions of loading such as certainty, risk or uncertainty and also taking into account the non-measurable uncertainties. No specific material or cross section has been proposed to use in this process, (except in the case of time-dependent behaviour) and instead certain behaviour idealizations like linearity, nonlinearity and inelasticity etc. have been made. Also, no uncertainty or randomness in the behaviour of the structural cross section is taken into account. Hence, the material and cross section of the members remain to be selected. This selection is to be done considering the uncertainty or randomness in the behaviour of material and also the economic aspects of the problem. The problem is stated in Section 10.2. Since the force-deformation relations can be anywhere on the conservative side of task-curve, the designer is free to choose any suitable cross section of any suitable material with behaviour identical to that of task curve. This point and certain other aspects about structural behaviour are discussed in Section 10.3.

In Section 10.4, the simplification achieved in optimization by this method is discussed. The direct design, unlike the existing optimization methods, allows a memberwise optimization (selection of structural material and cross section included) because of the separation of structural analysis from optimization. The choices under conditions of certainty, risk and uncertainty are discussed in Section 10.5-10.8. Section 10.9 contains a brief discussion on the methods suggested in this chapter.

Considering the volume of computational work involved, several simplified procedures are proposed to use depending upon the accuracy needed.

10.2 OUTLINE OF THE METHOD OF CHOICE

10.2.1 Statement of the Problem:

The decision problem involved in this process may be stated as follows.

Decision Variables: The material, shape and dimensions of the cross sections are the variables to be chosen in a decision process. In some problems, especially when time-dependent behaviour is considered, the type of material has to be chosen a priori. The choice, therefore, reduces to only that of the shape and dimensions of the cross sections.

Criterion: The cost-effectiveness criterion given by

$$C_T = C_S \left[1 + \sum_i p_i \frac{C_i}{C_S} \right] \tag{10.1}$$

is optimum. In this expression p_i and C_i are kept constant during this stage of design when C_i is other than C_s.

Constraints:

(i) The cross sections chosen should have their accepted force-deformation relations coinciding with the corresponding task curves fully corrected for all uncertainties.

(ii) The material should have the same sort of idealized behaviour, as the idealization made in the state equations.

(iii) Apart from structural considerations, the material chosen should be suitable for other requirements like aesthetics, durability etc.

(iv) The dimensions and shape chosen must be conformal to the aesthetic needs.

Decision Process: The decision is to be done under certainty, risk or uncertainty depending upon the information pattern available about the behaviour of the structural cross sections.

10.2.2 Method of Choice:

The problem stated above does not require a rigorous mathematical framework to solve it. Instead a graphical procedure as described below would be simpler. The choice of the cross section is based on a matching of the force-deformation relation of the section and the task curve required at the

section for safety, as shown in Fig. 10.1. The matching can be done not only with respect to forces but also with respect to deformations, stiffness and maximum capacity of deformation. In this matching, the designer gets enough freedom of choice, as described below.

(i) If the curve is coinciding with the task curve or lies on the conservative side <u>on all cross sections</u> structure is safe. Thus no point by point coincidence of the two is necessary.

(ii) Designer can choose any curve and the corresponding cross section on the conservative side of task curve, so long as the constraints stated in Section 10.2.1 are not violated. This would allow to choose the section from a wider class of materials. This choice no way affects the task curves.

(iii) So long as the behaviour of material matches with the behaviour idealization made in the design of task curve, any material can be chosen provided it satisfies other constraints stated in Section 10.2.1.

The method of solution is quicker and easier if force-deformation relations of various materials are made available in the form of graphs and charts. In such a case, the section can be chosen by matching the task curve to the curves provided in the chart. In precast construction work, the manufacturer can supply such information along with the

members marketed in the form of a specification. Actual stress-strain curves of several metallic materials (161) and reinforced plastics (162) and elements are published in the form of Handbooks (161) by the Defence Department, Washington. Similar charts and graphs for structural sections of different materials may be prepared for regular use.

10.3 MATERIAL AND BEHAVIOUR CONSIDERATIONS IN THE CHOICE OF SECTIONS

10.3.1 Materials in Design:

Traditionally, material consideration in design has been neither systematic nor rational. The designer is constrained to choose certain materials a priori and carry out the design, and there is not enough freedom to try the effectiveness of alternative materials with respect to cost or weight. This is because of the dependence of the entire analysis of structure on the force-deformation relation. As discussed in Chapter 2, any change on the relation would demand a revision of design. This difficulty is eliminated by the direct design concept introduced in this investigation. The design for external load is free of material consideration and the designer can choose the material on completion of the design for task analysis. Any alternative trials of materials will no way affect the design already carried out.

Several new materials and composites are being

developed the use of which in structural design need be investigated. The concept of composition of materials allows designer to have need-oriented materials and the direct design would promote the concept of composition of materials.

10.3.2 Interaction of Force-deformation Relations and Plastic Yield Condition:

The interaction of the various force-deformation relations at any section is to be taken into account in the computation of individual force-deformation relations. For example, the presence of axial forces or twisting moments affect the bending moment capacity of reinforced concrete sections. In certain cases like reinforced concrete, the order of development of various forces at a section has great influence on the deformation patterns and cracks. The worst order of development of forces is also known by the solution of differential game problem. Hence in an accurate analysis, this fact may be taken into account.

The plastic yield condition is another requirement to be satisfied by the forces at any section. In a nonlinearly inelastic material, the yield surface also expands with force level. The actual force-deformation relations are to be obtained for any plastic material satisfying the yield condition.

10.4 PROCEDURE OF OPTIMIZATION

A cost-effectiveness criterion has been proposed

earlier to select the optimal structure out of all the alternatives available. This optimum is achieved in two stages, as explained in Chapter 4. To obtain the task curves, the equation

$$K_T = 1 + \sum_i p_i K_i \qquad (10.2)$$

is minimized. Subsequently for the choice of the sections, the criterion in the form of Eq. 10.1 is made use of.

The sections are to be chosen such that their actual force-deformation relations coincide with the respective task curves or lie on the conservative side (above the task curves). If sections are chosen with its force-deformation relations coinciding with the corresponding task curves, the term $\Sigma p_i C_i$ in Eq. 10.1 remains practically unaffected. On the other hand, if a section having a conservative behaviour compared to that represented by task curves is chosen, the term $\Sigma p_i C_i$ may be reduced. During the selection of cross-sections since there is no chance of increase in the value $\Sigma p_i C_i$, one can write

$$\text{Min}\left\{C_s + \sum_i p_i C_i\right\} = \text{Min } C_s + \text{constant} \qquad (10.3)$$

The sections can be chosen such that <u>the cost of structure is minimum</u>. Finally, after the choice of a structure with minimum cost the accuracy of the costs of various chosen failure modes may be verified.

The cost of structure may be divided as

$$C_s = \text{Cost of members} + \text{Cost of connections} \quad (10.4)$$

The choice of cross sections of members cannot reduce the cost of connections. Hence,

$$\begin{aligned} \text{Min } C_s &= \text{Min Cost of members} + \text{Constant} \quad (10.5) \\ &= \text{Min} \sum_{i=1}^{M} \text{Cost of ith member} + \text{constant} \end{aligned}$$

As the cost of all the members are positive numbers, minimization of the cost of each member can lead to overall minimum. Hence,

$$\text{Min } C_s = \sum_{i=1}^{M} \underline{\text{Minimum}} \text{ cost of ith member} + \text{const.} \quad (10.6)$$

It shows that the cross sections of the members are to be chosen such that the cost of the member is a minimum. This memberwise minimization is possible only because of the separation of structural analysis from the optimization process.

10.4.1 Some Special Cases:

Maintenance Factor:

$$\begin{aligned} \text{Cost of a member} &= \text{Initial cost} + \text{Cost of maintenance} \\ &= \text{Initial cost}\,(1 + \text{maintenance factor}) \\ &= \text{Initial cost}\,(1 + \beta) \quad (10.7) \end{aligned}$$

The factor β represents maintenance cost as a fraction of the initial cost. These factors may be evaluated for different materials used for construction and for location. For example,

β may be more for a steel structure where corrosion possibility is high, than that of a concrete construction or steel structure itself when used in a place where corrosion is less. In comparing structures of different materials, this factor may also be introduced. However, when the material is chosen a priori, there is no necessity of the introduction of β, as it does not affect the choice. For aerospace structures, where weight is also an important consideration apart from cost, the use of similar factors to represent the equivalent operational cost due to the weight of a section as a fraction of the cost of member may be attempted.

Uniform Members: When the members are of uniform cross section, the cost of unit length of member will represent the cost of the entire length. In this particular case,

$$\text{Initial cost} = \text{Cost of unit length of member} \times \text{length} \tag{10.8}$$

Initial cost consists of cost of material and cost of construction. When complex shapes are compared for choice, the cost of construction may be an important factor to be considered.

10.5 CHOICE UNDER CERTAINTY, RISK OR UNCERTAINTY OF MATERIAL BEHAVIOUR

Depending upon the information of material behaviour, the choice of cross section is to be made under three

different situations.

(a) The material and cross sections chosen may have deterministic behaviour. In this case, the force-deformation relations of the cross sections will be uniquely determined. The choice of the section under such conditions is called <u>choice under certainty</u>. Under strict control of manufacture, especially for metals, a condition close to certainty may sometimes be obtained though such a possibility is very rare.

(b) The mechanical behaviour of the sections may be random with the probabilities known. This situation is said to lead to a <u>choice under risk</u>.

(c) The mechanical behaviour of the cross section may be random with unknown probabilities. This situation occurs when no sufficient information is available about the statistical nature of the behaviour of the cross section. This situation leads to a <u>choice under uncertainty</u>.

10.6 CHOICE UNDER CERTAINTY

This is the simplest condition of choice possible. The force-deformation relations may be uniquely obtained as the behaviour is unique. These curves may be compared with the corresponding task curves. If the charts of force-deformation relations of several cross sections are made available, the choice can be easily made by superposing the task curve and selecting the section having force-deformation

relations lying on the conservative side of the task curve.

10.6.1 Illustration of the Method of Choice:

The method of choice of the material and members is illustrated by means of the examples that are previously solved using optimal control formulation and cost-effectiveness model. Figs. 9.2 and 9.3 show the method of choice. The stress strain curves given in Ref. 161 are made use of to find out the force-deformation relations. Cost factors indicated against each of them are arbitrarily provided and are for purpose of illustration only.

Example 1: The two bar truss analyzed in Chapter 6 is chosen for purpose of illustration. Since the bars are of uniform diameter, the optimization of cost can be carried out at the level of cross section. The force-deformation relations of several materials are compared as shown in Fig.10.2. The section having least cost and its force-deformation relation lying on the conservative side of task curve is chosen.

Example 2: When the design is done for only normal load condition and a section is to be chosen using the task curve, the shifting may be done by using a greater margin. This method of choice is illustrated for tension bar in Fig. 10.3. The force-deformation relations of bars with different material are compared with the shifted task curve of tension bar. A factor 1.5 is chosen for shifting the curve.

10.7 CHOICE UNDER RISK (PROBABILITY OF MATERIAL BEHAVIOUR PARAMETERS KNOWN.)

When the behaviour of structural cross section is random and full information on probabilities is available, the choice is a 'choice under risk'. Unlike for loads, information about the statistical nature of the stress-strain curve of materials can be collected at relatively less cost. Therefore, statistical nature of the behaviour of cross sections may, in general, be considered as known phenomenon. The choice of sections in most cases may be treated as a choice under risk.

The nonlinear random behaviour of cross sections is a sequential process or stochastic process. The consideration of the process in this most general form is too complex to use in practical design. Hence, the process can be idealized to a Markov process introducing suitable assumptions.

10.7.1 Markov Idealization of Random Behaviour of Sections:

It is shown in Section 4.2.3 that the random process of deformational behaviour of sections can be idealized as a Markov process. Markov process is a special type of stochastic process distinguished by a certain Markov property. According to this property, the probability of transition to future states depends only on the state presently occupied by the system and not on the history of the system prior to that state. The Markov chain is a Markov process with denumerable number of states (163). The approach taken in this formulation

is based on the work of Howard (164) and Martin (92).

It is to be noted that the randomness under consideration is in the pure mechanical behaviour of the cross sections. It may be reasonable to assume that the mechanical behaviour of any cross section is statistically independent of that of adjacent sections. For example, the moment-curvature relation of one section may be statistically independent of that at another section. The force-deformation relations of any cross section cannot deterministically be specified as assumed in the optimal control formulation. However, if all the random curves are on the conservative side of the task curves, then the structure is safe. On the other hand, if one or more curves lie below the task curve, there is some associated probability of unsafety. Again, it is practically difficult to find a section that has all its random curves well above the task curves. Hence, it is aimed to have sections with force-deformation relations coinciding with the task curves below which the probability of occurrence is negligibly small. So, the ultimate aim in the study of statistical behaviour of a cross section must be to find such a curve with a specified probability level.which may be arbitrarily chosen quantity.

In the study of the statistical phenomenon of the force-deformation relations, the forces corresponding to any stated deformation or the deformations corresponding to any forces may be considered as random. The former approach is

followed here. Let the deformation y take the place of the stage variable time in a stochastic process. Consider that at any y, the force at the section is F^i. Let ΔF be a reasonably acceptable increment of force that divides the states of force into finite intervals. Let N be the number of states of force level the section can have. Thus the forces at states 1, 2, ... N are $(1-1)\Delta F$, $(2-1)\Delta F$, ... $(i-1)\Delta F$, ..., $(N-1)\Delta F$ respectively as shown in Fig.10.4. The force at ith state is therefore $(i-1)\Delta F$.

In a short interval dy of deformation after y, let a process at state i will make a transition to state j, with probability $p_{ij}(y)\, dy (i \neq j)$. The transition is assumed to have the Markov Property. Such a process having <u>transition rate matrices</u> $P(y) = [p_{ij}(y)]$ is called a continuous process. The diagonal elements are not yet defined. The transition rate matrix $[P]$ may in some problems be constant and do not change with y. In the random behaviour of structural members $[P]$ is not same throughout y. Let $\pi_i(y)$ be the probability that the section will have a force F^i at ith state at a deformation y. The state probabilities $\pi_i(y + dy)$ and $\pi_i(y)$ may be related as follows (164):

$$\pi_i(y + dy) = \pi_i(y)\left[1 - \sum_{j=1}^{N} p_{ij}\, dy\right] + \sum_{\substack{j=1 \\ j \neq 1}}^{N} \pi_j(y)\, p_{ji}(y)\, dy, \quad i = 1, \ldots, N \tag{10.9}$$

It shows that the system can occupy state i at $y + dy$ in two mutually exclusive ways.

(i) It could have been in state i itself at y and made no transition at $y + dy$. It corresponds to yielding of the section. The probability of these events are $\pi_i(y)$ as the probability of stage i at y, and $(1 - \sum_{i \neq j} p_{ij}\, dy)$ as transition probability. Let us introduce the diagonal element as

$$p_{ii} = -\sum_{j \neq i} p_{ij} \tag{10.10}$$

(ii) The second term in the Eq. 10.9 represent that the system is in the state j at $y + dy$ is to have been at state $i \neq j$ at y and to have made a transition from i to j during the interval dy. Eq. 10.9 can be written as,

$$\pi_i(y + dy) = \pi_i(y) \left[1 + p_{ii}\, dy\right] + \sum_{j \neq i} \pi_j(y)\, p_{ji}(y) dy$$

$$\text{or} \quad \pi_i(y + dy) - \pi_i(y) = \sum_{j=1}^{N} \pi_j(y)\, p_{ji}(y)\, dt \tag{10.11}$$

Dividing both sides by dt and taking the limit,

$$\frac{d\pi_i(y)}{dt} = \sum_{j=1}^{N} \pi_j(y)\, p_{ji}(y), \quad i = 1, \ldots, N \tag{10.12}$$

To obtain the solution of these N equations, the initial conditions $\pi_i(0)$ for $i = 1, \ldots, N$ must be specified. The solution of the differential equation gives the state probability at any y in terms of the initial state probability and the transition rate matrix $[p_{ij}(y)]$. The off-diagonal terms of the matrix $[p_{ij}(y)]$ are given by the transition rates. The diagonal element of $[P]$ are given by Eq. 10.10. As a result the rows of $[P]$ sum to zero

$$\sum_{j=1}^{N} p_{ij} = 0 \qquad (10.13)$$

10.7.2 Application:

Though the description of the random process is accurate enough, it requires considerable data for application. The essential statistical information needed are:

(i) The probability $\pi_i(0)$, $j = 1, \ldots, N$ of the starting point, and

(ii) The transition probability matrix $[p_{ij}(y)]$ as a function of the stage y. It may be noted that p_{ij} is not a fixed matrix, but is a function of the stage y.

Knowing these two data, the state probabilities can be found out for all stages y. Let there be a curve such that the probability of any curve lying below the one under consideration is an acceptable value p. Such a curve can be obtained by finding those states of forces below which the

total probability of occurrence of a curve is p and plotting them against y. The curve with such an acceptable probability margin p and that corresponds to a section of least cost may be chosen. Considerable experimental study is needed to obtain the data mentioned above for purpose of design.

10.7.3 Simplified Approach Due to Ferry Borges:

An approximate way of dealing with the problem of finding the random force-deformation relations $\pi_i(y)$, is suggested by Ferry Borges(113). He studied the statistical variation of Moment-curvature relations (M-θ). Fig. 10.5.a shows the application of such (M-θ) curves of any cross section. Corresponding to any M_O and any curvature θ_O. there is a statistical distribution of θ and M respectively. The mean value, standard deviation and co-efficient of variation for θ and M respectively are represented by $E(\theta|M)$, $D(\theta|M)$, $C(\theta|M)$ and $E(M|\theta)$, $D(M|\theta)$ and $C(M|\theta)$ respecively. Fig. 10.5.b indicates the values of $E(M|\theta)$ and $D(M|\theta)$ corresponding to Fig. 10.5.a. Such diagrams for each cross section may be obtained. As usual, the curve that has a probability level p can be chosen from the alternatives.

10.7.4 A Cost-effectiveness Model For Choice of Members:

In Section 10.7.2, the criterion of selection of a cross section proposed is the minimum cost corresponding to an

acceptable probability level p. Such an arbitrary specification of probability as p may be excluded by incorporating a cost-effectiveness analysis at the member level. This analysis is for the safety of the system relative to the task curves by assuming that failure of some kind occurs if task curve requirement is violated. Let the cost-effectiveness model at member level be as follows:

$$C_{Tm} = C_{sm} + \sum_{i} p_i \, C_{im} \qquad (10.14)$$

where

C_{Tm} = Cost-effectiveness of member m

C_{sm} = Cost of the member m given by Eqs. 10.7 or 10.8

C_{im} = Cost of ith type of failure of member m.

p_i = Probability of ith type of failure.

In general, failure of a member may lead to 3 types of losses. Failure is assessed by matching the actual force-deformation relation with the task curve.

(i) Failure of serviceability: If the actual force deformation relation lies on the unconservative side of task curve but the forces match within the normal load range the failure is said to be the unserviceability (Fig. 10.6). The probability p_1 of cost C_{1m} of unserviceability is the area of the probability density function that is less than that indicated by the task curve.

effectiveness model described in Section 10.7.4 with the Bayesian Markov process. As the Bayesian Markov process is still in the process of development, it is not attempted to make a detailed study of the applicability of the method. Instead only a brief outline is presented.

Bayesian decision theory is a convenient model to improve the decision maker's belief on the probability of a state of nature (subjectively assigned) by incorporating information from experimental evidence. Thus a prior distribution is subjectively assigned to the state of nature by the decision maker. Utilities are also assigned to the various action state combinations. This would give the expected utility of each action of decision maker. When additional experimental evidence is obtained, the prior distribution can be improved incorporating the new information by means of Bayes' theorem. This refinement is done consistent with the associated utilities. A prior posterior analysis and a preposterior analysis (92) may be carried out to find out the optimum action-experiment combination that maximizes the expected utility.

In the Bayesian Markov process, Markov process with alternatives and rewards is considered. In this case, each transition probability $p_{ij}(y)$ may have, say, K alternatives. A <u>policy</u> consists of the selection of one alternative in each state. The choice of such a policy brings in a reward r_{ij}^{k}

to the decision maker. The uncertainty in the stochastic matrix because of the alternative possibilities is expressed by regarding it as a random matrix with a prior probability distribution function. The initial state probability may be assumed known. A fixed sample size analysis may be carried out with the data obtained from a consecutive sampling of **n** state transitions. The prior posterior and preposterior analysis may be carried out. The above process of refinement of distributions may be carried out by code writers and the result transmitted to the designers. This will reduce the computations in an actual design process.

10.9 DISCUSSION

10.9.1 Comparison with Mathematical Programming Problem:

It is worth comparing the direct design with the problem of structural optimization based on mathematical programming. This would give a better understanding of the proposal and its merit over the existing mathematical programming approach to design. The mathematical programming problem in structural design can be stated in a general way as follows:

Decision Variables: The parameters defining the cross sections such as area, modulus of the section, breadth, depth etc.

Objective Function: A mathematically stated criterion like cost, weight, or any other utility that measures the system merit.

<u>Constraints</u>: The constraints generally chosen are listed below:

1. Constraints on shape, dimensions, and material of cross sections.
2. Constraints associated with safety or failure mode.
3. Equilibrium equations
4. Compatibility conditions
5. Serviceability constraints
6. Buckling constraints
7. Ductility constraints, if necessary.

The constraints 2 to 7 are required to satisfy either the laws of mechanics, or the values in design namely safety, serviceability and ductility.

A close examination of the problem will reveal that the link between the objective function and the constraint space is actually provided by the force-deformation relations. This fact is recognized in the proposed method of choice. The task curves are obtained by satisfying the constraints 2 to 7 for the load pattern given. Hence, they can replace the constraints 2 to 7. This is in this form that the problem is stated in Section 10.2. It will be clear from the following paragraphs that the process of optimization in this way is much simpler than that in a mathematical programming problem.

In the mathematical programming method of optimization, the optimal choice of the section and the structural analysis are combined together into a single operation. The alterations

made in the dimension of any one cross section may affect the stress distribution in the entire structure. Hence, the search for optimum requires the repeated calculation of forces or stresses for every change made in the cross sections. This difficulty is not present in the proposed choice of sections. It is only required to see whether the force-deformation relation of the section chosen agrees with the corresponding task curve. If not, a new section can be tried; so on and so forth until an agreeable one is found. This repeated trial no way affects the choice of other sections, nor affected by the choice of others as no stress analysis is carried out at this stage of decision.

10.9.2 The Problem of Interaction of Forces:

The matching of the actual force-deformation relation of a section and the task curve is easy in the case of members with only one active force-deformation relations provided the relations are available in the form of charts or graphs. When more than one force-deformation relation is present at a section, the matching may be a problem especially when interaction or yield condition is to be satisfied. No method is available in this direction as such a problem has never arisen in the conventional design. Systematic methods may be developed for this choice. It may be noted that an exact fitting of the two diagrams are not necessary for safety purpose. The actual force-deformation relations need be on the conservative side of task curve for purpose of safety.

The method of choice alone does not guarantee an absolute minimum cost design. The attainment of an absolute minimum depends on many factors. Economy has a greater dependence on the correctness of the alternatives chosen. Also, economy depends on the proper choice of the final cross sections. However, this drawback is not a speciality of this method. In many of the non-linear programming problems, absolute optimum is not guaranteed and the choice of proper initial starting point has great influence in the minimum obtained when the functions are not smoothly convex.

Consideration of cost as an objection in the usual optimization problems is very complex. This is due to the fact that cost may have to be related to area of cross sections and sometimes to forces even. This is a complex operation and cost may not be easily related to the cross section dimensions. In the present approach such a hypothetical relation is not necessary. Cross sections are chosen directly using the cost index such that the force-deformation relation matches with the task curve.

10.10 SUMMARY

The operation described in this chapter is the last step in a series of decisions regarding the structural design. It consists of the selection of the material and cross section of members. The choice is made such that the actual force-deformation relations coincide with the task curves and the cost or cost-effectiveness of member is a minimum.

CHAPTER ELEVEN

SUMMARY AND CONCLUSIONS

In the foregoing chapters, the framework of a methodology of optimal inelastic structural design for simultaneous satisfaction of strength, serviceability, ductility and economy has been described. The need to formulate a more rational methodology of inelastic structural design using theories of decision making so as to approach further to the goal of a well-balanced structure, has been the main motivation behind this investigation. With this intention, several decision techniques extensively used in optimal control theory, management and war sciences, and operations research have been studied for their feasibility of applying to the various decisions in structural design. Several decision models such as differential game, statistical decision game (Wald's minimax model) optimal control formulation (Pontryagin's maximum principle and variational approach), certain risk decision models based on preferences, check list, subjective programming, gaming simulation, Bayesian Markov process, cost-effectiveness analysis etc. have been applied to the various decisions in the proposed methodology of structural design and their suitability illustrated. The emphasis throughout the investigation has been more on the formulation of the problems in the framework of the proposed design methodology than on the computational aspects of the problem.

11.2 SUMMARY OF THE DESIGN METHODOLOGY PROPOSED IN THE THESIS

In order to identify the decisions involved in the structural design and the characteristics of the decisions, the nature of structural design decisions has been studied from the viewpoint of engineering systems design and theory of decision making. With this study the major decisions have been listed out, and a decision criterion has been proposed. Various decision models and acceptance rules have been reviewed with respect to their suitability for decisions of different character. Some of the major problems that deserve attention in a structural design methodology have also been listed out.

11.2.1 General Methodology:

Structural design, like any other design, is a creative decision process which starts with vague concepts and ideas and ends up with a finished product. In this process, the designer has to make several decisions, a simultaneous consideration of which is difficult. Hence, the basic philosophy in the proposed methodology is to divide the decisions in structural design to smaller levels where they can be dealt with in an easier and rational manner. The decisions are then embedded in the framework of a learning mechanism, which connects the decisions together to a global decision. Even in choosing the decision models, it is emphasized wherever possible to take the judgements to a still smaller level where the judgement can be more precisely

excercised. No empirical procedure is proposed, and all decisions are to be dealt with as decision theory problems. The method is iterative in nature.

In this investigation, structural design is divided into five major decisions which are grouped into three depending upon the nature of the decisions. A group decision method is proposed for the initial planning of the site and functional configuration of the general system of which the structure is a part. The structural designer takes part in the decision as a partner. Operational gaming is the model proposed for this decision. With the background information gathered and using his own judgement, structural designer selects a structural concept. Several methods like check list, subjective programming, decision tree etc. have been suggested, studied and illustrated through examples. The decision of structural concept is followed by the arrangement of members and types of connections and the selection of material and proportioning of the members. This decision is again subdivided into smaller decisions and studied in detail. A system-effectiveness criterion that trades off functional conformity, structural efficiency and economy is taken as the basic design criterion.

The interdependence of the decisions is considered to be taken into account by connecting them together by a learning mechanism or heuristic procedure. By repeated trials,

any degree of precision may be achieved. This aspect of the problem has not been dealt with and is beyond the scope of this investigation.

11.2.2 Selection of Materials and Proportioning of Members:

A direct design method is proposed in which the force-deformation relations of cross sections that are required to satisfy strength and serviceability consistent with economy are obtained as output of design as opposed to the use of them as linking relations in conventional method of analysis. Later, cross sections of suitable materials and dimensions are chosen such that they provide the necessary force-deformation relations obtained by the direct design method. A brief summary of the method of direct design is presented here. The essential data and steps of computation are also briefly described.

(a) Aim: The material, shape and dimensions of the member cross sections are to be chosen.

(b) Needs:

(i) At normal load conditions (defined in Chapter 4), the structure must be safe as well as serviceable.

(ii) The safety, ductility and cost are traded-off optimally for random loads and abnormal load conditions through a cost-effectiveness criterion. The consequences of failure associated with various modes of failure are taken

into consideration.

(iii) The proposed design for inelastic materials is equally suitable for both elastic and inelastic systems with or without time-effects.

(iv) The design is essentially a decision process under uncertainty. (But the method is flexible enough to be a design under certainty or risk as well).

(c) Data or Information Needed:

(i) The skeleton of the structure with the layout of the members to be proportioned and the manner of connections.

(ii) The possible loads and their positions: Any possible change in the position or direction of load may be specified before hand.

The loads may be deterministic or random in magnitude. If random, the probabilities may be known or known to be within two bounds.

The normal load conditions are to be specified. These are the magnitudes of the loads below which the structure is to be serviceable. They may be arbitrarily specified or if probabilities are known may be taken as the characteristic loads of Comitee Europeen du Beton (CEB)(26)

(iii) The type of idealization of the structural behaviour like elastic, inelastic, with or without creep effects.

(iv) Force-deformation relations of cross-sections of

various materials, shapes and dimensions. It is convenient if such relations for a vast number of cross sections in the form of charts or graphs are available. The force-deformation relations may be certain, random with Markov property (Chapter 10). The transition probabilities may be known or unknown.

(v) The cost per unit length and the maintenance factor β (given in Chapter 10) may also be entered as indices against each force-deformation relation in which case the choice of optimal sections become easier.

(vi) Serviceability requirements in terms of allowable deflections, permanent deformations, rotations etc.

(vii) The cost of the consequences of failure associated with each mode of failure. It may be a function of time or may be invariant with time.

(viii) Three marginal factors r_s, r_c and r_d for uncertainty are to be evaluated to compensate for the non-measurable uncertainties. They may be specified beforehand or may be decided in which case the rating of the various events by the designer is also needed.

(d) Preliminary Computations:

(i) The cross sections that are to be designed are chosen and numbered from 1 to N. In the case of distributed parameter systems (see Chapter 5), the cross sections at

regular intervals may be chosen as the representative ones and the size of any other section may be obtained after design by interpolation.

The force-deformation relations of the N cross sections may be listed out and numbered from 1 to n ($n \geq N$).

(ii) Find out the following constraints

1. Equilibrium Equations
2. Compatibility Conditions, if required.
3. Serviceability Constraints.

(iii) The performance index or pay-off which is a measure of energy potential may also be computed.

(iv) The possible failure modes that are to be considered and the associated costs of failure may also be listed out. The cost-effectiveness criterion may be formulated using the collected data.

(v) The state equations that represent the behaviour idealization may also be listed out.

(e) Problem Formulation: The problems for the various decisions may be formulated. Four decisions are involved:

(i) Stage 1. Design for serviceability under normal load condition (Chapters 5, 8).

(ii) Stage 2. Design for cost-effectiveness under abnormal loads or random loads. (Chapters 6, 9).

(iii) Stage 3. Correcting the force-deformation relations for non-measurable uncertainties (see Chapter 7).

(iv) Stage 4. Selection of cross sections (material, shape and dimensions (Chapter 10).

Depending upon whether the data is certain or random with probabilities known or unknown, different formulations may be chosen as shown in Table 10.1. Uncertainties of various types in loads, materials and non-measurable uncertainties due to other factors are isolated and dealt with separately.

(f) Solution of the Problem: The design formulated as a 4-stage decision problem may be solved in stages in the order given above. The decisions of each stage are the input data to the subsequent decisions. Algorithms may be developed for the various decisions mentioned above. The results of the decisions are the following:

(i) In Stage 1, the n force-deformation relations required for safety and serviceability under normal load conditions are obtained.

(ii) In Stage 2, the force-deformation relations are further extended for optimum cost-effectiveness under abnormal and random loading.

(iii) In Stage 3, the force deformation relations are corrected for the non-measurable uncertainties discussed in Chapter 7.

(iv) In Stage 4, the cross sections are chosen such that their force deformation relations match with the set obtained in Stage 3.

11.3 CONCLUSIONS

A formulation of structural design that links the conventional field of safety, the current trends in optimum design and the modern concepts of structural design processes considering the social and economical and other involvements of design is proposed in this investigation. A well-balanced outlook on the design needs like safety and serviceability, under normal load conditions and optimum cost-effectiveness under abnormal or random loading is also maintained in the proposed design. Many associated problems in structural design like subjectivity of design data, uncertainty and randomness of loads, materials and other design parameters etc.(listed in Chapter 3) have been given due consideration. The decision models chosen for various design decisions are selected in such a way that they fit in with the nature of the problem and design needs.

The special features and results of the various decision models like checklist method, decision tree, subjective programming, optimal control formulation, cost-effectiveness design, decision based on preferences, differential game formulation, statistical decision game and finally

Markov assumption and Bayesian Markov process are discussed in the corresponding chapters. Hence, they are not reproduced here. In this section, certain features of the general methodology are presented. Also, the concepts and methods of the proposed design procedure are compared with those of the existing methods of design.

11.3.1 Special Features of the Proposed Design Method:

Some of the features of the proposed method of design may be listed as follows:

(i) The present trend in coded structural design does not keep a well-balanced outlook on the needs of design on one side and the data available at other. The optimization must be consistent with the social acceptability. The decision model must be suitable to incorporate the data and goal which may be subjective or objective. This problem is given due attention in the proposed design decisions. The group decision activity for selection of site and functional configuration is a model that allows for the personal preferences of individuals and incorporates the decisions under conflicting interest.

(ii) For purpose of design, structure is considered as a component of the general engineered system which is also in constant action along with other components. The interaction of the structural component with other components of the system in terms of functional, aesthetic and other

requirements are taken in design through the system effectiveness concept, and the initial group decision procedure.

(iii) The social, psychological, political and economic considerations in design have been given due consideration by the choice of an operational gaming model for the group decision activity.

(iv) The nature of data available at various stages of design has been taken into account in the choice of decision models. The informations at the initial stage of selection of site etc. are vague and subjective and as the final stage of choice of members are reached, the data become more specific. Hence, the decision models for earlier stages are not so rigorous as that for the later decisions. The accuracy of design may not be improved so long as the input data is vague and inaccurate, however rational the decision model may be. Hence, there is no specific advantage in resorting to a rigorous model like mathematical programming for the first three decisions of the design.

(iv) The global decision of a structural component is decomposed into smaller decisions at which level they can be tackled in a rational way. Whenever judgement is involved, the decisions are still divided into smaller components and the judgement is made with respect to a limited number of factors. The piecemeal decisions and judgements and their synthesis through a design methodology are the special features of the proposed method.

11.3.2 Practical Applications:

(i) The method of design proposed may find considerable application in the design of reinforced concrete structures due to the following reasons.

(a) The serviceability controlled design limiting deflections and permanent deformations (which limits the crack-width) considering creep effect is very much suited for reinforced concrete. A direct procedure of this type is much simpler for this material.

(b) Reinforced concrete is a nonlinear material which has nonlinearity even within the working load conditions due to the development of cracks. Hence, the nonlinear inelastic design for normal load condition is very well suited to it.

(c) As discussed by Nair and Sridhar Rao (165), the non-conservative nature of structural concrete due to the progressive development and propagation of microcracks, the usual theories of continuum mechanics may not rationally predict the behaviour of reinforced concrete members. The sequence, timing and manner of loading (known as strategic uncertainties) are more important in reinforced concrete design especially when serviceability is to be satisfied. The optimal control and differential game are well suited models to deal with this problem. Further, a knowledge of the worst course of loading and the optimal force-deformation relations would be

very much helpful in finding the sections that are conservative in behaviour. In reinforced concrete the force-deformation relations of actual sections vary with the path of loading. Hence, a prior knowledge of the path of loading would be very much useful for better design.

(d) Force-deformation relations of various shape and requirement can be easily obtained by changing the amount of steel. Hence, a method of direct design is very helpful to design the sections with minimum reinforcements required or for minimum cost.

(ii) This method of design may suit very well to the modern practice of prefabrication and standardization of structural elements. The basic design computations are free from material and cross sectional considerations. Hence, on completion of design, the designer can demand the sections made of suitable materials that he wants. If such a section is not available, he is free to choose any other suitable section without altering the basic design requirements. This freedom of choice suits to the concept of standardization and mass production of structural elements.

(iii) The method is equally applicable for both conventional materials and modern composites and plastics. The concept of design of materials for specific use by composition of several elements is a fast developing field of study. The proposed design approach fits in very well to the philosophy.

(iv) The trade-off between weight, cost and safety to arrive at well-balanced structure is very important in aerospace structures. The design of such structures is to be carried out under conditions of uncertainty. The design may be carried out in such cases as a decision under uncertainty as dealt with in this thesis.

The design method covers a wide class of materials and structures both conventional and innovative and is applicable to all of them with equal rigor.

11.3.3 <u>Comparison with Existing Design Concepts</u>:

The proposed method is compared with the existing methods of design.

(a) <u>Comparison with the Conventional Non-Utilitarian Design</u>: The proposed method differs considerably from the conventional method of design. In conventional method of design, most of the decisions mentioned are taken empirically. The proposed method considers that all decisions are decision theory problems of one type or another. Further, the method is a direct logical design in contrast to the indirect intuitive character of the conventional design.

(b) <u>Comparison with the Modern Optimum Utilitarian Design Processes</u>: By nature, the proposed method is a utilitarian decision process. However, it differs from the existing methods of optimum design in the following

respects.

(i) The decision methods suggested in this design that makes use of subjective data, are not so rigorous as the mathematical programming methods of design.

(ii) The direct design approach separates the optimization and structural design as discussed in Chapter 10. This approach has got specific advantage in the incorporation of uncertainty, and gives enough freedom to the designer in the choice of members and materials.

(c) <u>Safety</u>: Design for safety is the matching of the load and strength. If the strength is greater than load, structure is theoretically safe. In the existing design, the matching is done at the level of stress, strength of a section or at the load level depending upon whether the design is stress design, strength design or limit design. The proposed method also considers safety by matching the two at the cross section level. The task curves may be considered as the load effect and the force-deformation relation of the section as the strength or capacity of the section. The matching of the two therefore corresponds to the matching of strength and load in conventional design. It may be noted that the matching in the proposed method is not only for strength alone but also for stiffness and ductility with the result that the design assures safety not only in terms of structural failure, but also in terms of functional failure.

(d) Statement of Safety: Safety in the existing methods of design is stated in terms of an arbitrarily specified factor of safety, load factor or probability of failure. In the proposed method this approach is dispensed with. Instead, safety and serviceability under normal load conditions are assured by a design first and safety is traded off to the consequences of failure and cost of structure under abnormal and random load conditions. Further, inelastic reserved strength is used under abnormal load conditions to share part of the excess load acting on the system.

(e) Uncertainty: In the existing design, uncertainties are taken empirically by means of safety factors. In the proposed method, design is treated as decisions under uncertainty by which the use of arbitrary safety factor is eliminated.

(f) Probabilistic Design: Though the random behaviour of the structure and loads are taken into account, the method of design is basically deterministic.

(g) Computational Feasibility: The proposed method may appear computationally involved. It is so because most of the decisions dealt with are not done in a conventional design. If all the decisions proposed here are to be done by conventional methods of design (not using empirical factors) for inelastic materials, the computations involved may not be less than that in the present method and perhaps it may be

even more. Inverse inelastic analysis and proportioning for similar conditions involves iteration and is computationally involved even for feasible solutions.

11.4 RECOMMENDATIONS FOR FURTHER RESEARCH

The method of design proposed in this thesis is new though the underlying philosophy is known to all structural engineers. The method has to be carefully studied in all aspects before making use of it as a design method. It gives greater scope to carry out considerable research. Further, the method suggested here is only one method of approach. Similar possibilities may be studied and the one that is more acceptable may be chosen as the design methodology.

The various possible extensions of the method have been discussed at the end of each chapter under discussion. Of these, three aspects of research deserves immediate attention to make use of it as a design methodology.

(i) Computational algorithm for all the decision methods suggested in this thesis.

(ii) Collection of relevant design data that are needed for use of this design process.

(iii) Preparation of charts and graphs of force-deformation relations of cross sections of various material of various shape and dimensions.

Further extensions of the proposed design method to complicated structures and practical problems with creep, thermal, fatigue and other effects and also for earthquake loading may be attempted.

The various decision models like differential game, optimal control, cost-effectiveness, statistical game, Bayesian Markov process etc. are all recently originated techniques and by themselves are not fully developed. With the further development of these basic decision models, improvement in the proposed design can also be made so that the method can be applied to practical inelastic structural design problems and can be used for improvement of codes of practice and specifications.

REFERENCES

1. Wasiutynski, F., and Brandt, A., 'The Present State of Knowledge in the Field of Optimum Design of Structures,' Applied Mechanics Reviews, Vol. 16, No. 5, 1963.

2. Pugsley, Sir, A., 'The Well-Balanced Structure', Fifty-fourth Wilbur and Orville Wright Memorial Lecture, Journal of the Royal Aeronautical Society, Vol.70, No. 662, Feb. 1966, pp. 304-311.

3. Gerard, G., 'Structural Interplay: Design and Materials', Aero/Space Engineering, Aug. 1969, pp. 58-62 (Also ref. 40).

4. Murthy, P.N., and Rao, J.K.S., 'Some Considerations in Structural Design Process,' Paper Presented at the Twentieth Annual General Meeting of the Aeronautical Society of India, at Bangalore, 1967.

5. Gregory, S.A., 'Design Science' Chapter 35 in The Design Methods, Ed. by Gregory, S.A., Plenum Press, Newyork, 1966.

6. Dixon, J.R., 'Design Engineering: Inventiveness, Analysis and Decision Making', McGraw Hill Book Co., N.Y., 1966.

7. Rosenstein, A.B., 'Design as a Basis for a Unified Engineering Curriculum,' Proceedings of the First Conference on Engineering Design Education, Case Institute of Technology, Sept. 1960.

8. Asimow, M., Introduction to Design, Prentice Hall, Inc. Englewood Cliffs, N.J., 1962.

9. Mesarović, M.D., Views on General System Theory, John Wiley and Sons, Inc., Newyork, N.Y. 1964.

10. Zwicky, F., 'The Morphological Method of Analysis and Construction in Studies and Essays Courant Anniversary Volume, Interscience Newyork, 1948.

11. Norris, K.W., 'The Morphological Approach to Engineering Design', Conference on Design Methods, Ed. by Jones, J.C. and Thornley, D.G., Pergammon, Oxford, 1963.

12. Krick, E.V., An Introduction to Engineering and Engineering Design, John Wiley and Sons, 1965.

13. Eder, W.E., 'Definitions and Methodologies' Ch.3 in The Design Methods, Ed. by Gregory, S.A., Plenum Press, Newyork, 1966.

14. Quade, E.S., 'Cost-Effectiveness: Some Trends in Analysis', Cost Effectiveness, Ed. by English, J.M., John Wiley, 1968.

15. Au, T., 'Heuristic Games for Structural Design', Journal of the Structural Division, Proceedings, ASCE Vol.92, No. ST.6, Dec., 1966.

16. Spillers, 'Artificial Intelligence in Structural Design', Journal of the Structural Division, ASCE, Vol.92 No. ST.6, Dec. 1966.

17. Khachaturian, N., 'Basic Concepts in Structural Optimization', An Introduction to Structural Optimization, Solid Mechanics Study No.1, University of Waterloo, Ontario, Canada, 1969.
Also, 'Optimization and Structural Design', ASCE Joint Specialty Conference on Optimization and Non-linear Problems, Chicago 1968, pp. 75-90.

18. Moe, A.J., 'Begriff der Sicher heit, Publication Preliminaire du 3ome congres de 'Association Internationale des ponts et Charpentes Liege, 1948.

19. Freudenthal, A.M., 'The Safety of Structures', Transactions, ASCE, Vol. 112, 1947.

20. Prot, M., 'Theorie Probabiliste de la Securite' Revue Generate de Chemins de Fer, June, 1951.

21. Pugsley, A.G., 'Concepts of Safety in Structural Engineering, Journal of the Institution of Civil Engineers, Vol. 36, No. 5, March, 1951. Also, 'The Safety of Structures', Edward Arnold, 1966.

22. Turkstra, C.J., 'A Formulation of Structural Design Decisions', Ph.D. Dissertation, University of Waterloo, Canada, 1962.

23. Benjamin, J.K., 'Probabilistic Structural Analysis and Design', Journal of the Structural Division, Proc., ASCE, Vol. 94, No. ST 7, July 1968, Discussions by Blume, April 1969, pp. 776-779, Sridhar Rao, J.K., July 1969, pp. 1572-73, Closure Jan. 1970, pp. 129-130.

24. Raiffa, H. and Schlaifer, R., Applied Statistical Decision Theory, Graduate School of Business Administration, Harward University, 1961.

25. Sexsmith, R.G., 'Structural Reliability Decisions Using Bayesian Technique', ASCE-EMD and Purdue University Speciality Conference Nov. 13, 1969.

26. Comite, European du Beton, 'CEB. Recommendations for an International Code of Practice for Reinforced Concrete', Cement and Concrete Association, London, 1964.

27. Ang, A.H.S., and Amin, M., 'Safety Factors and Probability in Structural Design, Journal of Structural Division, Proc. ASCE, Vol. 95, No. ST.7, pp. 1389-1405, July 1969. Discussion by Nair, N.G., and Sridhar Rao, J.K., Ibid, April 1970, pp. 853-856.

28. Ang, A.H.S., 'Probability Considerations in Design and Formulation of Safety Factors', Contribution to the 1969 (London) I.A.B.S.E. Symposium on Concepts of Safety of Structures and Methods of Design. Also, 'Extended Reliability Basis for Formulation of Design Criteria' Paper presented at the Speciality Conference on Probabilistic Concepts and Methods in Engineering, held at Purdue University, 1969.

29. Ang, A.H.S., 'Extended Reliability Basis of Structural Design Under Uncertainties', Paper for Presentation at SAE/AIAA/ASME 9th Reliability and Maintainability Conference, Michigan, 20-23 July 1970.

30. Lind, N.C., 'Deterministic Format for the Probabilistic Design of Structures', An Introduction to Structural Optimization, S.M. Study No.1, Solid Mechanics Division, University of Waterloo, Waterloo, Ontario, 1969.

31. Cornell, C.A., 'A Probability Based Structural Design', Presented at the 1968 Fall Convention of American Concrete Institute, 1968.

32. Ravindra, M.K., Heaney, A.C., and Lind, N.C., 'Probabilistic Evaluation of Safety Factors', Report No. 23, Solid Mechanics Division, University of Waterloo, Waterloo, Ontario, Canada, Oct. 1969.

33. Turkstra, C.J., 'Choice of Failure Probabilities', Journal of the Structural Division, Proc. ASCE, Vol. 93, No. ST.6, Dec. 1967, pp. 189-200.

34. Institution of Structural Engineers, 'Report on Structural Safety', The Structural Engineer, London, May 1955, pp. 141-149.

35. Knol, F., 'Fundamentals of Safety for Structures', ETH. Zurich Doctoral Thesis, Prom, Nr. 3701, 1965, 47p. Also, Technical Translation 1367. National Research Council of Canada, Ottawa, 1969.

36. Gerard, G., 'Optimum Structural Design Concepts for Aerospace Vehicles', Journal of Spacecrafts and Rockets, Vol. 3, No. 1, January 1966, pp. 5-18.

37. Barnett, R.L., 'Survey of Optimum Structural Design', Experimental Mechanics, Vol. 6, Dec. 1966, pp. 19A-26A.

38. Prager, W., 'Optimization in Structural Design Chapter 13, Mathematical Optimization Techniques, Ed. by Bellman, R., University of California Press, Berkeley and Los Angeles, 1963.

39. Sheu, C.Y., and Prager, W., 'Recent Developments in Optimal Structural Design', Applied Mechanics Reviews, Vol. 21, No. 10, Oct. 1968, pp. 985-992.

40. Scipio, L.A., 'Structural Design Concepts', NASA SP-5039 National Aeronautical and Space Administration Washington, D.C., 1967.

41. Mick, W.R., 'Bibliography of Literature on Optimum Design of Structures and Related Topics, 'The Rand Corporation, RM-2304, ASTIA-AD-215771, Dec. 15, 1958.

42. Borges, J.F., and Castanhita, M., 'Structural Safety', Laboratorio Nacional De Engengaria Civil, Lisbon Nov. 1968.

43. Maxwell, J.C., 'Scientific Papers II Cambridge University Press, Cambridge, England 1890, pp. 175-177.

44. Michell, A.G.M., 'The Limit of Economy in Framed Structures', Philosophical Magazine, London, Series 6, Vol. 8, No. 47, Nov. 1904, pp. 589-597.

45. Forsell, C., Jordtryck kekol Handels'f publ. No. 41, Stockholm 1920.

46. Heyman, J., 'On the Absolute Minimum Weight Design of Framed Structures', Quarterly Journal of Mech. and Applied Maths., Vol. 12, 1959, pp. 314-324.

47. Foulkes, J.D., 'Minimum Weight Design and the Theory of Plastic Collapse', Quarterly of Appl. Maths., Vol. 10, 1953, pp. 347-358.

48. Prager, W. and Shield, R.T., 'A General Theory of Optimal Plastic Design', Journal of Appl. Mechs., ASME, March 1967, pp. 184-187.

49. Shield, R.T., 'On the Optimum Design of Shells', Journal of Appl. Mechs., ASME, 27, 1960, pp. 316-322.

50. Save, M., 'Some Aspects of Minimum Weight Design', Engineering Plasticity, Ed. by Heyman, J. and Leckie, F.A., University Press, Cambridge, 1968, pp. 611-626.

51. Drucker, D.C. and Shield, R.T., 'Bounds on Minimum Weight', Quarterly Appl. Maths, Vol. 15, 1957, pp. 269-281.

52. Taylor, J.E., 'Maximum Strength Elastic Structural Design', Journal of the Engineering Mechanics Division, ASCE, Vol. 95, No. EM.3, Proc. Paper 6617, June 1969, pp. 653-663.

53. Prager, W., 'Optimality Criteria Derived from Classical Extremum Principles', An Introduction to Structural Optimization Ed. by M.Z. Cohn, University of Waterloo, Canada, 1969, pp. 165-178.

Also, Prager, W., and Taylor, J.E., 'Problems of Optimal Structural Design', Journal of Applied Mechanics, Vol. 35, 1968, pp. 102-106.

54. Sridhar Rao, J.K., and Murthy, P.N., (Eds.)., 'Intensive Course on Optimization in Structural Design', Vol. I and Vol. II, I.I.T.Kanpur, March, 1969.

55. Khachaturian, N., 'Optimization by Dynamic Programming of Trusses', Intensive Course on Optimization in Structural Design, Vol. II, Ed. by J.K.Sridhar Rao and P.N. Murthy, Indian Institute of Technology Kanpur, March 1969.

56. Porter Goff, R.F.D., 'Decision Theory and the Shape of Structures', Technical Note in the Journal of the Royal Aeronautical Society, Vol. 70, No. 663, March 1966, pp. 448-452.

57. Schmit, L.A., and Kicher, T.P., 'Synthesis of Material and Configuration Selection', Journal of Structural Division, ASCE, Vol. 88, No. ST.3, June 1962, pp. 79-102.

58. Dobbs, M.W., and Felton, L.P., 'Optimization of Truss Geometry', Journal of Structural Division, ASCE, Vol. 95, ST. 10, Oct. 1969, pp. 2105-2118.

59. Moses, F., 'Approaches to Structural Reliability and Optimization', Structural Synthesis Summer Course Notes, Vol. II, Case Institute of Technology, Cleveland, Ohio, U.S.A., July 1965.

60. Schmit, L.A., 'Structural Design by Systematic Synthesis Proc. 2nd Natl. Conf. on Electronic Computation, Structural Div., Am. Soc. Civil Engrs., Pittsburgh, Penna. 105-132 (1960).

61. Fox, R. L., 'Mathematical Methods in Optimization', An Introduction to Structural Optimization, Ed. by Cohn, M.Z., University of Waterloo, Waterloo, 1969, pp. 47-80.

62. Rozvany, G.I.N., 'Optimal Design of Axisymmetric Slabs', Engineering Conference, Brisbane 1968, Paper No. 2418. Also, 'A Review of Optimization Techniques and Applications to Structural Analysis and Design' in Intensive Course on Optimization in Structural Design', Vol. II Ed. by Sridhar Rao, J.K., and Murthy, P.N., Indian Institute of Technology, Kanpur, 1969.

63. Hilton, H.H. and Feign, H., 'Minimum Weight Analysis Based on Structural Reliability', Journal of Aerospace Science, Vol. 27, Sept., 1960, pp. 641-652.

64. Kalaba, R., 'Design of Minimum Weight Structures for Given Reliability and Cost', Journal of Aerospace Science, Vol. 29, 1962, pp. 355-356.

65. Moses, F., 'Approaches to Structural Reliability and Optimization', An Introduction to Structural Optimization Ed. by M.Z. Cohn, University of Waterloo, Canada, 1969, pp. 81-120.

66. Ghista, D.N., 'Structural Optimization With Probability of Failure Constraints', NASA TN. D-3777. Dec. 1966.

67. Haider, S.G., 'Structural Design as an Optimal Decision Process,' Ph.D. Thesis, University of Illinois, 1969.

68. Blake, R.E., 'Predicting Structural Reliability for Design Decisions', Journal of Spacecrafts and Rockets, Vol. 4, No. 3, March 1967, pp. 392-398.

69. Haug, E.J., and Kirmser, P.G., 'Minimum Weight Design of Beams With Inequality Constraints on Stress and Deflection', Journal of Applied Mechs., Dec. 1967, pp. 999-1004.

70. Krokosky, E.M., 'The Ideal Multifunctional Constructoral Materials', Journal of Structural Division, ASCE, ST. 4, Vol. 94, 1968.

71. Chamis, C.C., 'The Impact of Optimization on the Materials Research Structural Design Cycle,' ASCE Conference on Optimization and Nonlinear Problems, Chicago, April, 1820, 1968.

72. Smolenski, C.P., and Krokosky, E.M., 'Optimal Multifactor Design Procedure for Sandwich Panels', Journal of Structural Division, ASCE, Vol. 96, No. ST.4, Proc. 7243, April 1970, pp. 823-837.

73. Kline, M.W., and Lifson, M.W., 'System Engineering', Chapter 2 in Cost-Effectiveness Ed. by English, J.M., John Wiley, New York, 1969.

74. Boulding, K.E., 'General System Theory - the Skeleton of Science', Management Science, April 1956, p. 197.

75. Webster, L.R., 'Optimum System Reliability and Cost-Effectiveness', Proceedings of the 1967 Annual Symposium on Reliability held at Washington D.C., Jan. 1967, pp. 489-500.

76. Nervi, P.L. Structures, McGraw Hill Book Co., 1956.

77. Sawyer, H.A., 'The Status and Potentialities of Nonlinear Design of Concrete Members', Flexural Mechanics of Reinforced Concrete, Proc. of the International Symposium, Miami, Florida, 1964, pp. 7-28.

78. English, J.M., 'Concepts of System Resources Requirements', Ch. 5 in Cost-Effectiveness, Ed. by J.M. English, John Wiley 1968, pp. 64-78.

79. Micholas, A.C., Principles of Logic, Prentice Hall Inc. 1969.

80. Luce, R.D., and Raiffa, H., Games and Decisions, John Wiley and Sons, 1957.

81. Rozvany, G.I.N., 'The Role of Optimization in Design', Key note speech Intensive Course on Optimization in Structural Design, Vol. I, Ed. by Sridhar Rao, J.K., Murthy, P.N., Indian Institute of Technology, Kanpur, 1969.

82. Hadley, G., 'Nonlinear and Dynamic Programming', Addison-Wesley Publishing Company, Inc. Mass.1964.

83. Pierre, D.A., 'Optimization Theory With Applications', John Wiley, 1969.

84. Aumann, R.J., 'Subjective Programming', Human Judgement and Optimality ed. by Shelly, M.W., and Bryan, G.L. John Wiley, 1964, p. 217.

85. Von Neumann, J., and Morgenstern, O., 'Theory of Games and Economic Behaviour, Princeton University Press, Princeton, 1955.

86. Sengupta, J.K., Tintner, G., and Millhan, C., 'On Some Theorems of Stochastic Linear Programming with Application,' Management Science, Oct., 1963.

87. Naslund, B., 'Decisions Under Risk', The Economic Research Institute of the Stockholm School of Economics, 1967.

88. Wald, A., Statistical Decision Functions, New York, John Wiley, 1950.

89. Blackwell, D., and Girshick, M.A., 'Theory of Games and Statistical Decisions', John Wiley, New York, 1954.

90. Savage, L.J., 'The Foundations of Statistics,' John Wiley, 1954.

91. Pratt, J.W., Raiffa, H., and Schlaifer, R., 'Introduction to Statistical Decision Theory, McGraw Hill Book Co., 1965.

92. Martin, J.J., 'Bayesian Decision Problems and Markov Chains', John Wiley, New York 1967.

93. Raiffa, H., 'Decision Analysis', Addison Wesley, 1968.

94. Dyckman, T.R., Smidt, S., and Mc Adams, A.K., 'Management Decisions Making Under Uncertainty, Macmillan Co., London, 1969.

95. Warner, R.F., and Kabaila, A.P., 'Monte-Carlo Study of Structural Safety', Journal of Structural Division, ASCE, Vol. 94, No. ST-12, Proc. Paper No. 6275, Dec., 1968, pp. 2847-2859.

96. Zuk, W., 'Kinetic Structures', Civil Engineering ASCE, Dec. 1968, pp. 62-64.

97. Yao, J.T.P., 'Adaptive Structural System', University of New Mexico, Technical Report CE-9, June 1968.

98. Schilling, C.H., 'Optimization in Site Selection', Problem No. 13, in Computers in Engineering Design Education, Vol. III, Civil Engineering, University of Michigan, 1966, pp. 111-134 to 111-146.

99. Aguilar, R.J., 'Decision Making in Building Planning', Problem No. 7, in Ibid, pp. III, 26-III 33.

100. Au, T., and Recker, W.W., 'Engineering Synthesis Game- Simple Structural Framing in a Lunar Environment', Report No. LS-1, Carnegie Institute of Technology, Pennsylvania, July 1966.

101. Wei, M.L.C., Christiano, P.P., and Au, T., 'Bridge Design Game Administrators Manual for Simplified Version, 'Report No. BD-2, Carnegie Institute of Technology, Pennsylvania, August, 1966.

102. Mac Queen, J., and Miller, R.G., Jr., 'Optimal Persistence Policies', Operations Research, Vol. 10, 1962, pp. 463-470.

103. Meier, R.L., and Duke, R.K., 'Gaming Simulation for Urban Planning', American Institute of Planners Journal, V. 32, Jan. 1966, pp. 3-17.

104. Feldt., A.G., 'Operational Gaming in Planning Education', American Institute of Planners Journal, V. 32, Jan. 1966, pp. 17-23.

105. Benjamin, S., 'Operational Gaming in Architecture', Architecture Canada, Vol. 45, No. 2, February 1968, pp. 57-59, (Also Ekistics, Vol. 26, No. 157, Dec., 1968 pp. 525-529).

106. Shepard, R.N., 'On Subjectively Optimum Selections among Multiattribute Alternatives', In Human Judgement and Optimality, Ed. by Shelly, M.W. and Bryan, G.L., Wiley, 1964, pp. 257-81.

107. Matousek, R., 'Engineering Design', A Systematic Approach', Interscience Publishers, 1963.

108. Alger, J.R.M., and Hays, C.V., 'Creative Synthesis in Design', Prentice Hall, Inc., Englewood Cliffs, N.J. 1964.

109. Adams, E.W., and Fagot, R., 'A Model of Riskless Choice', Behaviour Science, Vol. 4, (1959) pp. 1-10, Also in Decision Making Selected Reading, Ed. by Edwards, W., and Tversky, A., Penguin Books, 1967.

110. Freudenthal, A.M., 'Inelastic Analysis and Design', Proceedings of the Second Conference on Dimensioning and Strength Calculations, Akademiaikiado, Budapest, 1965, pp. 9-21.

111. Prager, W., and Symonds, P.S., 'Stress Analysis in Elastic Plastic Structures', Proceedings of Symposium in Applied Mathematics, Vol. III McGraw Hill Book Co., 1950, pp. 187-197.

112. Horne, M.R., 'The Stability of Elastic Plastic Structures', Progress in Solid Mechanics, Vol. II, Ed. by Sneddon, I.N., and Hill, R. North Holland Publishing Co., Amsterdam, 1961, pp. 279-322.

113. Borges, J.F., 'Structural Behaviour and Safety Criteria', Seventh Congress of the IABSE RIO-de-Janerio, Nov. 10-16, 1964.

114. Kjellman, W., 'Sakerhetsproblemet ur Principiell och teoretisk Sympunkt Ingeniors Vetenskaps Akademien, Handlingar nr 156, Stockholm, 1940. Also, see ref. 42.

115. Page, A.N., 'Utility Theory: A Book of Reading', John Wiley, 1968.

116. Kazanowski, A.D., 'Cost-effectiveness Fallacies and Misconceptions Revisited', Cost-effectiveness, Ed. by English, J.M., Wiley, 1969, pp. 151-165.

117. Oldenburger, R., 'Optimal Control', Applied Mechanics Surveys, Ed. by Abramson, H.N., et al. Spartan Books, 1966, pp. 127-148.

118. Paiewonsky, B., 'Optimal Control: A Review of Theory and Practice,' AIAA Journal, Vol. 3, Nov. 1965, pp. 1985-2006.

119. Athans, M., 'The Status of Optimal Control Theory and Applications for Deterministic Systems', IEEE Transactions on Automatic Control, AC-11, July 1966, pp. 580-596.

120. Lanczos, C., 'The Variational Principles of Mechanics', University Press, Toronto, 1949.

121. Pontryagin, L.S., et al., 'Mathematical Theory of Optimal Processes, (English Translation by K.N. Trirogoff), Interscience, New York, 1962.

122. Berkovitz, L.D., 'Variational Methods in Problems of Control and Programming', Journal of Mathematical Analysis and Applications, Vol. 3, 1961, pp.145-169.

123. Hodge, P.G., 'Numerical Application of Minimum Principles in Plasticity', Engineering Plasticity, Ed. by Heyman, J., and Leckie, F.A., Cambridge University Press, 1968, pp. 237-256.

124. Bellman, R., 'Dynamic Programming, Princeton University Press, Princeton, New Jersey, 1957.

125. Troitskii, V.A., 'On Variational Problems of Optimization of Control Processes', PMM Journal of Applied Mathematics and Mechanics, Vol. 25, 1961, pp.994-1010.

126. Troitskii, V.A., 'Variational Problems of Optimization of Control Processes for Equations with Discontinuous Right Hand Sides', Ibid, Vol. 26, 1962, pp. 336-355.

127. Troitskii, V.A., 'Variational Problems in the Optimization of Control Processes in Systems with Bounded Coordinates,' Ibid, Vol. 26, pp. 641.

128. Berkovitz, L.D., 'On Control Problems with Bounded State Variables', Journal of Mathematical Analysis and Applications, Vol. 5, 1962, pp. 488-498.

129. Krasovski, N.N., 'On the Theory of Optimum Control', Automatika i Telemekhanika, Vol. 18, No. 11, 1957, pp. 960-970.

130. Rozonoer, L.I., 'The Maximum Principle of L.S., Pontryagin in Optimal System Theory', Part III, Automation and Remote Control, Vol. 20, No. 12, 1959, pp. 1517-1532.

131. Pagurek, B., and Woodside, C.M., 'The Conjugate Gradient Method for Optimal Control Problems with Bounded Control Variables', Automatica, Vol. 4, pp. 337-349.

132. McGill., 'Optimum Control, Inequality Constraints and Generalized Newton Raphson Algorithm', SIAM Journal on Control, Ser.A., Vol. 3, pp. 291-298, 1965.

133. Paul, R.J.A., and Legge, C.G., 'Direct Sensitivity Method of Solving Boundary Value Problems in Optimal Control Studies', Proc. of IEEE, Feb. 1969, pp. 273-280.

134. Breakwell, Speyer and Bryson, 'Optimization and Control of Nonlinear System Using Second Variation', J SIAM on Control, Vol. 1, No. 2, pp. 193-223.

135. Sage, A.P., 'Optimum Systems Control', Englewood Cliffs, N.J., Prentice Hall 1968.

136. Kaldjian, M.J., 'Moment Curvature of Beams as Ramberg-Osgood Functions,' Journal of the Structural Division, ASCE, Vol. 93, No. ST5, Proc. Paper No. 5488, Oct. 1967, pp. 53-65.

137. Vind, K., 'Control Systems With Jumps in the State Variables', Econometrica, Vol. 35, No. 2, April 1967.

138. Eringen, A.C., 'Mechanics of Continua', John Wiley and Sons, 1967.

139. Prager, W., 'Recent Developments in the Mathematical Theory of Plasticity,' Journal of Applied Physics, Vol. 20, No. 3, Mar. 1949, pp. 235-241.

140. Reissner, E., 'On a Variational Theorem in Elasticity', Journal of Maths and Physics, Vol. 29, 1950, pp. 90-95.

141. Wang, A.J., and Prager, W., 'Thermal and Creep Effects in Work Hardening Elastic-plastic Solids', Journal of Aeronautical Sciences, Vol. 21, 1954, pp. 343-344.

142. Hult, J.A.H., 'Creep in Engineering Structures', Blaisdell Publishing Company, Mass., 1966.

143. Prager, W., 'On Ideal Locking Materials', Transactions of the Society of Rheology, Vol. 1, 1957. (See also Quarterly of Appl. Maths., Providence R.I., April 1970).

144. Alfrey, T., 'Mechanical Behaviour of High Polymers', John Wiley - Sons, 1947.

145. Anderson, T.W., 'An Introduction to Multivariate Statistical Analysis', John Wiley - Sons., London, 1958.

146. Owen, D.B., 'Handbook of Statistical Tables', Addison-Wesley Publishing Co., 1962.

147. Moses, F. and Stevenson, J.D., 'Reliability-Based Structural Design', ASCE Annual Meeting and National Meeting on Structural Engineering, Louis Ville, Ky. 1969. (Also, J. of Structural Division, Proc. 7072, Feb. 1970).

148. Radner, R., 'Mathematical Specification of Goals for Decision Problems' in Human Judgement and Optimality, Ed. by Shelly, M.W., and Bryan, G.L., John Wiley, 1964.

149. Issacs, R., 'Differential Games', New York, Wiley 1965.

150. Berkovitz, L.D., 'A Variational Approach to Differential Games', Advances in Game Theory, Annals of Math., Study 52, Princeton University Press, 1964, pp. 127-174.

151. Berkovitz, L.D., 'Necessary Conditions for Optimal Strategies in a Class of Differential Games and Control Problems', Journal SIAM on Control, Vol.5, No. 1, 1967, pp. 1-24.

152. Berkovitz, L.D., 'A Survey of Differential Games', Math Theory of Control, Ed. by Balakrishnan and Neustadt, Academic Press, 1967, pp. 342-372.

153. Pontryagin, L.S., 'On Some Differential Games', Journal SIAM on Control, Vol. 3, No. 1, 1965, pp. 49-52.

154. Petrosjan, L.A., 'On a Family of Differential Games of Survival in the Space R^n', Dokl. Akad. Nauk. USSR, Vol. 161, 1965, pp. 52-54.

155. Ragade, R.K. and Sarma, I.G., 'A Game Theoretic Approach to Optimal Control in the Presence of Uncertainty', IEEE Transactions on Automatic Control, Vol. AC-12, No. 4, August, 1967, pp. 396-401.

156. Sarma, I.G., and Ragade, R.K., 'Some Considerations in Formulating Control Problems as Differential Games', International Journal of Control, Vol.4, No. 3, 1966 pp. 265-279.

157. Proceedings of the First International Conference on the Theory and Application of Differential Games', University of Massachusetts, Mass., Sept.- Oct.,1969.

158. Hoff, N.J., 'Buckling and Stability', Journal of the Royal Aeronautical Society, Vol. 58, 1959, pp.3-52.

159. Ziegler, H., '<u>Principles of Structural Stability</u>', Flaisdell Publishing Co., London, 1968.

160. Kendall, M.G., '<u>The Advanced Theory of Statistics</u>' Vol. I, London, Charles Griffin and Company, Ltd., 1948.

161. Military Handbook, '<u>Metallic Materials and Elements for Aerospace Vehicle Structures</u>', Dept. of Defense Washington 25, D.C., Feb. 1966.

162. Military Handbook, '<u>Plastics for Flight Vehicles, Part I Reinforced Plastics</u>', Armed Forces Supply Support Center, Washington 25, D.C., Nov. 1959.

163. Chung, K.L., '<u>Markov Chains With Stationary Transition Probabilities</u>', Springer Verlag, Berlin (1960).

164. Howard, R.A., '<u>Dynamic Programming and Markov Processes</u>', John Wiley and Sons, New York, 1960.

165. Nair, N.G. and Sridhar Rao, J.K., 'On Continuum Mechanics and Structural Concrete', Paper presented at the <u>13th Annual Meeting</u> of the Indian Society of Theoretical and Applied Mechanics, Kharagpur, India, December, 1968.

166. Leitmann, G., and Mon, G., 'Some Geometric Aspects of Differential Games', <u>The Journal of the Astronautical Sciences</u>, Vol. XIV, No. 2, p. 56-65, 1967.

STATES OF NATURE

	No.	Thickness	Load / strength →	1	2	3	4	5
				$\frac{30 \text{ psft}}{13 \text{ ksi}}$	$\frac{40}{13}$	$\frac{30}{15}$	$\frac{40}{15}$**	$\frac{40}{16}$
Alternative Slabs.	1	3 ins.		u_{11}	u_{12}	u_{13}	u_{14}	u_{15}
	2	4 ins.		u_{21}	u_{22}	u_{23}	u_{24}	u_{25}
	3	5 ins.		u_{31}	u_{32}	u_{33}	u_{34}	u_{35}
	4	6 ins.		u_{41}	u_{42}	u_{43}	u_{44}	u_{45}
Probabilities*				p_1	p_2	p_3	p_4	p_5

*Probabilities are known (decision under risk)

*Probabilities are unknown (decision under uncertainty)

**Assumed true state of nature in deterministic design (Decision under certainty).

TABLE 2.2 - UTILITY MATRIX TO ILLUSTRATE DECISION UNDER CERTAINTY. RISK AND UNCERTAINTY

Optimisation criterion \ Decision variables →	Parameters: Continuous	Parameters: Integer (Discrete)	Functions
Function	Function optimization problems (See Table 2.4)	Integer programming	
Functional (continuous, convex)			1. Calculus of variation. 2. Dynamic programming. 3. Pontryagin's maximum principle
Functional (Discontinuous non-smooth non-convex)			Non-Eulerian calculus

TABLE 2.3 - MATHEMATICAL METHODS OF OPTIMIZATION UNDER CERTAINTY.

Nature of Objective Function	Nature of constraints	Problem	Computational Method
Linear	Linear	Linear Programming	Simplex Algorithm
Non-linear	No constraints	Classical optimization problems	1. Indirect methods using differential calculus. 2. Direct methods or search methods (See Table 2.5). 3. Combined use of indirect and direct methods. a. Equation solution by search. b. Reduction of dimensionality.
Non-linear	Equality constraints	Classical optimization problems	Lagrangian Multiplier method. Jacobian method
Non-linear	Constraints are equalities, inequalities or both Linear or Non-linear	Non-linear programming	1. Method of feasible directions. 2. Penalty function approach. (i) Outside penalty (ii) SUMT (Fiacco and McKormick) 3. Slack variable method
SPECIAL TYPES OF NONLINEAR PROGRAMMING PROBLEMS			
Non-linear	Linear equalities	-	Gradient projection method.
Quadratic function	Linear inequalities	Quadratic programming	-
Convex functions	Convex inequalities	Convex programming	SUMT, using Kuhn-Tucker theorem
Non-convex, discontinuous	Discontinuous	Concave programming	-
Geometric progression in decision variables.	Inequalities	Geometric programming	-

Group	Method
One dimensional search	(1) Newton-Raphson search, (2) Cubic convergent search without derivatives, (3) Quadratic convergent search without derivatives, (4) Fibonacci search, (5) Search by Golden section, (6) One dimensional search in n-dimensional space.
None Sequential Methods	(1) Non-sequential random search (Monte Carlo search), (2) Non-sequential factorial search
Univariate and Relaxation search	(1) Univariate search, (2) Southwell's relaxation search, (3) Southwell Synge search
Gradient Methods	(1) Continuous steepest ascent (descent), (2) Discrete steepest ascent (descent) (3) n-dimensional Newton-Raphson search
Acceleration-Step Search	(1) Two dimensional acceleration step search, (2) PARTAN (n-dimensional acceleration-step search)

TABLE 2.5 - SEARCH METHODS OF OPTIMIZATION

Table 2.5 (Continued)

Conjugate Direction Methods	(1) Conjugate gradient method of Hestenes and Stiefel. (2) Method of Fletcher and Reeves, (3) DFP method (Davidson method via Fletcher and Powell)
Other Search Methods	(1) Pattern search, (2) Search by directed array, (3) Bunny-Hop search, (4) Creeping random methods, (5) Centroid method

TABLE 2.5 (CONTINUED)

NON UTILITARIAN DECISIONS	UTILITARIAN DECISIONS
<u>PROBABILITY ASSIGNED A PRIORI</u> 1. Rule of high probability 2. Rule of max. probability 3. Rule of high weight 4. Rule of max. weight <u>PROBABILITY NOT KNOWN A PRIORI</u> 5. Rule of max. likelihood 6. Method of least squares.	<u>DECISION UNDER CERTAINTY</u> Optimum utility <u>DECISION UNDER RISK</u> 1. Laplace utility criterion 2. Expected utility criterion <u>DECISION UNDER UNCERTAINTY</u> a. <u>Game situation</u> Minimax (saddle value) theory. b. <u>Complete ignorance of probability</u> 1. Minimax loss (Wald's theory) 2. Minimum loss 3. Maximin gain 4. Maximax gain 5. Minimax regret rule (Savage) 6. Optimism-pessimism criterion (Hurwicz's rule) c. <u>Probability can be found by experiments</u> Bayesian rule

TABLE 2.6 - ACCEPTANCE RULES IN DECISION MAKING.

Step	Nature of Decision	Decision Made	Decision Maker	Model	Criterion
1	Group decision	Selection of site or position	Group consisting of client financier, society, structural engineer, construction engineer, architect and others	Operational gaming.	System effectiveness (Functional conformity, structural efficiency, economy)
2	Group decision	Selection of functional configuration			
3	Individual decision	Choice of structural concept	Structural engineer	Check list Method and other subjective decision models	
4	Individual decision of course of action	Arrangement of members and types of connections	Structure	Game	Cost-effectiveness (safety, serviceability, ductility, economy).
5		Selection of material and geometry of members	Structure	Game and decision theories	

TABLE 3.1 - DETAILS OF PROPOSED DESIGN METHODOLOGY.

Attribute (Factors) →		1	2	3	4	5	
Weightage →		w_1	w_2	w_3	w_4	w_5	$\sum_j w_j u_{ij}$
Alternatives	1	u_{11}	u_{12}	u_{13}	u_{14}	u_{15}	U_1
	2	u_{21}	u_{22}	u_{23}	u_{24}	u_{25}	U_2
	3	u_{31}	u_{32}	u_{33}	u_{34}	u_{35}	U_3
	4	u_{41}	u_{42}	u_{43}	u_{44}	u_{45}	U_4
	5	u_{51}	u_{52}	u_{53}	u_{54}	u_{55}	U_5
	6	u_{61}	u_{62}	u_{63}	u_{64}	u_{65}	U_6
	7	u_{71}	u_{72}	u_{73}	u_{74}	u_{75}	U_7

TABLE 3.2 - DECISION TABLE OF CHECK LIST METHOD.

Attributes or Factors	1*	2	3	4	5	6	7	8	9	U
Weightage → Alternative	0.2665	0.2665	0.1335	0.1335	0.066	0.0532	0.0399	0.0266	0.0133	
One way slab	2	1	10	10	10	10	6	10	10	5.3049
Two way slab	8	2	10	10	10	10	6	10	10	7.1734
Beam and girder slab B_2	8	10	6	4	5	6	6	10	6	7.3659
I-beam floor	5	8	7	6	6	8	6	10	6	6.6023
Grid floor (Rect.) B_1	10	10	10	10	[illegible]	6	6	10	8	$\boxed{9.1975}$
Grid floor (Diag.) B_3	8	6	10	8	4	6	6	10	8	7.3350
Cellular plate B_4	7	3	10	10	3	10	10	10	10	6.866
Precast concrete slab	3	3	8	10	10	10	6	10	10	5.8379

*1. Structural efficiency, 2. Cost, 3. Aesthetics, 4. Function,
5. Ease of construction, 6. Fire resistance, 7. Heat insulation
8. Acoustics, 9. Lighting.

TABLE 3.3 - DECISION TABLE FOR THE SELECTION OF OPTIMUM TYPE OF ROOF

	Attributes of Factors	1*	2	3	4	5	6	7	8	9	Σ
	Vertical Load Bearing Members										
	Weightage w	0.2665	0.2665	0.1335	0.1335	0.0666	0.0532	0.0399	0.0266	0.0133	1.0
Alternatives	Wall system C_1	9	10	8	10	10	10	10	10	10	9.4625
	Column system C_2 with partitions	10	8	10	10	10	10	8	9	10	9.3566
	Rigid frame supports C_3 with partitions	8	7	10	10	10	10	2	9	10	8.3177
	Truss supports C_4 with partitions	7	6	6	8	10	10	2	6	8	6.8758
	Foundations										
	Weightage w	0.35	0.35	0	D.0	0.2	0.10	0	0	0	1
Alternatives	Spread footing L_1	10	10	-	-	10	8	-	-	-	9.8
	Pile foundation D_2	8	8	-	-	2	10	-	-	-	7.0
	Raft foundation D_3	6	6	-	-	4	9	-	-	-	5.9

*Numbers refer to the attributes listed in Table 3.3.

TABLE 3.4 - DECISION TABLE TO FIND THE UTILITY OF STRUCTURAL MEMBERS

	Weightage for systems	Roof	Support	Foundation
1.	0.3	0.3	0.35	0.35
2.	0.4	0.5	0.3	0.2
3.	0.3	0.5	0.5	0
Factor		0.44	0.38	0.18

1. Structural efficiency
2. Cost
3. Aesthetics

TABLE 3.5 - EVALUATION OF GRADATION FACTOR FOR THE COMBINATION OF VERTICAL ROOF + VERTICAL SUPPORT + FOUNDATION.

Sl. No.	Alternative combinations	B	C	D
1.	B_1 C_1 D_1	1 0.33	1 x 1 0.33	1 0.33
2.	B_1 C_1 D_2	1 0.4	1 x 0.5 0.2	1.0 0.4
3.	B_1 C_2 D_2	1.5 0.375	2 x 1 0.5	0.5 0.125
4.	B_1 C_2 D_2	1.5 0.333	2 x 1.0 0.444	1.0 0.223
5.	B_1 C_3 D_1	1 0.375	2 x 1 0.5	0.5 0.125
6.	B_1 C_3 D_2	1 0.333	2 x 1.0 0.444	1.0 0.223
7.	B_1 C_4 D_1	1 0.286	2 x 1.0 0.572	0.5 0.143
8.	B_2 C_1 D_1	1 0.4	0.5 x 1.0 0.2	1.0 0.4
9.	B_2 C_1 D_2	1 0.445	0.5 x 0.5 0.11	1.0 0.445
10.	B_2 C_2 D_1	0.8 0.348	1.0 x 1.0 0.435	0.5 0.218
11.	B_2 C_2 D_2	0.8 0.286	1.0 x 1.0 0.357	1.0 0.357
12.	B_2 C_3 D_1	0.8 0.348	1.0 x 1.0 0.435	0.5 0.218
13.	B_2 C_4 D_1	0.8 0.348	1.0 x 1.0 0.435	0.5 0.218
14.	B_3 C_1 D_1	1 0.333	1 x 1.0 0.333	1.0 0.333
15.	B_3 C_2 D_1	1.5 0.375	2 x 1.0 0.5	0.5 0.125
16.	B_3 C_3 D_1	1 0.286	2 x 1.0 0.572	0.5 0.143

TABLE 3.6 - EVALUATION OF INTERACTION FACTORS

Sl.No.	Alternative Combination		U(B)	U(C)	U(D)	Σ
	Gradation factors	g	0.44	0.38	0.18	1.0
1.	B_1 C_1 D_1	U	9.1980	9.4630	9.800	
		m_f	0.33	0.33	0.33	2.795
2.	B_1 C_1 D_2	U	9.1980	9.4630	7.0	
		m_f	0.40	0.20	0.40	
			0.1615	0.719	0.504	1.3845
3.	B_1 C_2 D_1	U	9.198	9.3566	9.80	
		m_f	0.375	0.50	0.125	
			1.52	1.78	0.2205	3.5205
4.	B_1 C_2 D_2	U	9.198	9.3566	7.0	
		m_f	0.333	0.444	0.223	
			1.01	1.58	0.281	2.871
5.	B_1 C_3 D_1	U	9.198	8.318	9.80	
		m_f	0.375	0.5	0.125	
			1.52	1.58	0.2205	3.3205
6.	B_1 C_3 D_2	U	9.1975	8.3177	7.0	
		m_f	0.333	0.444	0.223	
			1.01	1.405	0.281	2.696
7.	B_1 C_4 D_1	U	9.198	6.878	9.80	
		m_f	0.286	0.572	0.143	
			1.16	1.50	0.232	2.912
8.	B_2 C_1 D_1	U	7.3659	9.4625	7.00	
		[illegible]	0.4	0.2	0.4	
			1.298	0.72	0.705	2.723

TABLE 3.7 - EVALUATION OF UTILITY FOR VARIOUS COMBINATIONS OF ROOF + VERTICAL SUPPORTS + FOUNDATION.

Sl.No.	Alternative combination		U(B)	U(C)	U(D)	Σ
		g	0.44	0.38	0.18	1.0
9.	B_2 C_1 D_2		7.3659	9.4625	7.0	
			0.445	0.11	0.445	
			1.441	0.396	0.561	2.398
10.	B_2 C_2 D_1		7.3659	9.3566	9.8	
			0.348	0.435	0.218	
			1.130	1.551	0.385	3.066
11.	B_2 C_2 D_2		7.3659	9.3566	7.0	
			0.286	0.357	0.357	
			0.895	1.270	0.450	2.615
12.	B_2 C_3 D_1		7.3659	8.3177	9.8	
			0.348	0.435	0.218	
			1.130	1.1375	0.385	2.6525
13.	B_2 C_4 D_1		7.3659	6.8758	9.8	
			0.348	0.435	0.218	
			1.130	1.135	0.385	2.650
14.	B_3 C_1 D_1		7.3350	9.4625	9.8	
			0.333	0.333	0.333	
			1.080	1.260	0.588	2.928
15.	B_3 C_2 D_1		7.3350	9.3566	9.8	
			0.375	0.5	0.125	
			1.215	1.785	0.2205	3.2205
16.	B_3 C_3 D_1		7.3350	8.3177	9.8	
			0.286	0.572	0.143	
			0.895	1.810	0.252	2.957

TABLE 3.7 (CONTD.)

Trial No.	Item	s=0	s=1	s=2	s=3	s=4	
1	α	440	440	440	440	440	
	y	0	0.002295	0.00459	0.006885	0.00918	
	H	0.918	0.918	0.918	0.918	0.918	
2	α	430	430	430	430	430	
	y	0	0.00232	0.004645	0.006965	0.00929	
	H	0.929	0.929	0.929	0.929	0.929	
3	α	420	420	420	420	420	
	y	0	0.00238	0.00476	0.00714	0.00952	
	H	0.952	0.952	0.952	0.952	0.952	
4	α	410	410	410	410	410	
	y	0	0.00244	0.00488	0.00732	0.00976	
	H	0.976	0.976	0.976	0.976	0.976	
5	α	400	400	400	400	400	Optimal
	y	0	0.0025	0.0050	0.0070	$\boxed{0.01}$	Limit
	H	1.0	1.0	$\boxed{1.0}$ Maximum H	1.0	1.0	

TABLE 5.1 - EXAMPLE OF TENSION BAR ELASTIC CASE

Sl.No.	Item	s=0	s=1	s=2	s=3	s=4	
1	α	444	444	444	444	444	Upper bound to α
	y	0	0.00225	0.0045	0.00675	0.009	
2	α	444.0	436.5	426.0	418	409	
	y	0	0.00225	0.00454	0.00689	0.00928	
3	α	440.0	431.5	423.0	414.0	405	
	y	0	0.00227	0.00458	0.00694	0.00936	
4	α	430.0	422.0	412.5	404	396	
	y	0	0.00233	0.00470	0.00713	0.00960	
5	α	420.0	412.0	403.0	395.0	386.0	
	y	0	0.00238	0.00480	0.00729	0.00982	
6	α	415	406	398	390	382	Lower bound to α
	y	0	0.00241	0.00486	0.00738	0.00995	

TABLE 5.2 - UPPER AND LOWER BOUNDS TO THE VALUE OF $\alpha(s)$ FOR THE TENSION BAR WITH NONLINEAR MATERIALS.

Basic Data		s = 0	s = 1	s = 2	s = 3	s = 4	
v		1.0	1.0	1.0	1.0	1.0	
λ_1		-400	-300	-200	-100	0	
Upper bound to α		444	444	444	444	444	
Lower bound to α		415	406	398	390	382	
Trial	**Item**						
1	α	444	444	444	444	444	
	y	0	0.00225	0.00450	0.00675	0.0090	
	λ_2	0	0	0	0	0	
	H	-0.901	-0.901	-0.900	0.900	0.900	
2	α	430	422	412.5	404	396	
	y	0	0.00233	0.00470	0.007125	0.00960	
	λ_2	-0.0286	-0.0286	-0.0152	-0.0049	0	
	H	-0.9486	-0.9716	-0.9702	-0.9654	-0.960	
3	α	420.0	412.0	403.0	395.0	386.0	
	y	0.0	0.00238	0.00480	0.007285	0.009815	
	λ_2	-0.02993	-0.02993	-0.01583	-0.00533	0	
	H	-0.98493	-0.99593	-0.99183	-0.98683	-0.9815	
4	α	415	406	398	390	382	Optimal control $\alpha^*(s)$
	y	0	0.00241	0.00486	0.00738	$\boxed{0.00995}$	
	λ_2	-0.03257	-0.03257	-0.01622	-0.00550	0	
	H	-0.99757	-1.01157	-1.00422	-0.999590	-0.995	

TABLE 5.3 - ITERATIVE SOLUTION OF OPTIMAL CONTROL TENSION BAR (INELASTIC CASE).

Trial	Item	s = 0	s = 1	s = 2	s = 3	s = 4^-	s = 4^+	s = 5	s = 6	s = 7	s = 8
1	α^*	667	668	667	667	667					
	y	0	0.0015	0.003	0.0045	0.006	0.006	0.0045	0.003	0.0015	0.00
2	α	667	534	400	267	133.5	667	600	534	467	400
	y	0	0.0015	0.003375	0.005875	0.009655	0.009655	0.008155	0.006485	0.00471	0.00257
3	α	660	528	396	264	132.0	660	593	528	462	396
	y	0	0.001515	0.003410	0.00594	0.00973	0.00973	0.008215	0.00653	0.00464	0.00248
4	α	650	520	390	260	130.0	650	585	520	456	390
	y	0	0.00154	0.003462	0.006022	0.009872	0.009872	0.008332	0.006652	0.00473	0.00254
5	α^{**}	640	512	384	256	128.0	640	576	512	448	384
	y	0	0.001562	0.003512	0.006112	0.010022	$\boxed{0.010022}$	0.008460	0.006725	0.004775	0.002545

*Upper limit of α ** Lower limit of α.

TABLE 5.4 – BOUNDS TO THE ADMISSIBLE RANGE OF α.

Trial No.	Item	s=0	s=1	s=2	s=3	$s=4^-$	$s=4^+$	s=5	s=6	s=7	s=8
Upper bound to α		667	667	667	667	667					
Lower bound to α		640	512	384	256	128	640	576	512	448	384
		-400	-300	-200	-100	0	+400	300	200	100	0
	1										
	v	1	1	1	1	1	-1	-1	-1	-1	-1
						CALCULATION OF HAMILTONIAN					
1	α	660	528	396	264	132.0	660	593	528	462	396
	y	0	0.001515	0.003410	0.00549	0.00973	0.00973	0.008215	0.00653	0.00464	0.00248
	λ_2	-0.5000	-0.5000	-0.3580	-0.1895	0	+0.1352	+0.1352	+0.0781	+0.0308	0
	H	-1.1060	-1.2195	-1.2040	-1.1175	-0.9730	0.2318	0.1803	0.2069	0.2172	0.2480
							MINIMUM H NOT ACCEPTED SINCE $y(4) < 0.01$				
2	α	650	520	390	260	130	650	585	520	456	390
	y	0.0	0.00154	0.003462	0.006022	0.009872	0.009872	0.008332	0.006622	0.00470	0.00251
	λ_2	-0.5070	-0.5070	-0.3630	-0.1920	0	+0.1344	+0.1349	+0.0791	+0.0310	0
	H	-1.1230	-1.2380	-1.2412	-1.1792	-0.9872	0.2363	0.1853	0.1991	0.2200	0.251

TABLE 5.5 ITERATIVE SOLUTION OF OPTIMAL CONTROL, EXAMPLE OF TENSION BAR LOADED AND UNLOADED

		s=0	s=1	s=2	s=3	s=4⁻	s=4⁺	s=5	s=6	s=7	s=8
	α^*	640	512	384	256	128	640	576	512	448	384
3	y	0	0.001562	0.003512	0.00612	$\boxed{0.010022}$	0.010022	0.008460	0.006725	0.004775	0.002595
	λ_2	-0.5185	-0.5185	-0.3720	-0.1950	0	+0.1435	+0.1435	+0.0807	+0.0319	0
	H	-1.1435	-1.2607	-1.2432	-1.1962	-1.0022	0.2331	0.1815	0.2018	0.2226	0.2545

MINIMUM H ACCEPTED SINCE $y(4) = 0.01$

		s=0	s=1	s=2	s=3	s=4⁻	s=4⁺	s=5	s=6	s=7	s=8
	α	640	512	384	256	128	640	580	520	460	390
	y	0	0.001562	0.003512	0.006112	0.010022	0.010022	0.008462	0.006737	0.005812	0.003642
4	λ_2	-0.5185	-0.5185	-0.3720	-0.1950	0	+0.1262	+0.1262	+0.0726	+0.02830	0
	H	-1.1435	-1.2607	-1.2432	-1.1962	-1.0022	0.2500	0.2030	0.2171	0.2162	0.3642

*Optimal Policy

TABLE 5.5 (CONTD.)

s	F_1	F_2	v_1	v_2	1	2
0	0	0	0.884	0.884	-636.4	-353.6
1	0.884	0.884	0.884	0.884	-548.6	-265.2
2	1.768	1.768	0.884	0.884	-459.6	-176.4
3	2.652	2.652	0.884	0.884	-371.2	-88.4
4^-	3.536	3.536	0.884	0.884	-282.8	0
4^+	3.536	3.536	-0.177	-1.591	-282.8	282.8
5	3.359	1.945	-0.177	-1.591	-300.5	123.7
6	3.182	0.354	-0.177	-1.591	-318.2	-35.4
7	3.005	-1.237	-0.177	-1.591	-335.9	-194.5
8^-	2.828	-2.828	-0.177	-1.591	-353.6	-353.6
8^+	2.828	-2.828	0.884	0.884	-353.6	-353.6
9	3.712	-1.944	0.884	0.884	-265.2	-265.2
10	4.596	-1.06	0.884	0.884	-176.8	-176.8
11	5.480	-0.176	0.884	0.884	-88.4	-88.4
12	6.363	0.708	0.884	0.884	0	0

TABLE 5.6 TRIAL AND ERROR SOLUTION OF OPTIMAL CONTROL. EXAMPLE OF TWO BAR TRUSS.

Trial 1

s	α_1	α_2	y_1	y_2	λ_3	λ_4	H
0	580	500	0.0	0	-0.2655	-0.08585	-1.906
1	555	478	0.00150	0.00177	-0.2169	-0.05465	-1.892
2	530	456	0.00311	0.00362	-0.1613	-0.02905	-1.861
3	503	433	0.004775	0.00556	-0.1195	-0.01035	-1.862
4^-	478	411	0.006525	0.0076	-0.0818	00	-1.845
4^+	580	500	0.006525	0.00760	-0.0818	0	0.527
5	580	500	0.00622	0.00442	-0.818	0	0.527
6	580	500	0.005915	0.00124	-0.0818	0	0.5242
7	580	500	0.005610	-0.00194	-0.0818	0	0.524
8^-	580	500	0.005305	-0.00512	-0.0818	0	0.525
8^+	580	500	0.005305	-0.00512	-0.0818	0	-1.2525
9	464	500	0.005610	-0.00335	-0.0549	0	-1.2215
10	440	500	0.007515	-0.00158	-0.03835	0	-1.2245
11	414	500	0.009020	0.00019	-0.01505	0	-1.1735
12	388	500	0.01160	0.00196	0	0	-1.1985

TABLE 5.6 (CONTD.)

s	α_1^*	α_2^*	y_1	y_2	λ_3	λ_4	H
0	510	560	0	0	-0.2930	-0.07655	-1.989
1	488	536	0.00173	0.001575	-0.2477	-0.04875	-1.990
2	466	511	0.00354	0.003223	-0.1958	-0.02595	-1.967
3	443	486	0.00543	0.004951	-0.1481	-0.00925	-1.948
4^-	420	461	0.00742	0.006771	-0.1056	0	-1.943
4^+	510	560	0.00742	0.006771	-0.1056	0	0.5235
5	510	560	0.007073	0.003921	-0.1056		0.519
6	510	560	0.006726	0.001071	-0.1056	0	0.519
7	510	560	0.006379	-0.001779	-0.10506	0	0.519
8^-	510	560	0.006032	-0.004629	-0.10505	0	0.5188
8^+	510	560	0.006032	-0.004629	-0.10505	0	-1.385
9	416	560	0.006205	-0.003054	-0.07445	0	-1.333
10	394	560	0.008325	-0.001579	-0.04005	0	-1.3285
11	371	560	0.010565	-0.000004	-0.01445	0	-1.299
12	348	560	0.012945	0.001571	0.00	0	-1.298

*Optimal control $\underline{\alpha}$

TABLE 5.6 (CONTD.)

Trial	Item	s=0	s=1	s=2	s=3	s=4
1			<u>Trial With Single Bar</u>			
	v_1	10	10	10	10	10
	α	4740	4260	3780	3320	2850
	y	0	0.0210	0.00446	0.00700	0,01001
	λ_1	-4000	-3000	-2000	-1000	0
	μ_1	-0.00178	-0.00165	-0.00140	-0.000908	0
	λ_4	-0.5454	-0.3788	-0.2224	-0.0900	0
	H	-11.128	-11.036	-10.782	-10.470	-10.00
2			<u>Trial With Two Bars</u>			
	$\frac{dw}{ds}$	10	10	10	10	10
	v_2	5	5	5	5	5
	v_3	0	0	0	0	0
	v_1	5	5	5	5	5
	α_1	2370	2130	1890	1660	1425
	y	0.0	0.00211	0.00446	0.00700	$\boxed{0.01001}$ Limit of y.
	α_2	2370	2130	1890	1660	1425
	3	0	0	0	0	0
	λ_1	-2000	-1500	-1000	-500	0
	λ_2	-2000	-1500	-1000	-500	0
	λ_3	0	0	0	0	0
	μ_1	-0.00178	-0.00165	-0.00140	-0.000908	0

TABLE 5.7 TRIAL AND ERROR SOLUTION OF THREE BAR SYSTEM (NONLINEAR CASE)

Trial Item	s=0	s=1	s=2	s=3	s=4
μ_2	-0.00178	-0.00165	-0.00140	-0.000908	0
μ_3	0	0	0	0	0
λ_4	-0.2727	-0.1894	-0.1112	-0.0450	0
λ_5	-0.2727	-0.894	-0.1112	-0.0450	0
λ_6	0	0	0	0	0
n	-11.128	-11.036	-10.782	-10.470	-10.00

TABLE 5.7 - TRIAL AND ERROR SOLUTION OF THREE BAR SYSTEM (NONLINEAR CASE)

Item	s=0	s=1	s=2	s=3	s=4
			Trial With Three Bars		
$\frac{dw}{ds}$	10	10	10	10	10
v_1	3.33	3.33	3.33	3.33	3.33
v_2	3.33	3.33	3.33	3.33	3.33
v_3	3.33	3.33	3.33	3.33	3.33
α_1	1600	1440	1280	1120	960
α_2	1600	1440	1280	1120	960
α_3	1600	1440	1280	1120	960
y_1	0	0.00208	0.00439	0.00699	0.00996
y_2	0	0.00208	0.00439	0.00699	0.00996
y_3	0	0.00208	0.00439	0.00699	0.00996
λ_1	-1333	-1000	-667	-333	0
λ_2	-1333	-1000	-667	-333	0
λ_3	-1333	-1000	-667	-333	0
μ_1	-0.001735	-0.00161	-0.001358	-0.000912	0
μ_2	-0.001735	-0.00161	-0.001358	-0.000912	0
μ_3	-0.001735	-0.00161	-0.001358	-0.000912	0
λ_4	-0.1794	-0.1239	-0.0722	-0.0288	0
λ_5	-0.1794	-0.1239	-0.0722	-0.0288	0
λ_6	-0.1794	-0.1239	-0.0722	-0.0288	0
H	-10.144	-10.269	-10.342	-10.248	-9.96

TABLE 5.7 (CONTD.)

s	$\frac{dw_1}{ds}$	$\frac{dw_2}{ds}$	λ_1 (of y_1)	λ_2 (of y_2	λ_3 (of y_3)	v_1 (Assumed)	v_3	v_2
0	1	0.5	-282.8	-282.8	-141.4	0.9	0.193	0.2274
1	1	0.5	-247.45	-212.1	-106.5	0.9	0.193	0.2274
2	1	0.5	-212.10	-141.4	-70.7	0.9	0.193	0.2274
3	1	0.5	-176.75	-70.7	-35.35	0.9	0.193	0.2274
4^-	1	0.5	-141.4	0	0	0.9	0.193	0.2274
4^+	-1	0.5	-141.4	282.8	141.4	-0.167	-0.874	-0.406
5	-1	0.5	-106.05	212.1	106.05	-0.167	-0.874	-0.406
6	-1	0.5	-70.7	141.4	70.7	-0.167	-0.874	-0.406
7	-1	0.5	-35.35	70.7	35.35	-0.167	-0.874	-0.406
8	-1	0.5	0	0	0	-0.167	-0.874	-0.406

TABLE 5.8 EXAMPLE OF THREE BAR TRUSS

s	α_2	α_1	α_3	y_1	y_2	y_3	H
0	90.96	400	147.5	0	0	0	-1.666
1	90.96	400	147.5	0.00225	0.0025	0.0013	-1.387
2	90.96	400	147.5	0.00450	0.005	0.0026	-1.5117
3	90.96	400	147.5	0.00675	0.0075	0.0039	-1.4918
4	90.96	400	147.5	0.0090	0.01	0.0052	-1.4535
4	90.96	400	147.5	0.0090	0.01	0.0052	-1.006
5	90.96	400	147.5	0.00858	0.00553	-0.00072	-.6466
6	90.96	400	147.5	0.00816	0.00106	-0.00664	-.4085
7	90.96	400	147.5	0.00774	-0.00341	-0.01256	0.5417
8	90.96	400	147.5	0.00732	-0.00788	-0.01808	1.688

TABLE 5.83 (CONTD.)

s	v_1 in tons	v_2	v_3	λ_1 (of F_1)	λ_2 (of F_2)	λ_3 (of F_3)	α_1	α_2	α_3
0	79	67	33.5	-0.001594	-0.001978	-0.00001923	$13x10^5$	$18x10^5$	$13x10^5$
1	79	67	33.5	-0.001594	-0.001978	-0.00001608	$13x10^5$	$18x10^5$	$13x10^5$
2^-	79	67	33.5	-0.001157	-0.001542	-0.00001008	$13x10^5$	$18x10^5$	$13x10^5$
2^+	15.5	32	16	-0.001157	-0.001542	-0.00001008	$13x10^5$	$18x10^5$	$13x10^5$
3	15.5	32	16	-0.0000432	-0.000671	-0.0000045	$13x10^5$	$18x10^5$	$13x10^5$
4^-	15.5	32	16	0	0	0	0	0	0
4^+	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

TABLE 5.9 TRIAL AND ERROR SOLUTION OF SIMPLY SUPPORTED BEAM WITH CREEP EFFECTS.

Trial 1 Contd.

s	y_1	y_2	y_3	H
0	0	0	0	-0.78195
1	6.17×10^{-5}	3.72×10^{-5}	2.58×10^{-5}	-0.8121075
2^-	12.38×10^{-5}	7.741×10^{-5}	5.1978×10^{-5}	-0.81825
2^+	12.38×10^{-5}	7.741×10^{-5}	5.1978×10^{-5}	-0.33100
3	16.73×10^{-5}	11.931×10^{-5}	6.7278×10^{-5}	-0.32391
4^-	23.12×10^{-5}	18.28×10^{-5}	8.5298×10^{-5}	-0.30210
4^+	23.12×10^{-5}	18.28×10^{-5}	8.5298×10^{-5}	0
5	29.88×10^{-5}	26.05×10^{-5}	9.4998×10^{-5}	0
6	36.64×10^{-5}	34.80×10^{-5}	10.4698×10^{-5}	0
7	43.40×10^{-5}	42.55×10^{-5}	11.43×10^{-5}	0
8	50.16×10^{-5}	50.30×10^{-5}	12.4098×10^{-5}	0

TABLE 5.9 (CONTD.)

Trial 2

α_1	α_2	α_3	y_1	y_2	y_3	H
$18x10^5$	$13x10^5$	$13x10^5$	0	0	0	-0.78155
$18x10^5$	$13x10^5$	$13x10^5$	$4.39x10^{-5}$	$5.15x10^{-5}$	$2.58x10^{-5}$	-0.80170
$18x10^5$	$13x10^5$	$13x10^5$	$9.273x10^{-5}$	$10.3302x10^{-5}$	$5.1978x10^{-5}$	-0.81135
$18x10^5$	$13x10^5$	$13x10^5$	$9.273x10^{-5}$	$10.3302x10^{-5}$	$5.1978x10^{-5}$	-0.28201
$18x10^5$	$13x10^5$	$13x10^5$	$14.075x10^{-5}$	$15.30x10^{-5}$	$6.7278x10^{-5}$	-0.28201
$18x10^5$	$13x10^5$	$13x10^5$	$20.157x10^{-5}$	$22.33x10^{-5}$	$8.5298x10^{-5}$	-0.28100
0	0	0	$20.157x10^{-5}$	$22.33x10^{-5}$	$8.5298x10^{-5}$	0
0	0	0	$26.887x10^{-5}$	$30.08x10^{-5}$	$9.4998x10^{-5}$	0
0	0	0	$33.617x10^{-5}$	$37.83x10^{-5}$	$10.4698x10^{-5}$	0
0	0	0	$40.347x10^{-5}$	$45x58x10^{-5}$	$11.4398x10^{-5}$	0
0	0	0	$47.107x10^{-5}$	$53.33x10^{-5}$	$12.4098x10^{-5}$	0

TABLE 5.9 (CONTD.)

		s=0	s=1	s=3	s=5
	α	600	500	0	0
1	y	0	0.00666	0.00794	0.00922
	H	5.33	5.328	0	0
	α	590	590	0	0
2	y	0	0.00678	0.00806	0.00934
	H	5.42	5.424	0	0
	α	580.	580	0	0
3	y	0	0.00690	0.00818	0.00946
	H	5.51	5.52	0	0
	α	570	570	0	0
4	y	0	0.00702	0.00838	0.00966
	H	5.61	5.616	0	0
	α	560	560	0	0
5	y	0	0.00714	0.00842	0.00960
	H	5.72	5.712	0	0
	α	550	550	0	0
6	y	0	0.00727	0.00855	0.00983
	H	5.82	5.816	0	0
	α	540	540	0	0
7	y	0	0.0074	0.00868	$\boxed{0.00996}$
	H	5.92	5.92	0	0

TABLE 5.10 SOLUTION OF TWO BAR TRUSS PROBLEM WITH JUMPS IN LOADING FUNCTION.

Trial No. \ w_i / p_i	STATE OF NATURE (LOADS IN TONS) <40	40	41	42	43	44	45	46	47	$\geq$ 48	Cost-effectiveness factor
	0.90	0.035	0.025	0.017	0.010	0.005	0.003	0.002	0.001	0.002	$= 1+\Sigma p(w)K(w)$
1	0	13.5	13.5	13.5	13.5	13.5	13.5	13.5	13.5	13.5	2.35
2	0	0.227	0.227	0.227	0.227	12.35	12.35	12.35	12.35	12.35	1.18075
3	0	0.208	0.208	0.208	0.208	0.208	0.208	0.208	0.208	11.4	1.0432
4	0	0.454	0.454	0.454	0.454	12.35	12.35	12.35	12.35	12.35	1.2000
5	0	0.416	0.416	0.416	0.416	0.416	0.416	0.416	0.416	11.4	1.0635
6	0	0.217	0.217	0.217	0.217	0.478	0.478	11.85	11.85	11.85	1.08200
7	0	0.208	0.208	0.208	0.208	0.208	0.208	0.208	0.208	11.4	1.0432
8	0	0.454	0.454	0.454	0.454	12.35	12.35	12.35	12.35	12.35	1.2000
9	0	0.208	0.208	0.208	0.208	0.208	0.208	0.208	0.208	11.4	1.0432
10	0	0.416	0.416	0.416	0.416	0.416	0.416	0.416	0.416	1.0	1.0427
11	0	0.208	0.208	0.208	0.208	0.208	0.376	0.376	0.376	1.0	1.04365
12	0	0.666	0.666	0.666	0.666	0.666	0.666	0.666	0.666	1.0	1.0671

TABLE 6.1 - COST-EFFECTIVENESS DESIGN OF THREE BAR SYSTEM

Trial No. \ w_i / p_i	STATE OF NATURE (LOADS IN TONS) <40	40	41	42	43	44	45	46	47	≥ 48	Cost-effectiveness factor
	0.90	0.035	0.025	0.017	0.010	0.005	0.003	0.002	0.001	0.002	$= 1+\Sigma p(w)K(w)$
1	0	13.5	13.5	13.5	13.5	13.5	13.5	13.5	13.5	13.5	2.35
2	0	0.227	0.227	0.227	0.227	12.35	12.35	12.35	12.35	12.35	1.18075
3	0	0.208	0.208	0.208	0.208	0.208	0.208	0.208	0.208	11.4	1.0432
4	0	0.454	0.454	0.454	0.454	12.35	12.35	12.35	12.35	12.35	1.2000
5	0	0.416	0.416	0.416	0.416	0.416	0.416	0.416	0.416	11.4	1.0635
6	0	0.217	0.217	0.217	0.217	0.478	0.478	11.85	11.85	11.85	1.08200
7	0	0.208	0.208	0.208	0.208	0.208	0.208	0.208	0.208	11.4	1.0432
8	0	0.454	0.454	0.454	0.454	12.35	12.35	12.35	12.35	12.35	1.2000
9	0	0.208	0.208	0.208	0.208	0.208	0.208	0.208	0.208	11.4	$\boxed{1.0432}$
10	0	0.416	0.416	0.416	0.416	0.416	0.416	0.416	0.416	1.0	1.0427
11	0	0.208	0.208	0.208	0.208	0.208	0.376	0.376	0.376	1.0	1.04365
12	0.	0.666	0.666	0.666	0.666	0.666	0.666	0.666	0.666	1.0	1.0671

TABLE 6.1 – COST-EFFECTIVENESS DESIGN OF THREE BAR SYSTEM

$F_{1\,Ton}^{*}$	$F_{2\,Ton}^{*}$	$-F_{2\,Ton}^{*}$	Cost C_s Rs	S_1^* p_1	S_1^* K_1	S_2 p_2	S_2 K_2	S_3 p_3	S_3 Case 1 K_3	S_3 Case 2 K_3	S_4 Case 1	S_4 Diff.	S_4 Case 2	S_4 Diff.
6.363	3.535	2.828	99	0.8	0.0	0.1940	5.56	0.0059	6.55	66.4	2.11728	-	2.4704	-
7.0	4.0	3.4	110	0.8	0.0	0.1957	4.55	0.00429	5.55	51.0	1.91317	0.20411	2.10718	0.36322
7.5	4.50	4.0	120	0.8	0.0	0.19946	4.16	0.00058	5.16	46.8	1.83258	0.080632	1.85494	0.25224
7.75	4.50	4.0	123	0.8	0.0	0.199984	4.08	0.00159	5.08	46	1.815742	$\boxed{0.016796}$	1.82325	0.03169
8.0	4.50	4.0	125	0.8	0.0	0.199954	4.00	0.000046	5.00	45	1.800046	0.015696	1.801826	$\boxed{0.021424}$

*S_1 = Serviceable state; S_2 = Unserviceable state; S_3 = Collapse state; S_4 indicates $K_T = 1 + \Sigma\, p_i K_i$

TABLE 6.2 ALTERNATIVE TRIALS IN DESIGN OF A TWO BAR TRUSS FOR COST-EFFECTIVENESS.

Sl.No.	EVENTS	RATING			
1.	Scientific knowledge and technological skill with which the structure is designed	Vg	g	f	p
2.	Accuracy of assumptions made, modelling chosen for design, design technique used	Vg	g	f	p
3.	Accuracy of computation	Vg	g	f	p
4.	Accuracy in the estimation of needs, serviceability and cost of failure	Vg	g	f	p
5.	Possibility of loads that are not accounted for	VH	H	f	N
6.	Possibility of new positions and directions of loads that are unaccounted	VH	H	f	N
7.	Possibility of loading in manners in which they are unaccounted (fatigue, impact, vibration etc.)	VH	H	f	N
8.	Possibility of environment effects of destructive nature not considered, like temperature, shrinkage, settlement, fire, earthquake etc.	VH	H	f	N
9.	Workmanship (skilled craftsmanship, efficient supervision)	Vg	g	f	p
10.	Inspection and control of materials in accordance with codes of practice	Vg	g	f	p
11.	Constructional accuracy of shape and dimensions of members	Vg	g	f	p
12.	Future maintenance, periodic inspection	Vg	g	f	p

Vg very good f fair VH very high f fair

g good p poor H high N no

TABLE 7.1 = UNCERTAINTIES TO BE COMPENSATED BY r'_s

Sl. No.	EVENTS	RATING			
1.	Scientific knowledge and technological skill with which the structure is designed	Vg	g	f	p
2.	Accuracy of assumptions made, model chosen for design, design technique used	Vg	g	f	p
3.	Accuracy of computation	Vg	g	f	p
4.	Accuracy of the criteria chosen for serviceability	Vg	g	f	p
5.	Possibility of loads that are not accounted for	VH	H	f	N
6.	Possibility of change in loading process and manner of loading	VH	H	f	N
7.	Possibility of other environmental effects causing deflection and cracking	VH	H	f	N
8.	Workmanship (skilled craftsmen, efficient supervision)	Vg	g	f	p
9.	Inspection and control of material in accordance with codes of practice and design specification	Vg	g	f	p
10.	Constructional accuracy in the choice of material and dimensions of members	Vg	g	f	p
11.	Future maintenance and periodic inspection	Vg	g	f	p
12.	Importance of unserviceability on the overall usefulness of the system	VH	H	f	N

TABLE 7.2 - UNCERTAINTIES TO BE ACCOUNTED IN THE EVALUATION OF r_c.

Sl. No.	EVENTS	RATING			
1.	Seriousness of a sudden collapse without warning	VH	H	f	M
2.	Ductility of the materials chosen	Vg	g	f	p
3.	Degree of redundancy provided	VH	H	f	M
4.	Chances that all the critical sections will reach maximum capacity simultaneously	VH	H	f	N

TABLE 7.3 - UNCERTAINTIES TO BE CONSIDERED IN THE EVALUATION OF r_d.

Event* →		1	2	3	4	5	6	7	8	9	10	11	12	Σ
Rating	1.	Vg	g	Vg	f	N	N	H	f	Vg	Vg	Vg	g	
	2.	g	Vg	g	g	f	f	VH	N	g	g	g	Vg	
	3.	f	f	f	p	H	H	f	H	f	f	f	f	
	4.	p	p	p	Vg	VH	VH	N	VH	p	p	p	p	
Event Personal probability		$\frac{1}{24}$	$\frac{1}{6}$	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{6}$	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{24}$	1
Pay off matrix	**0.1	4	3	4	2	1	1	3	2	4	4	4	3	$\frac{17.25}{6}$
	0.2	3	4	3	3	2	2	4	1	3	3	3	4	$\frac{17.75}{6}$
	0.3	2	2	2	1	3	3	2	3	2	2	2	2	$\frac{13.00}{6}$
	0.4	1	1	1	4	4	4	1	4	1	1	1	1	$\frac{12.00}{6}$

* The numbers 1 - 12 refer to the events listed in Table 7.1

** 0.1, 0.2, 0.3, 0.4 are the alternative actions.

TABLE 7.4 - EVALUATION OF r'_s - FOR A THREE BAR SYSTEM

Structural Design Problem	Optimal Control Problem	Differential Game Problem
Structure	Decision maker	Minimizing player (Player 2)
Nature	(Not considered)	Maximizing player (Player 1)
Structural action	Control process	Game
Loading process $\underline{u}(s)$	(Given function)	Maximizing control
Load $\underline{w}(s)$	(Given function)	State of Player 1
Force $\underline{F}(s)$	State of minimizing control	State of Player 2
Force rate $\underline{v}(s)$	Minimizing control	Minimizing strategy
Stiffness $\underline{\alpha}(s)$	Minimizing control	Minimizing strategy
Deformation $\underline{x}(s)$	State of controlled system	State of the controlled system
Behaviour idealization	State equation	State equation
Equilibrium equations	Constraints on $\underline{F}$ or $\underline{v}$	State variable constraints
Compatibility condition	State variable equality constraints	State variable equality constraints
Limit of normal load condition	(Given)	State variable inequality constraints
Serviceability constraints	State variable inequality constraints	State variable inequality constraints
Energy potential	Performance index	Pay-off

TABLE 8.1 - CORRESPONDENCE IN TERMINOLOGY IN STRUCTURAL DESIGN, OPTIMAL CONTROL AND DIFFERENTIAL GAME PROBLEMS.

Trial		s=0	s=1	s=2	s=3	s=4	s=5	s=6	s=7	s=8
1	u^*	1	1	1	1	− 1 | + −1	−1	−1	−1	−1
	w	0	1	2	3	4	3	2	1	0
	α^*	640	512	384	256	128 640	576	512	448	384
	H	−1.1435	−1.2607	−1.2432	−1.1962	0.2331+ −1.0022	0.1815	0.2018	0.222	254!
2	u^*	1	1	1	1 −1	−1	−1	−1	−	−
	w	0	1	2	3	2	1	0	−	−
	α^*	390	312	234	390	380	370	−	−	−
	y	0	0.00256	0.00577	0.01005	0.00749	0.00486	0.00216	−	−
	λ_1	−300	−200	−100	0 300	200	100	0		
	λ_2	−0.305	−0.305	−0.145	0 0.0215	0.0215	0.00732	0	−	−
	H	−1.075	−1.202	−1.150	−1.005 0.2138	0.2008	0.11568	0.216		
3	u^*	1	1	1 −1	−1	−1	−	−	−	−
	w	0	1	2	1	0	−	−	−	−
	α^*	224	179	− 224	224.0	−	−	−	−	−
	y	0	0.00446	|0.01004	0.00558	0.00112	−	−	−	−
	λ_1	−200	−100	0 200	100	0	−	−	−	−
	λ_2	−1.141	−0.141	0 0	0	0	−	−	−	−
	H	−1.033	−1.155	−1.004 0.112	0.112	0.112	−	−	−	−
4	u^*	1	1	1	1	1	−	−	−	−
	w	0	1	2	3	4	−	−	−	−
	α^*	640	512	364	256	128	−	−	−	−
	H	−1.1435	−1.2607	−1.2432	−1.1962	−1.002	−	−	−	−

TABLE 8.2 SOLUTION OF THE EXAMPLE OF TENSION BAR (4 TRIALS SHOWN HERE)

Trial 1*

s	u_1	u_2	α_1	α_2	H
0	1.25	0	510	560	-1.989
1	1.25	0	488	536	-1.990
2	1.25	0	466	511	-1.967
3	1.25	0	443	486	-1.948
4^-	1.25	0	420	461	-1.943
4^+	1.25	1	510	560	0.5235
5	-1.25	1	510	560	0.5243
6	-1.25	1	510	560	0.519
7	-1.25	1	510	560	0.5190
8^-	-1.25	1	510	560	0.5188
8^+	1.25	0	510	560	-1.385
9	1.25	0	416	560	-1.3330
10	1.25	0	394	560	-1.3285
11	1.25	0	371	560	-1.294
12	1.25	0	348	560	-1298

*Same as Trial 2 in Table 5.6

TABLE 8.3 EXAMPLE OF TWO BAR TRUSS
(TWO TRIALS ARE SHOWN).

s	u_1	u_2	v_1	v_2	λ_1	λ_2	α_1	α_2	y_1	y_2	λ_3	λ_4	H
0	0.83	0.67	1.060	0.113	-638	-67.8	510	560	0	0	-0.1618	-0.01545	-1.52405
1	0.83	0.67	1.060	0.113	-530	-56.5	488	536	0.00208	0.000201	-0.1618	-0.01545	-1.57525
2	0.83	0.67	1.060	0.113	-424	-45.2	466	511	0.00434	0.000411	-0.1018	-0.00965	-1.5685
3	0.83	0.67	1.060	0.113	-318	-33.9	443	486	0.00651	0.000631	-0.0437	-0.00455	-116025
4	0.83	0.67	1.060	0.113	-212	-22.6	510	560	0.00890	0.000862	0	0	-1.4562
5	0.83	0.67	1.060	0.113	-106	-11.3	510	560	0.01098	0.001003	0	0	-1.4383
6^-	0.83	0.67	1.060	0.113	0	0	510	560	0.01302	0.001204	0	0	-1.4224
6^+	-0.83	-0.67	-1.060	-0.113	636	67.8	510	560	0.01302	0.001204	0	0	0.0837
7	-0.83	-0.67	-1.060	-0.113	530	56.5	510	560	0.01094	0.001003	0	0	0.0840
8	-0.83	-0.67	-1.060	-0.113	424	45.2	510	560	0.00890	0.000862	0	0	0.0872
9	-0.83	-0.67	-1.060	-0.113	318	33.9	510	560	0.00682	0.000660	0	0	0.0897
10	-0.83	-0.67	-1.060	-0.113	212	22.6	510	560	0.00474	0.000458	0	0	0.0853
11	-0.83	-0.67	-1.060	-0.113	106	-11.3	510	560	0.00266	0.000256	0	0	0.0890
12	-0.83	-0.67	-1.060	-0.113	0	0	510	560	0.00058	0.000055	0	0	0.0635

TABLE 8.3 (CONTD.)

STRATEGY OF PLAYER 1, LOADING PATHS	STRATEGY OF PLAYER 2 - ALTERNATIVE STRUCTURES								Minimum of each row indicates best structure for each path of loading.
	1	2	3	4	5	...	...	ℓ	
1	p_{11}	p_{12}	p_{13}	p_{14}	p_{15}	...	...	$p_{1\ell}$	
2	p_{21}	p_{22}	p_{23}	p_{24}	p_{25}	...	...	$p_{2\ell}$	
3	p_{31}	p_{32}	p_{33}	p_{34}	p_{35}	...	...	$p_{3\ell}$	
4	p_{41}	p_{42}	p_{43}	p_{44}	p_{45}	...	...	$p_{4\ell}$	
5	p_{51}	p_{52}	p_{53}	p_{54}	p_{55}	...	...	$p_{5\ell}$	
⋮	⋮	⋮	⋮	⋮	⋮	...	...	⋮	
k	p_{k1}	p_{k2}	p_{k3}	p_{k4}	p_{k5}			$p_{k\ell}$	

Maximum of each column indicates worst course of loading for each type of structure.

(Saddle point exist if Maximum of the rows coincide with minimum of the columns - There is one element that is the maximum of its column and minimum of its row).

TABLE 8.2 - EXISTENCE OF SADDLE POINT ILLUSTRATED.

Cases	Cost of structure Rs.	Serviceable state Max p_1	K_1	One member fails Max p_2	K_2	Collapse Max p_3	K_3	Max K_T
1. Distribution $N(40,5)$	500	0.655422	0.2	0.321828	0.60	0.022750	1.0	0.3469312
$36 \le \quad \le 40$ $\theta = 5$	500	0.884930 (L.L.)	0.2	0.321828 (U.L.)	0.60	0.022750 (U.L.)	1.0	0.3928328
$\mu = 40$ $5 \le \theta \le 10$	500	0.655422 (L.L.)	0.2	0.321828 (L.L.)	0.60	0.158655 (U.L.)	1.0	0.4828362

L.L. - Values correspond to lower limit.
U.L. - Values correspond to upper limit.

C_1 = Cost of no damage Rs. 100/-

C_2 = Cost of one member fail Rs. 300/-

C_3 = Cost of collapse Rs. 500/-

TABLE 9.1 - MAXIMIZATION OF COST-EFFECTIVENESS FACTOR K_T FOR - STRATEGIES AND θ-STRATEGIES ILLUSTRATED FOR A THREE BAR SYSTEM.

Type of Decision	Conditions under which decision is made		
	Certainty	Risk	Uncertainty
Stage 1. Design for serviceability	Optimal control (Ch. 5)	-	Differential game (Ch. 8)
Stage 2. Design for cost-effectiveness	-	Cost-effectiveness design (Ch.6)	Minimax game
Stage 3. Decision of uncertainty factor	Not needed	Not needed	Decision theory (Ch. 7)
Stage 4. Final choice of structural member	Force-deformation relation certain	Markov idealization or cost-effectiveness design	Bayesian Markov process

TABLE 10.1 - SECTION OF DECISION MODELS FOR VARIOUS STAGES.

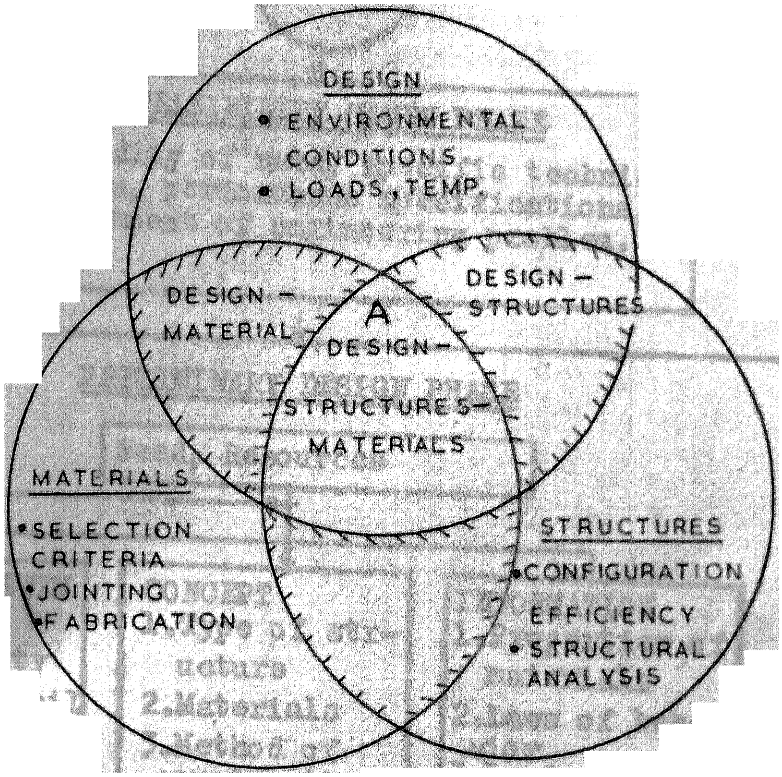


FIGURE 1.1 NATURE OF THE INTERPLAY IN STRUCTURAL DESIGN

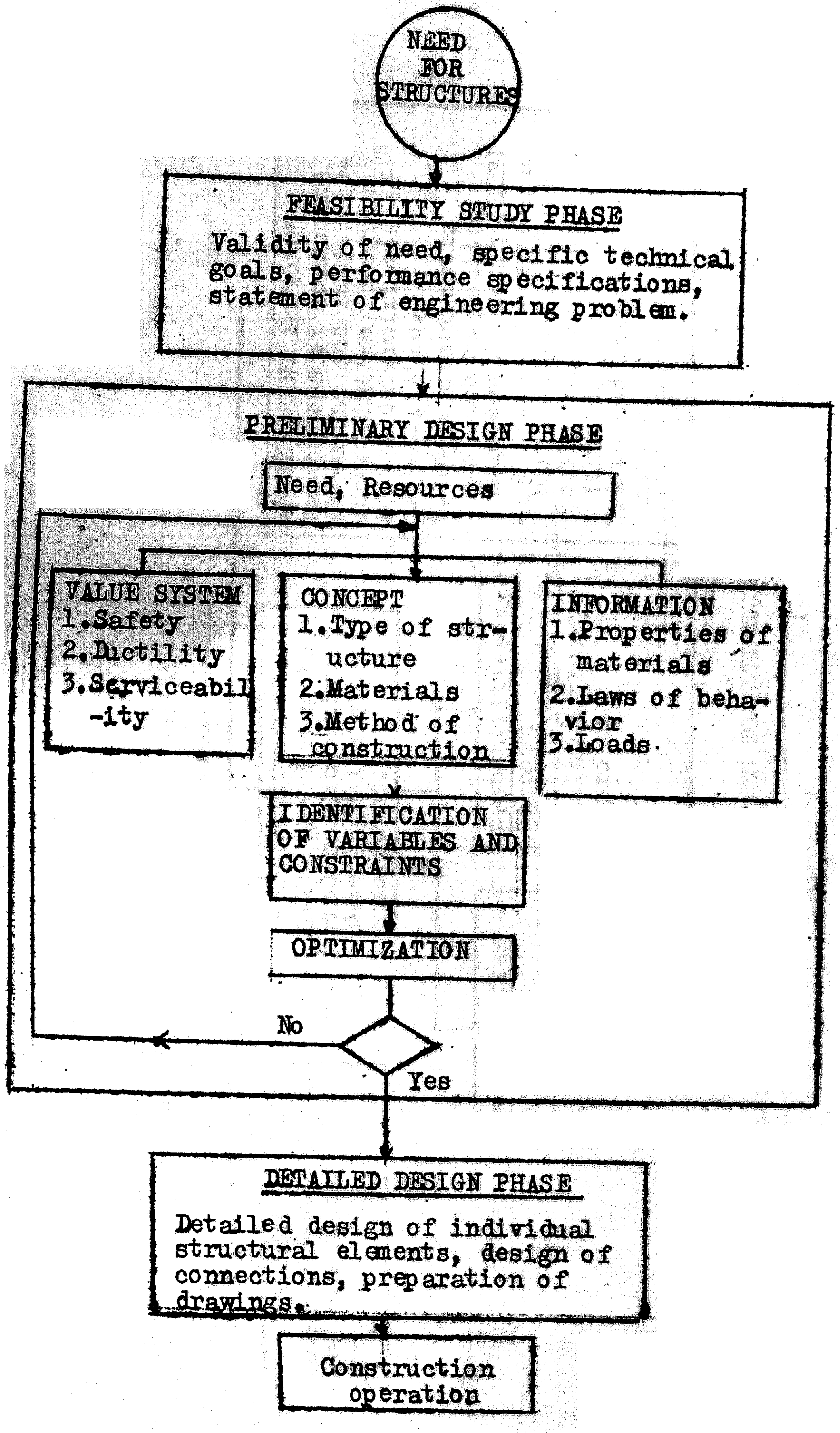


FIG. 1.2. - THE THREE PHASES OF DESIGN (17)

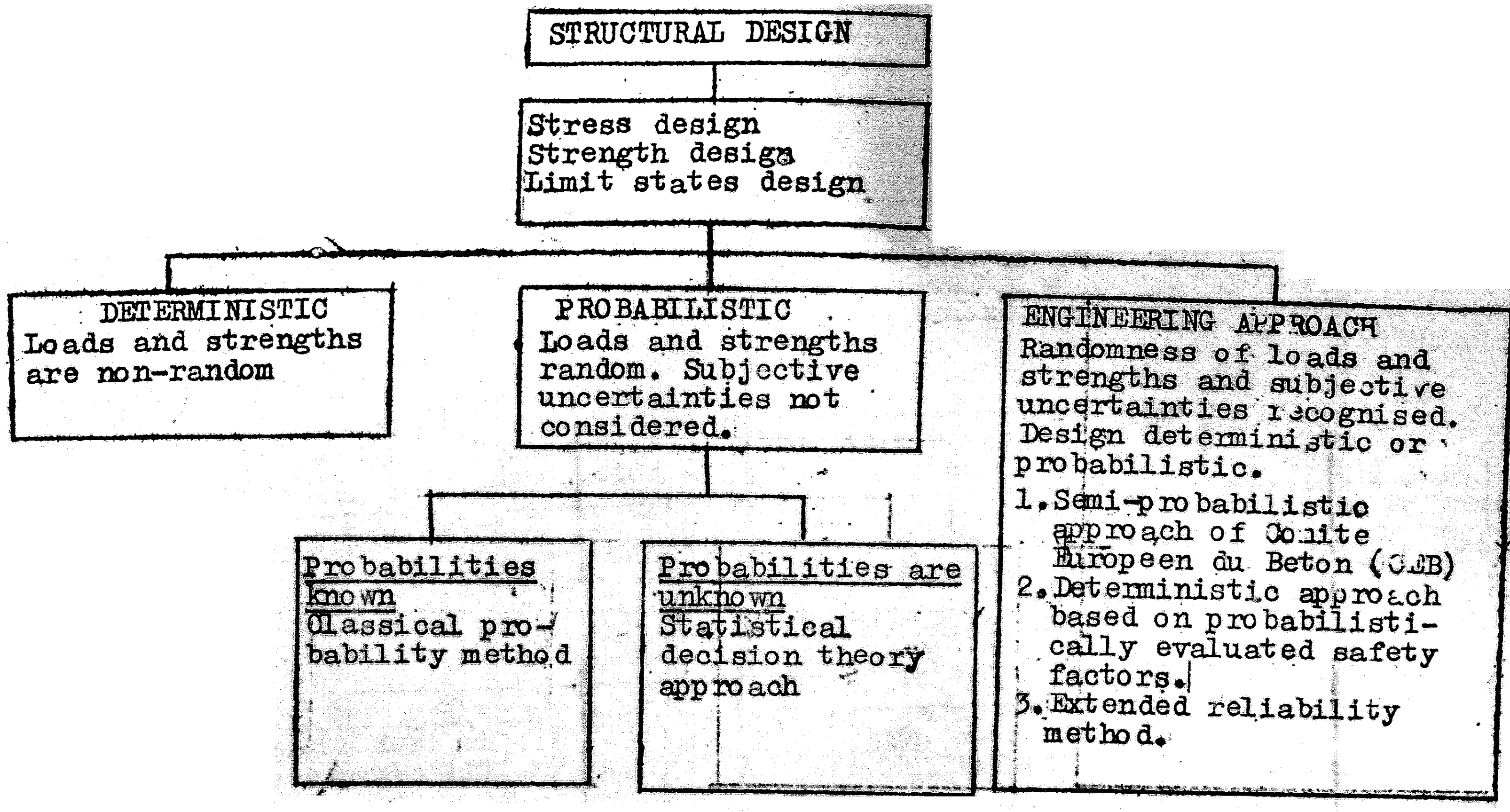


FIG. 1.3. - APPROACHES TO STRUCTURAL DESIGN.

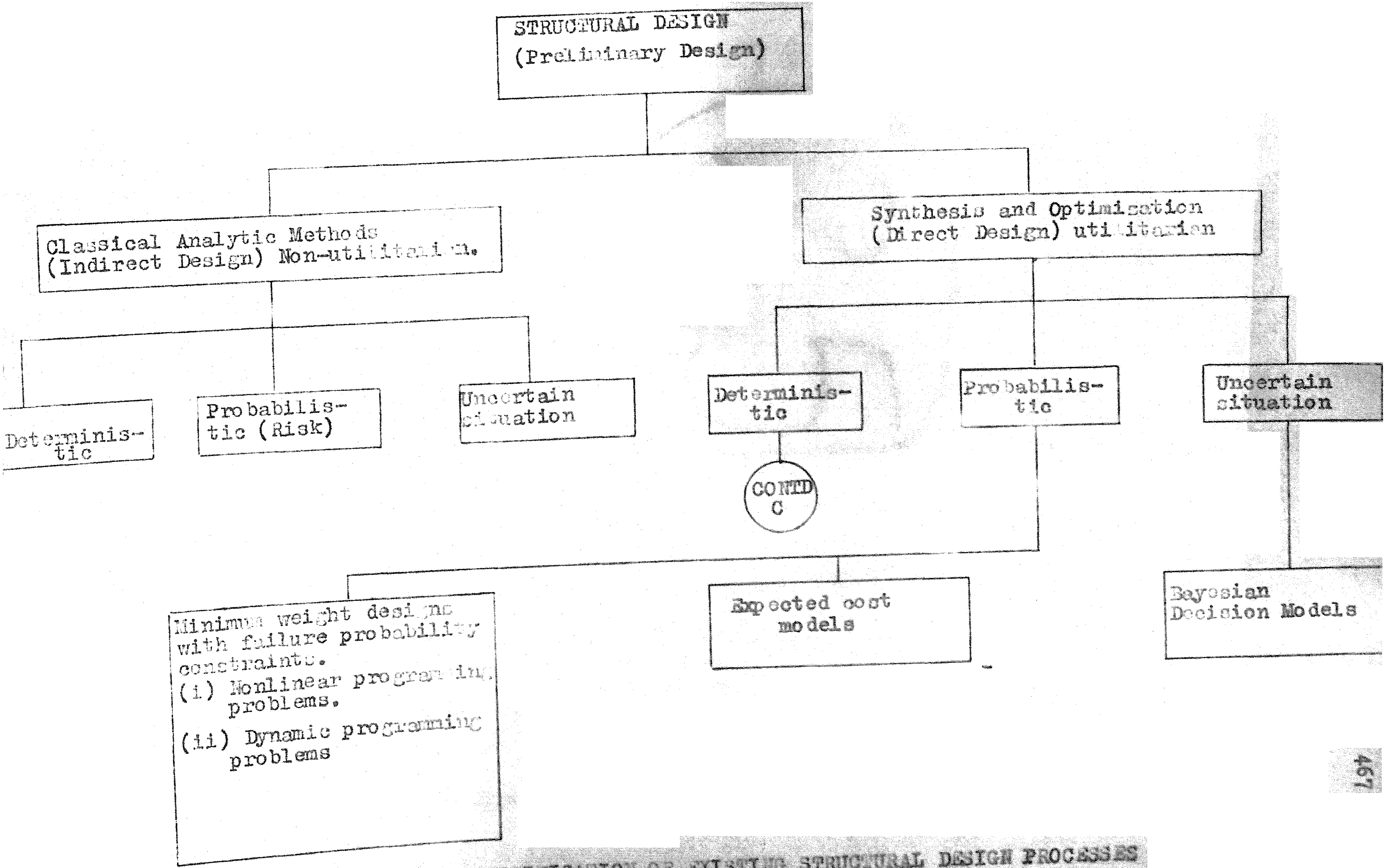


FIGURE 1.4 CLASSIFICATION OF EXISTING STRUCTURAL DESIGN PROCESSES (BASED ON FORMULATION).

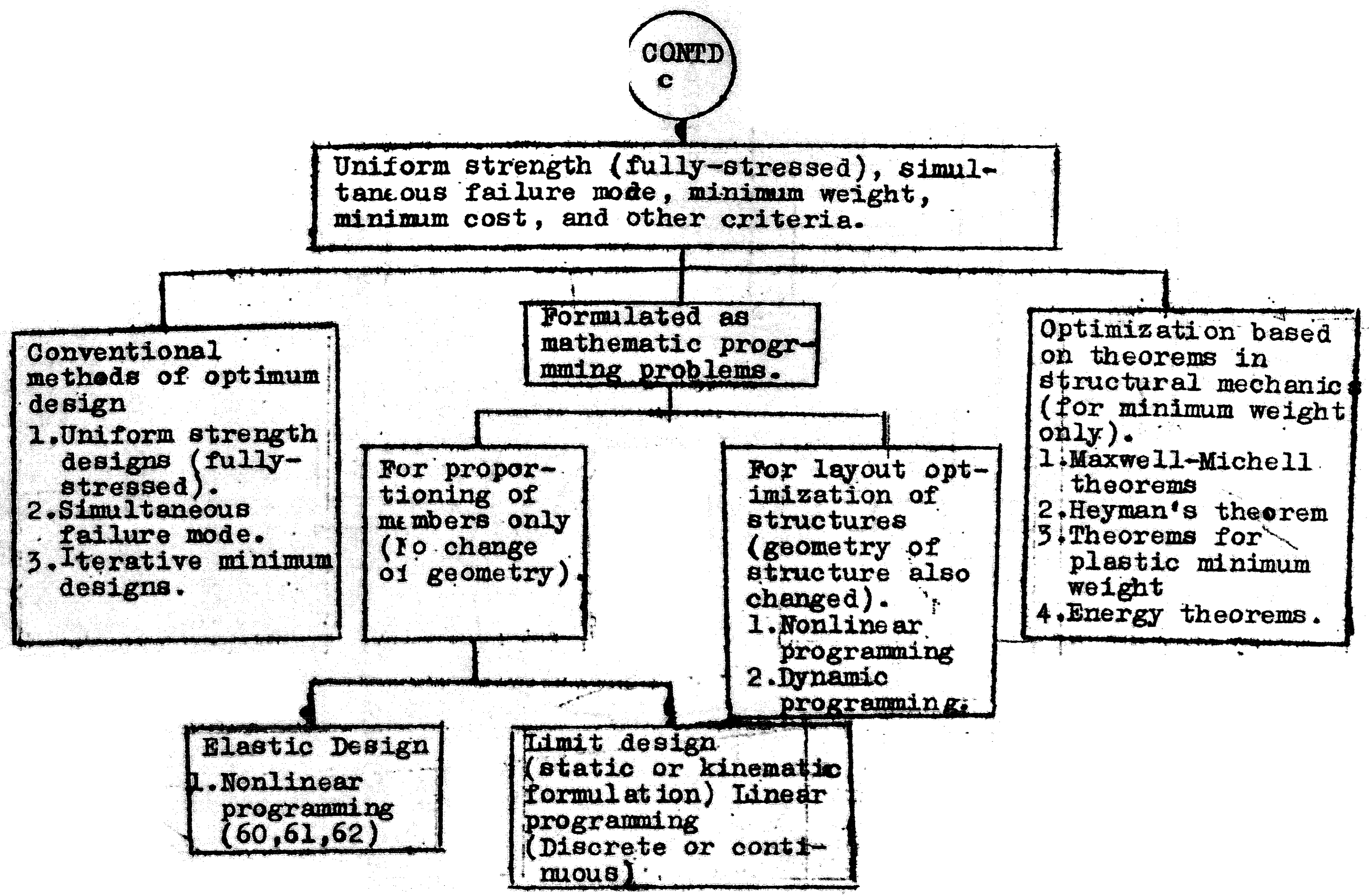


FIG. 1.4 (CONTINUED)

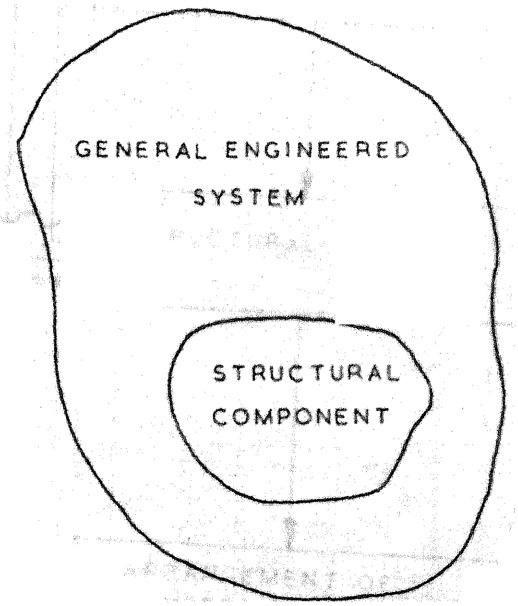


FIGURE 2.1 STRUCTURE SHOWN AS A COMPONENT OF GENERAL SYST

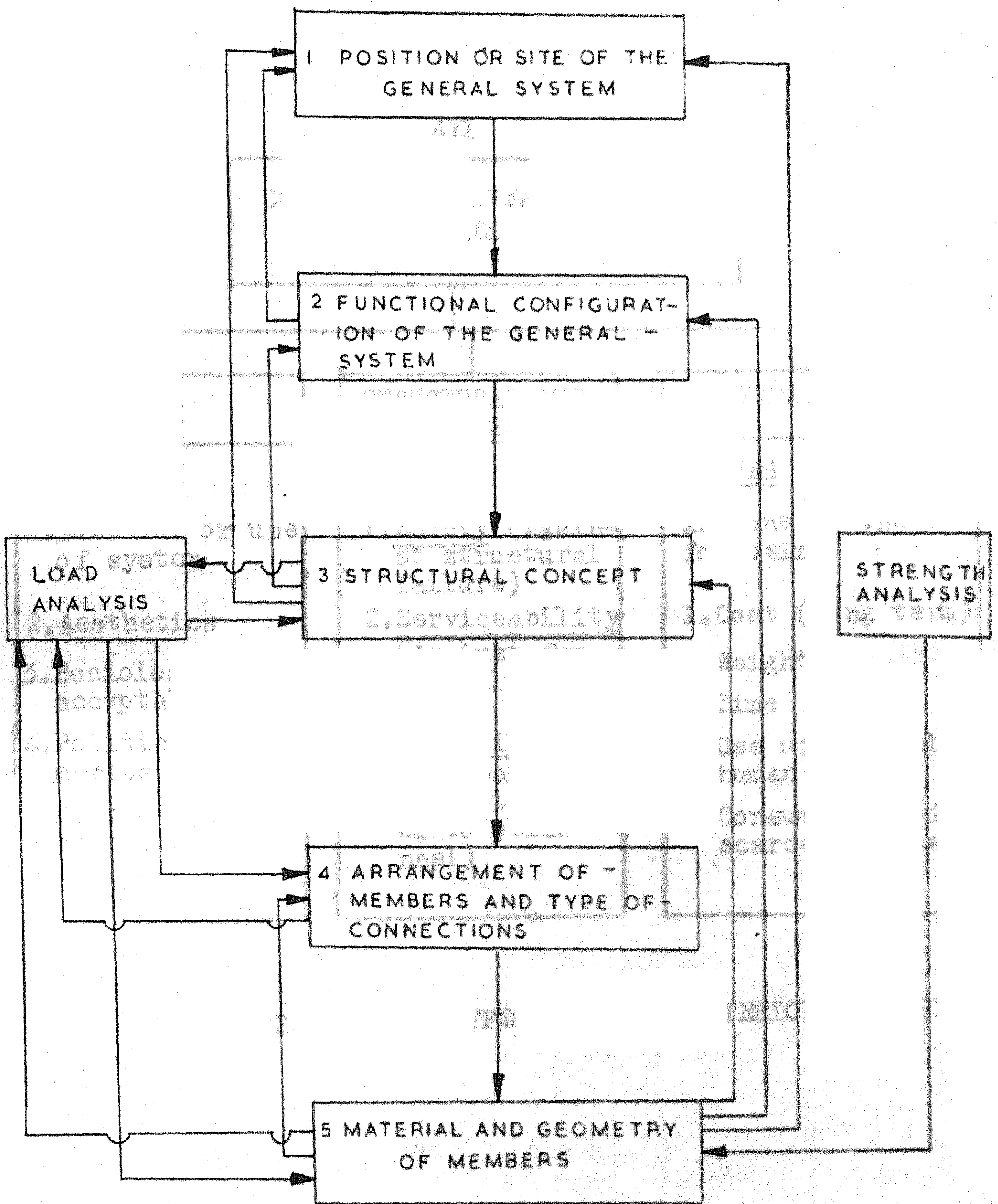


FIGURE 2.2 SCHEME OF DECISIONS RELATED TO STRUCTURAL COMPONENT AT THE PRELIMINARY STAGE OF DESIGNS (Arrows indicate the flow of information)

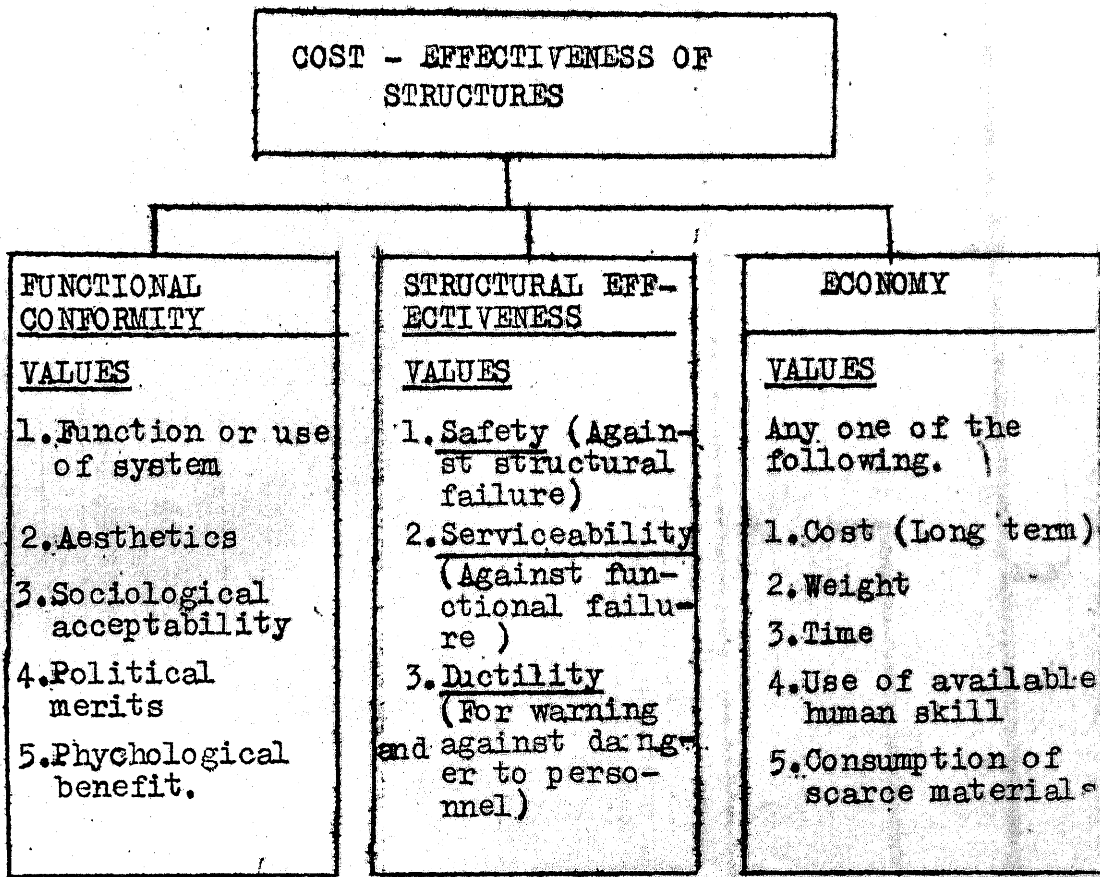


FIG. 2.3 - COST EFFECTIVENESS CRITERION OF DESIGN.

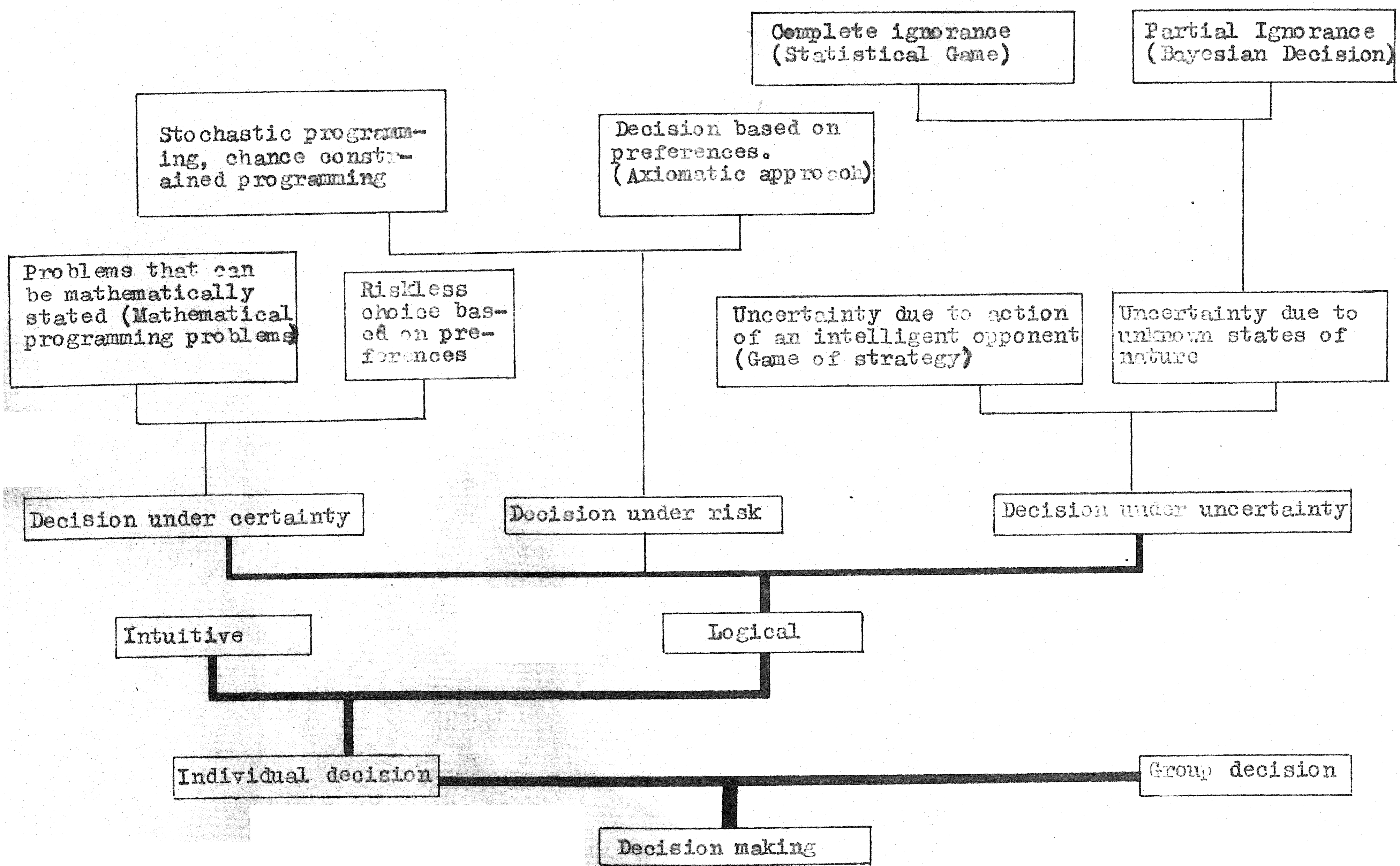


FIGURE 2.4 TREE OF DECISION MAKING

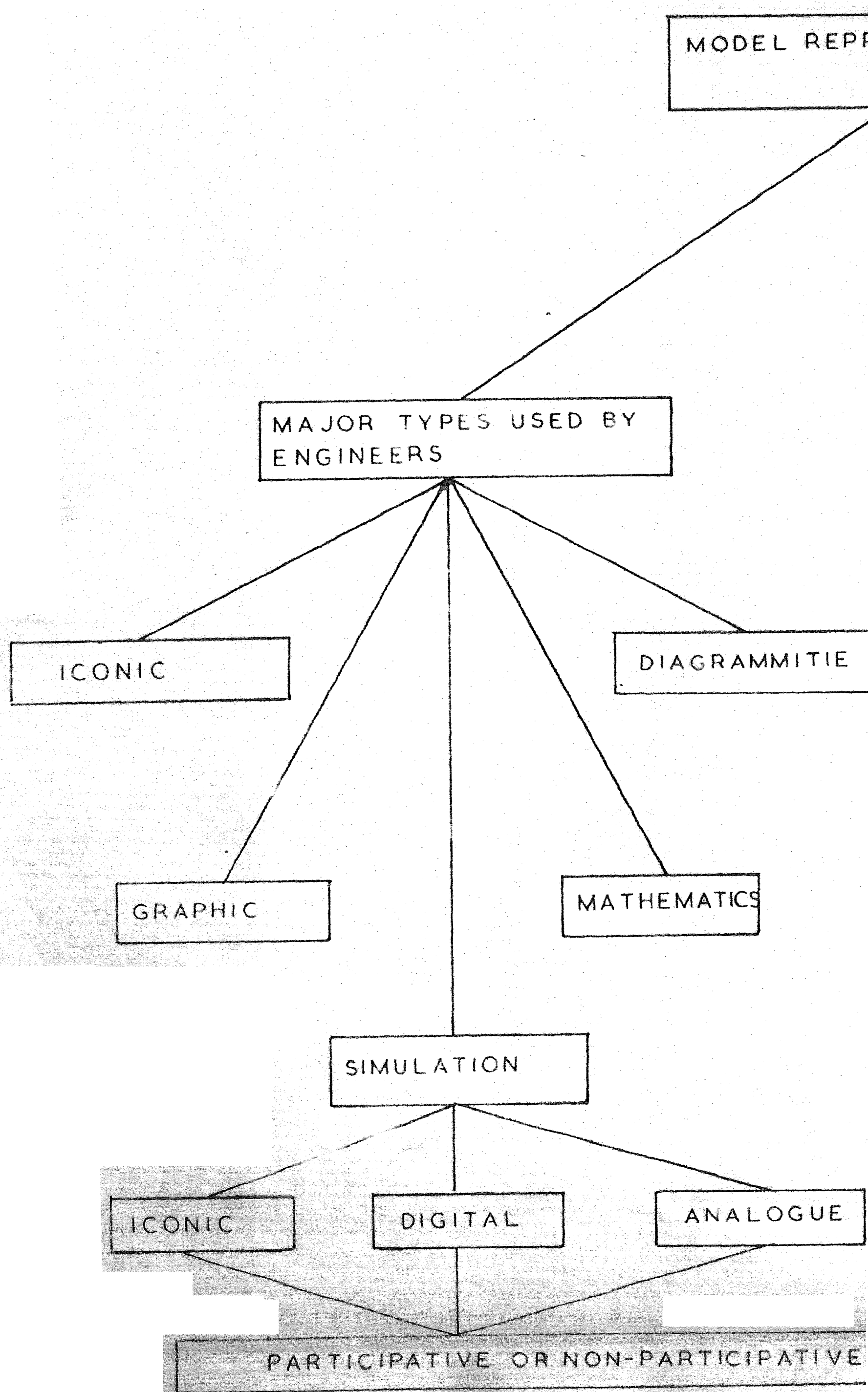


FIGURE 2.5 A DIAGRAMMATIC REPRESENTATION OF MODELLING (Ref.12).

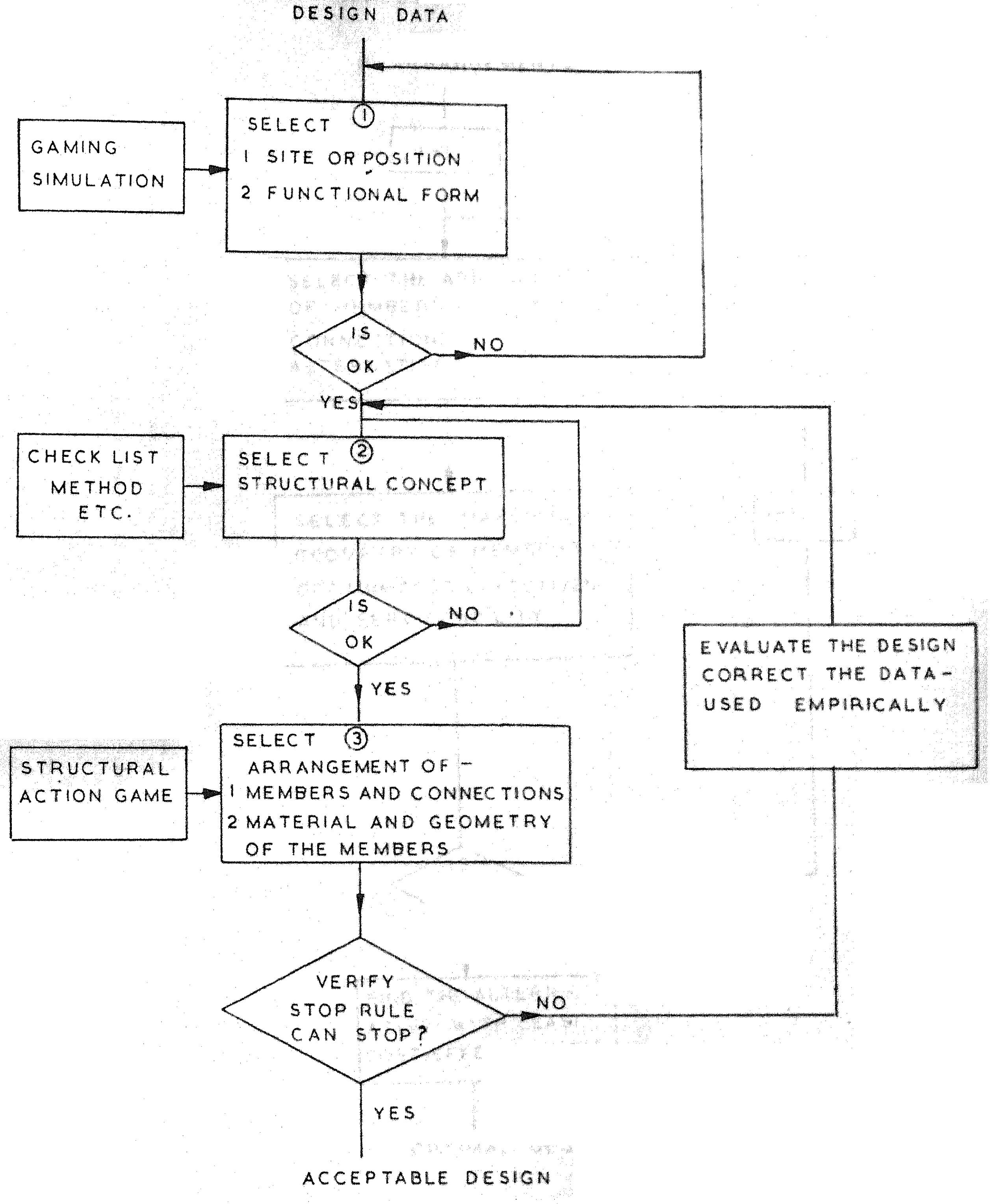


FIGURE 3.1 A FLOW CHART OF THE PROPOSED DESIGN METHODOLOGY

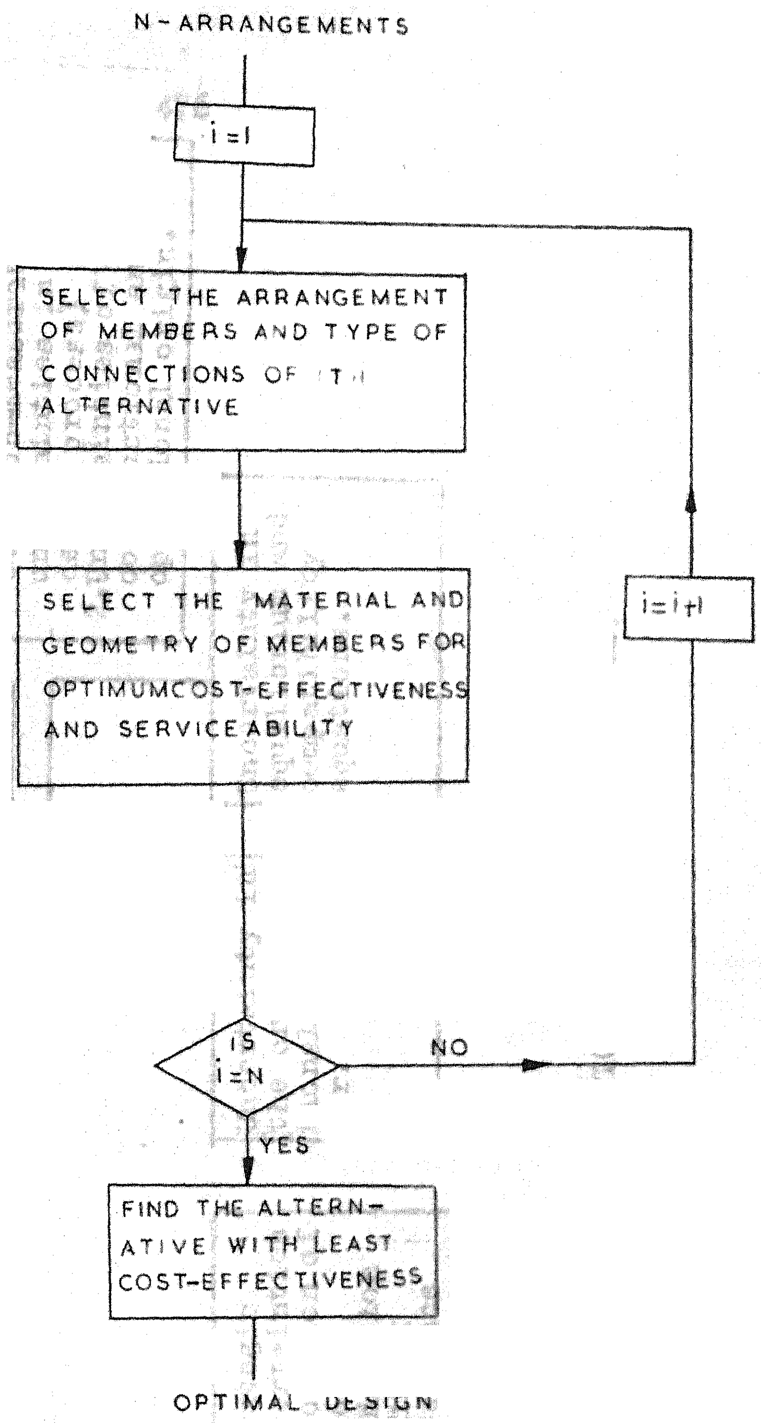


FIGURE 3.2 FLOW CHART OF THE SELECTION OF THE ARRANGEMENT OF MEMBERS, TYPES OF CONNECTIONS AND MATERIAL AND GEOMETRY OF MEMBERS.

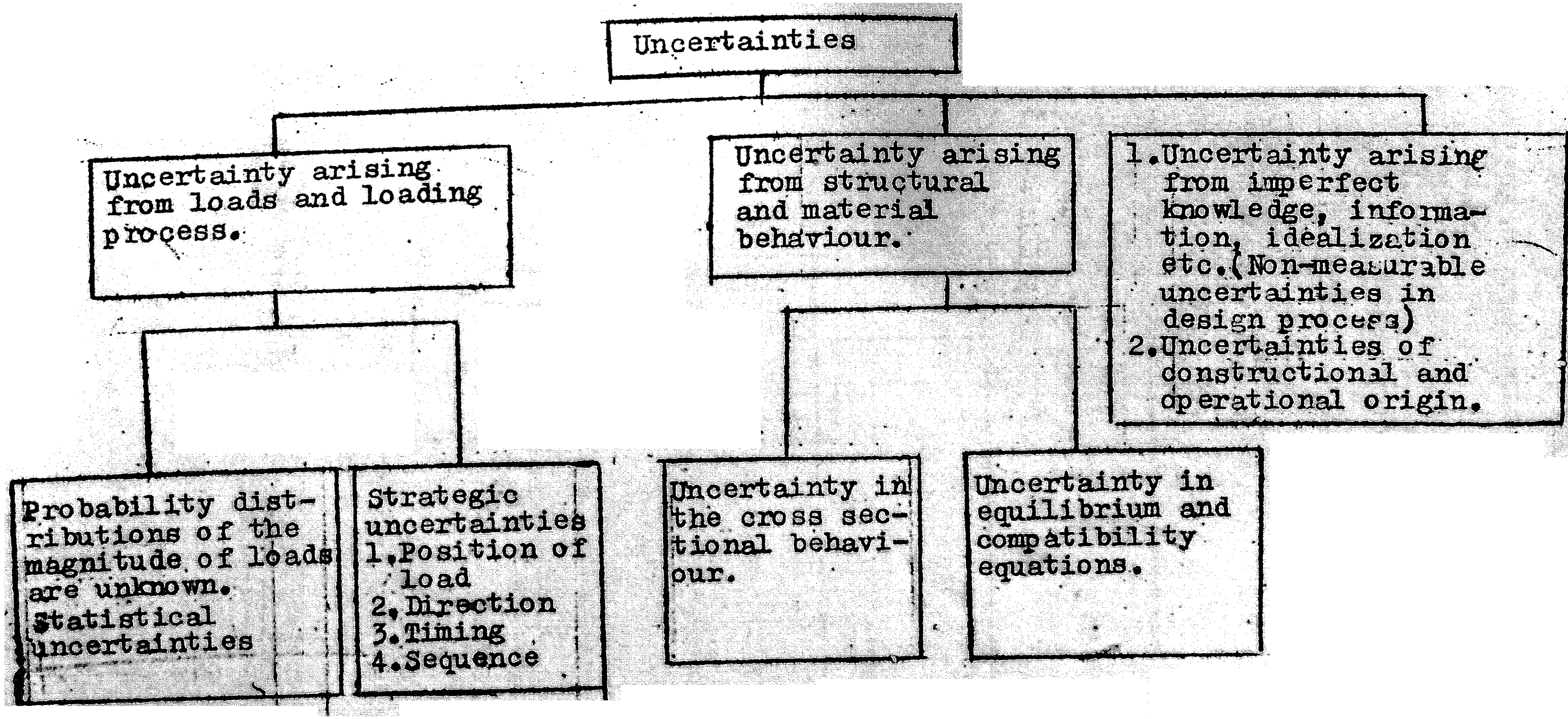
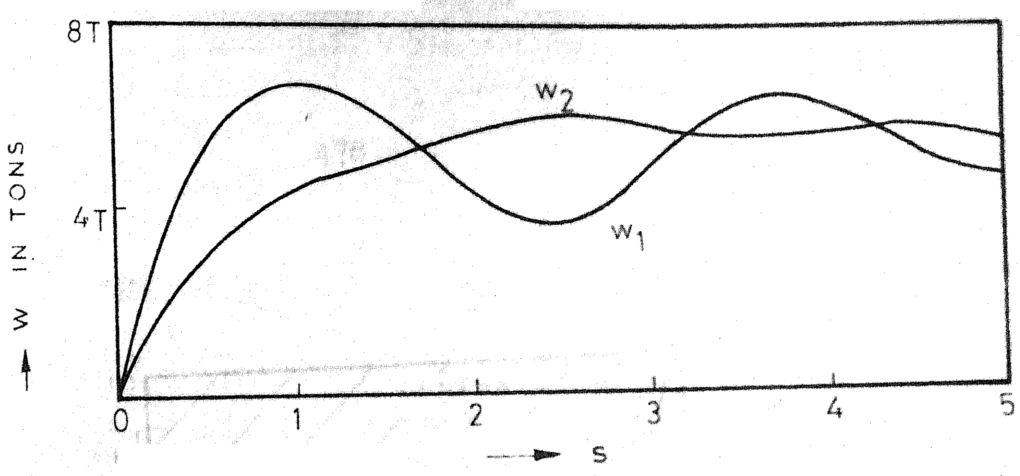
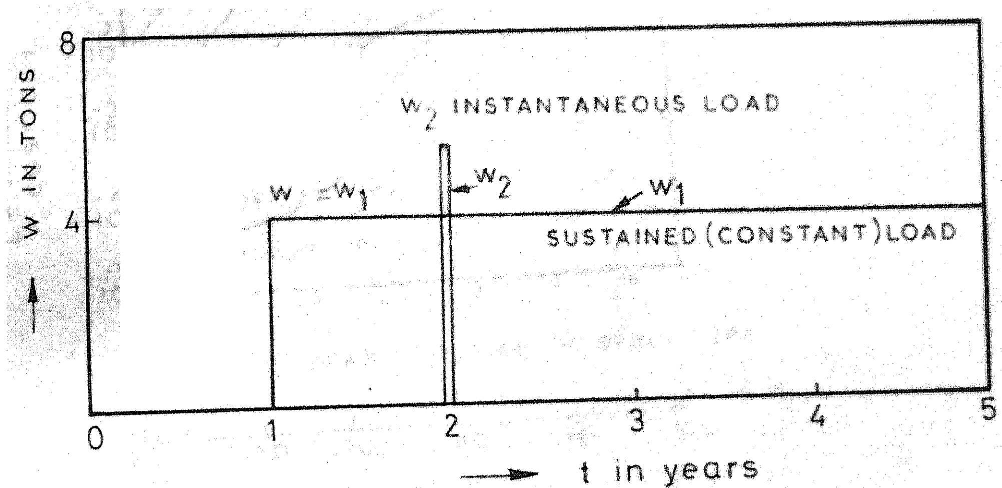


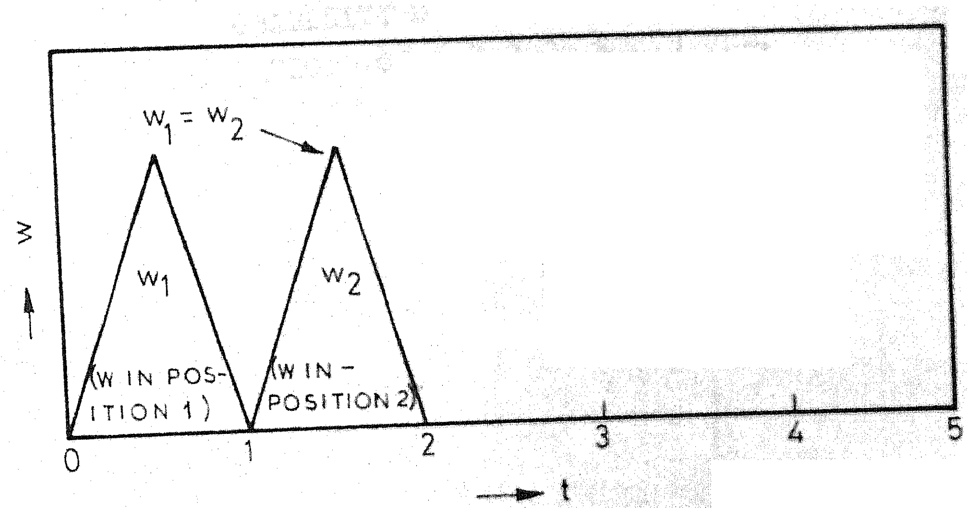
FIG. 4.1 - CLASSIFICATION OF UNCERTAINTIES.



(a) Sequence of loading expressed functionally



(b) Instantaneous and sustained loads expressed functionally



(c) Change in position considered as two loads applied one after another

FIGURE 4.2 LOADING FUNCTIONS.

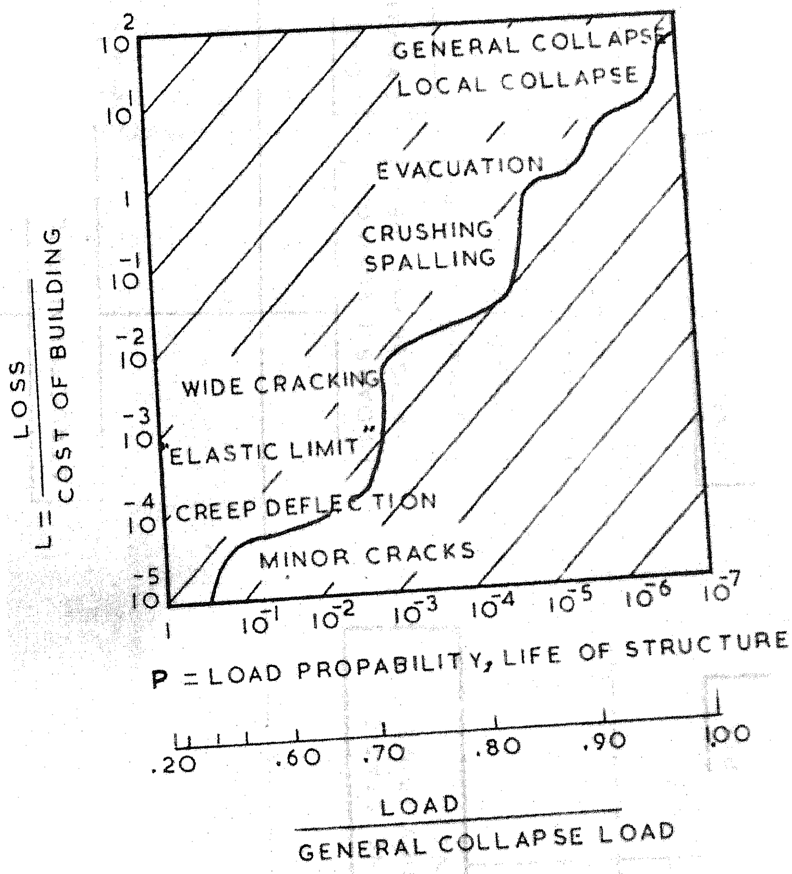


FIGURE 4.3 PROBABILITY OF FAILURE vs LOAD vs COST RELATION FOR A R.C.C. STRUCTURE.

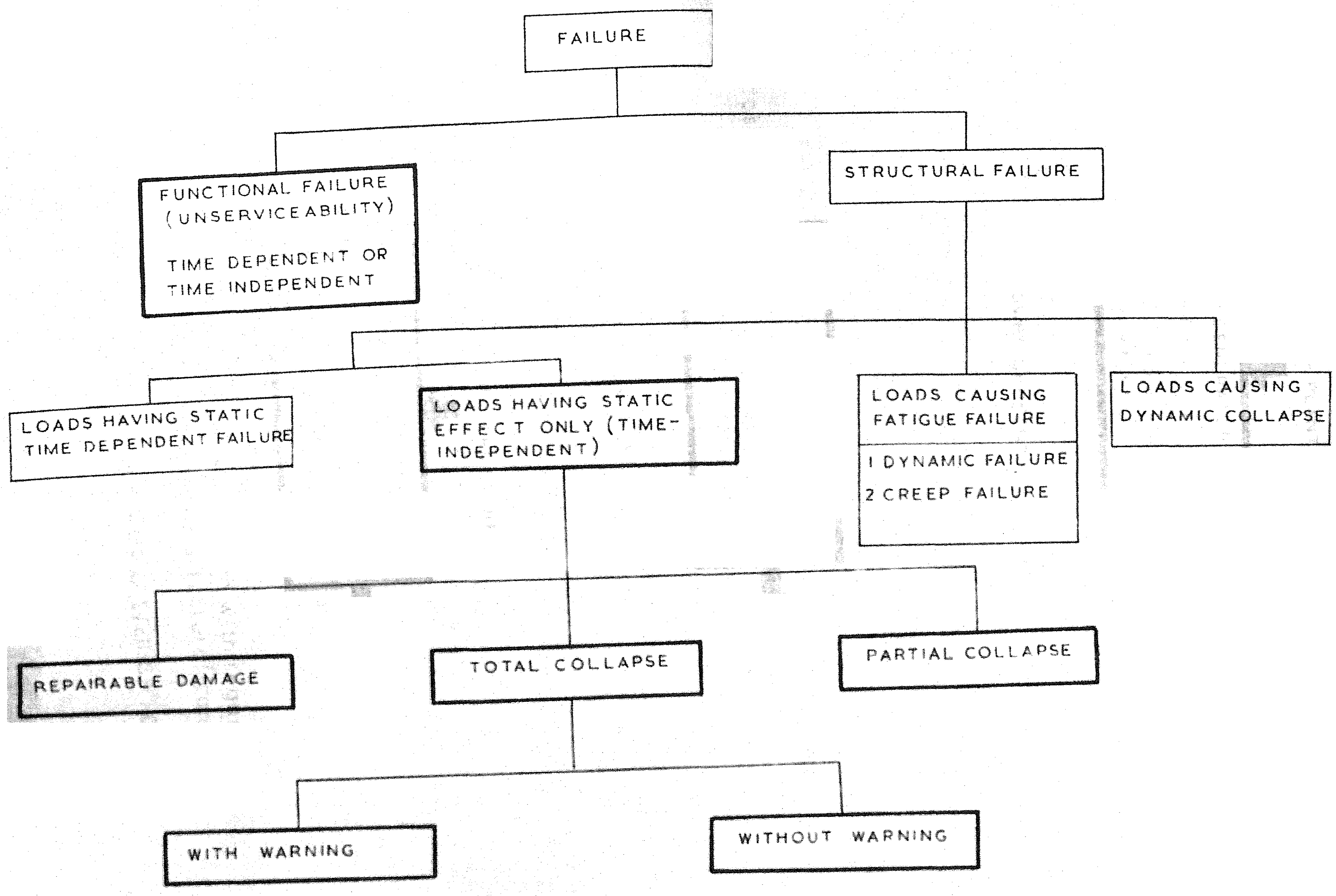
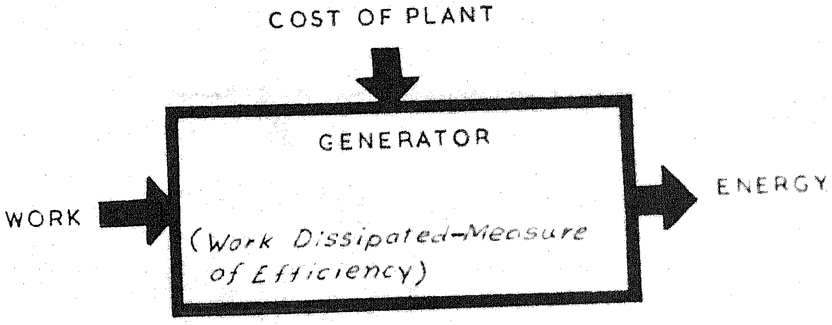
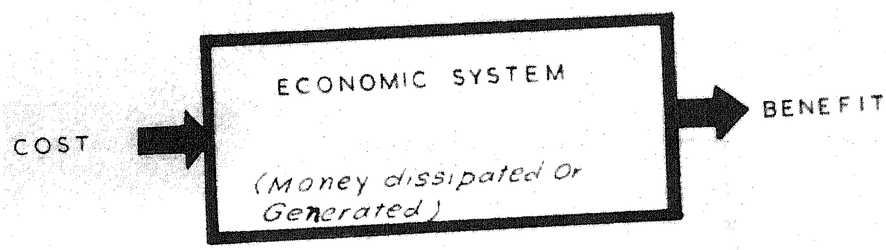


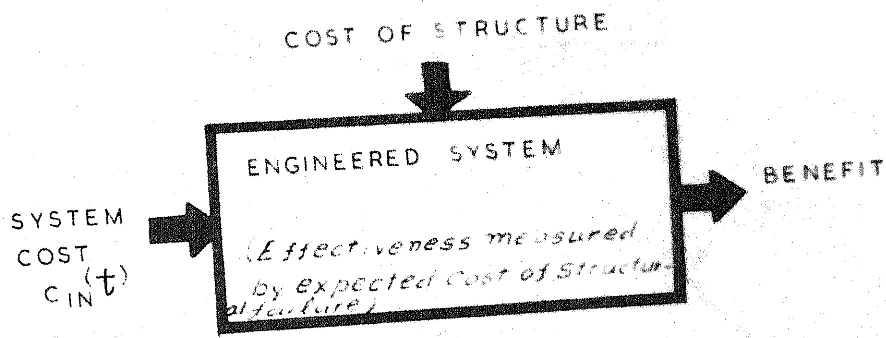
FIGURE 4.4 CLASSIFICATION OF FAILURE MODES.



(a) Power generating system



(b) Economic system



(c) Study of general system with regard to the efficiency of structural component

FIGURE 4.5 COST-EFFECTIVENESS MODEL.

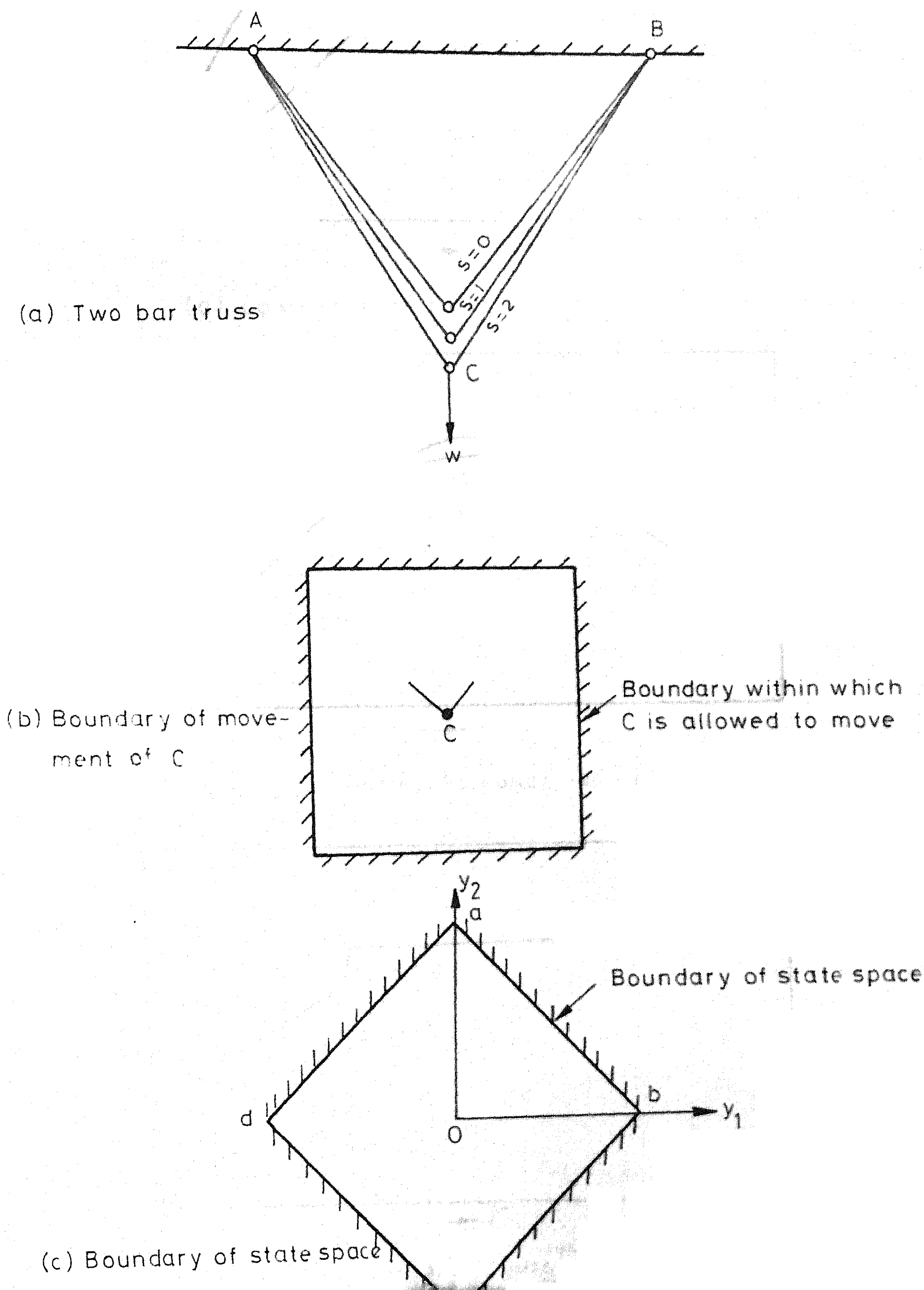
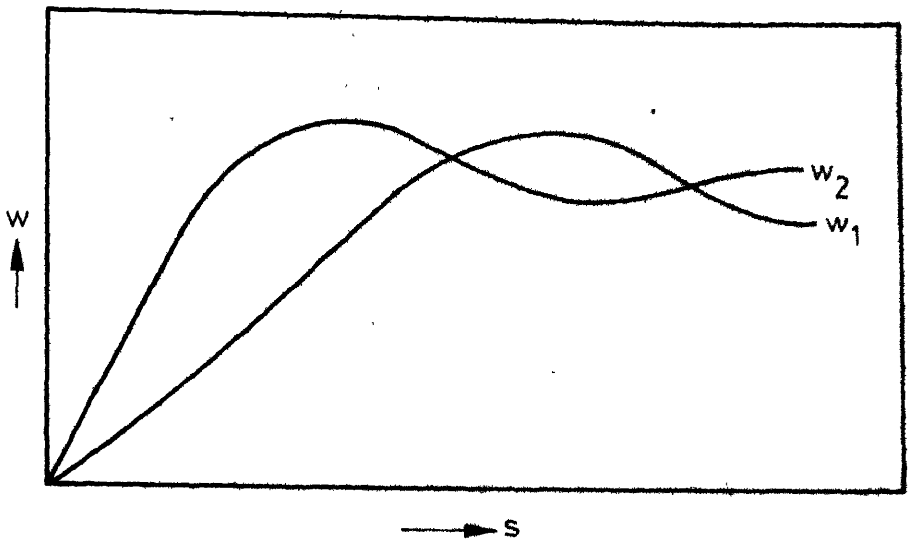
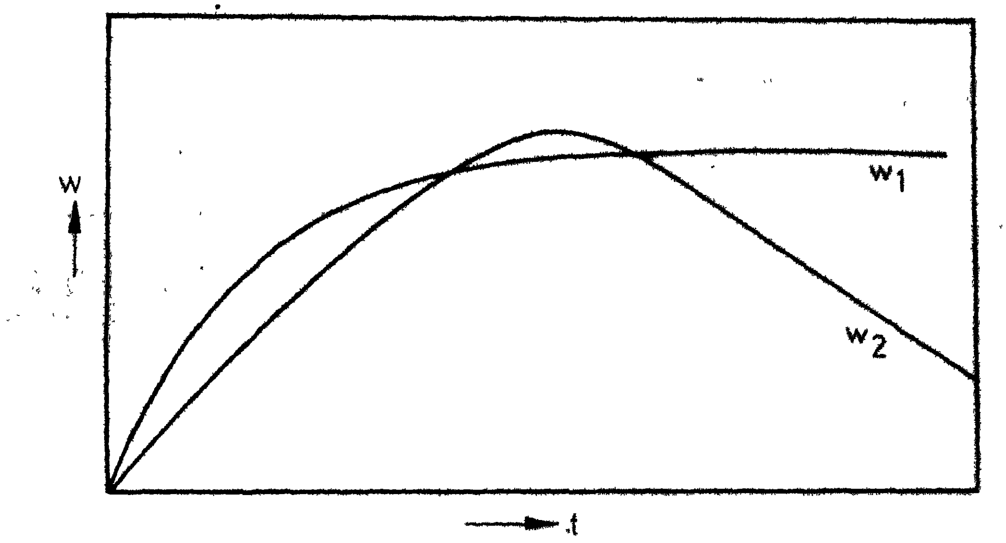


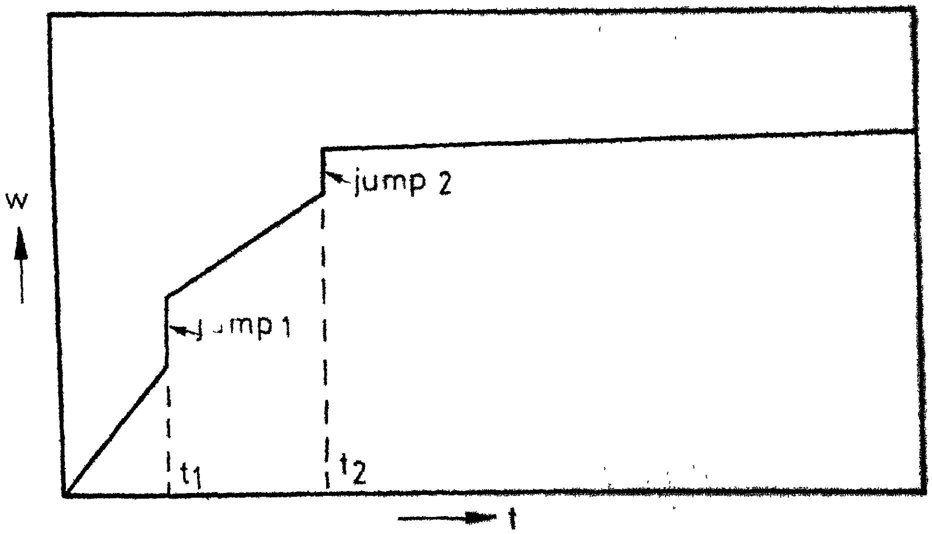
FIGURE 5.1 EXAMPLE OF TWO BAR TRUSS TO ILLUSTRATE THE PRINCIPLES OF OPTIMAL CONTROL FORMULATION OF STRUCTURAL DESIGN



(a) LOADS AS FUNCTIONS OF S



(b) LOADS AS CONTINUOUS FUNCTIONS OF TIME t



(c) LOADS WITH INTERMITTENT JUMPS WITH RESPECT TO TIME t

FIGURE 5.2 LOAD AS FUNCTION OF STAGE VARIABLE s OR t.

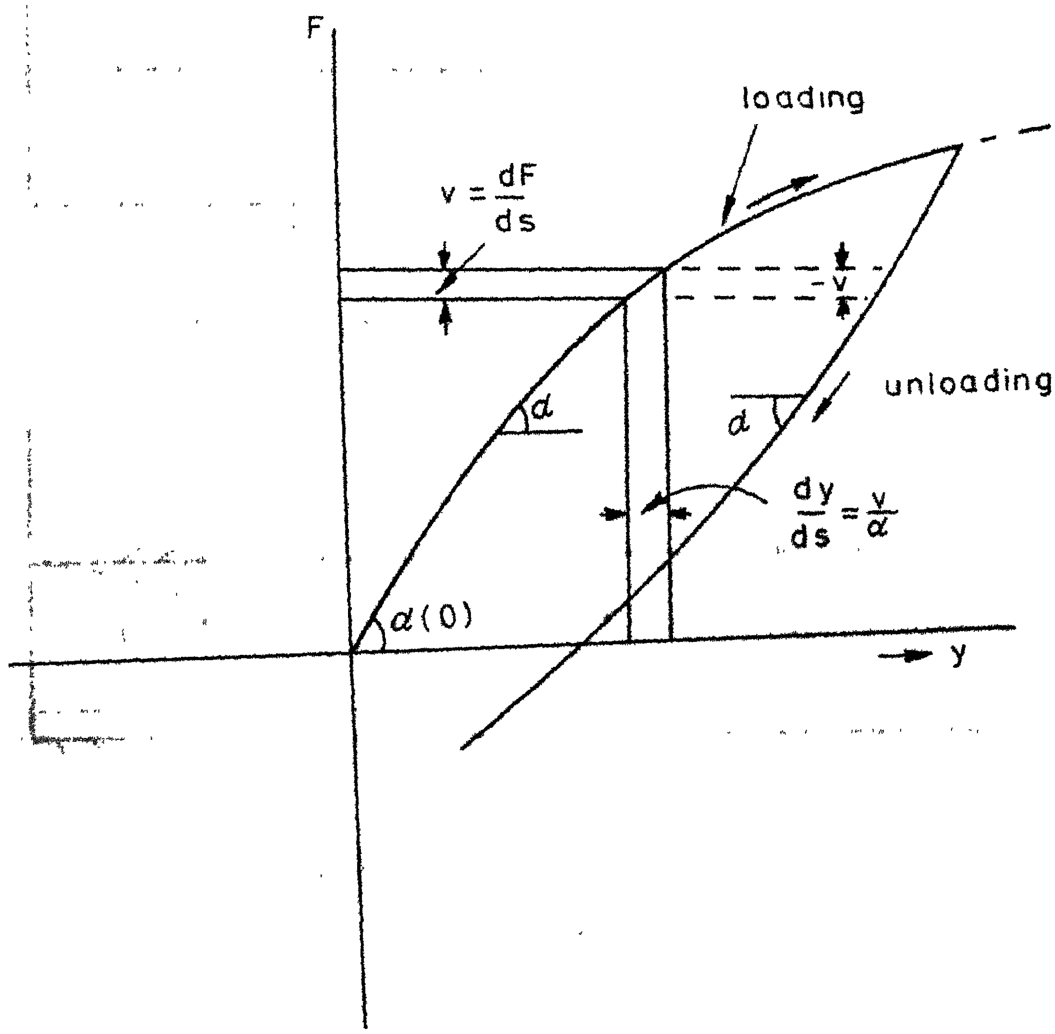
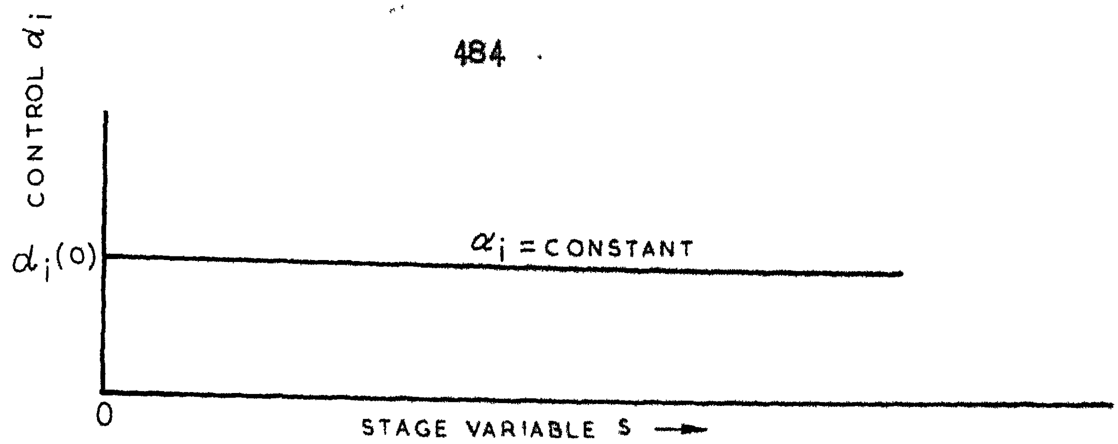
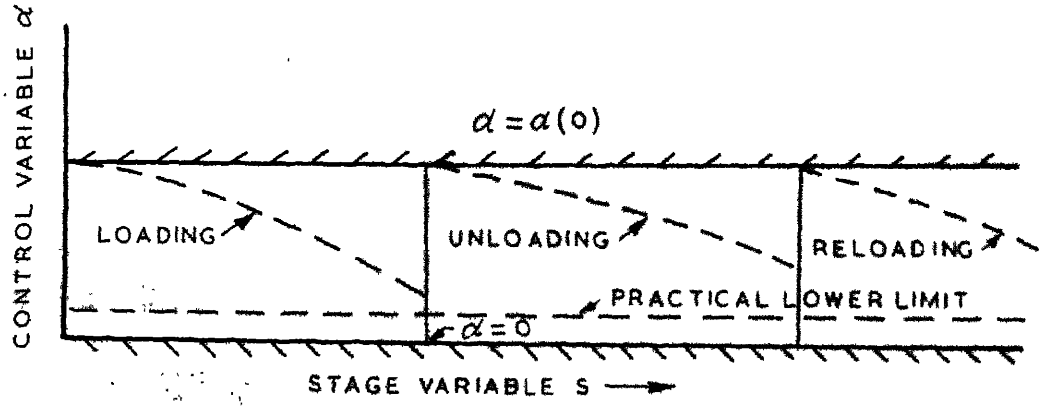


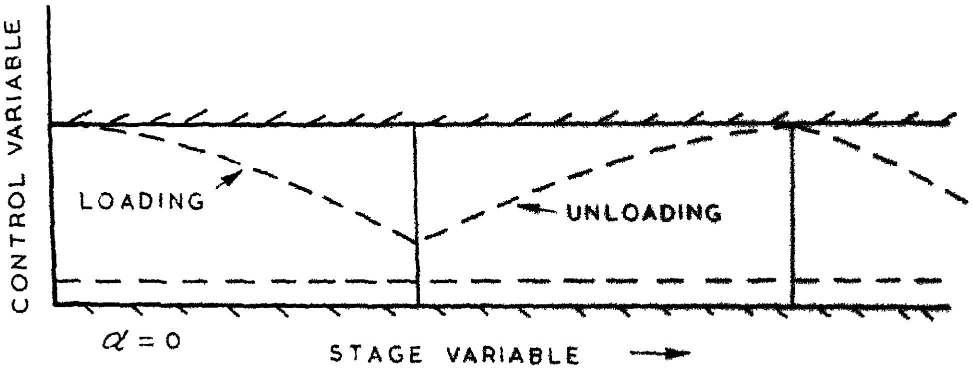
FIGURE 5.3 PATTERN OF FORCE-DEFORMATION RELATIONS WITH THE CONTROL AND STATE VARIABLES ILLUSTRATED



(a) Linearly elastic material



(b) Inelastic material



(c) Non linear elastic material

FIGURE 5.4 ADMISSIBILITY OF CONTROL α ILLUSTRATED.

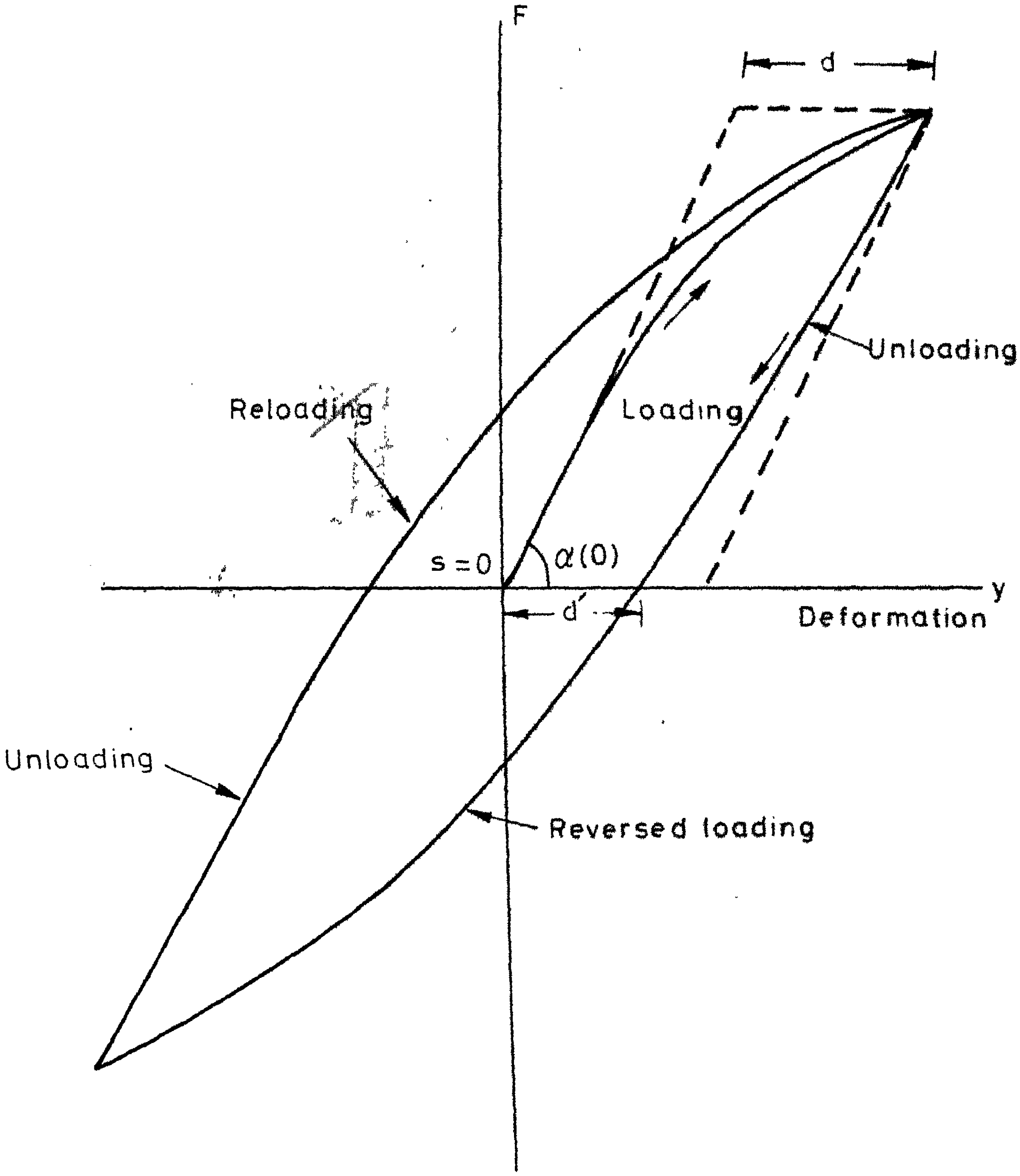


FIGURE 5.5 CONSTRAINTS ON PERMANENT DEFORMATION ILLUSTRATED.

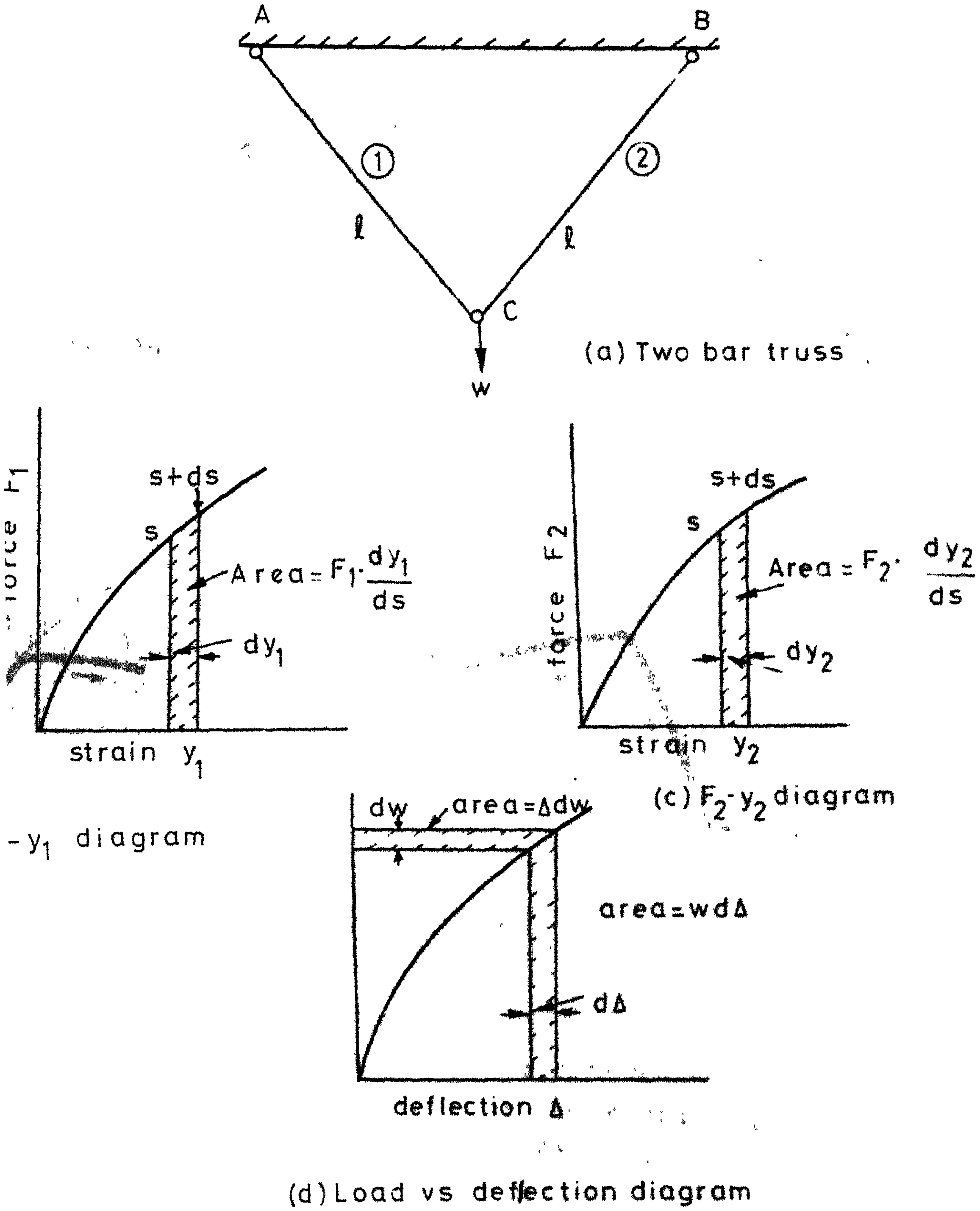


FIGURE 5.6 EVALUETION OF PERFORMANCE INDEX (EXAMPLE OF TWO BAR TRUSS).

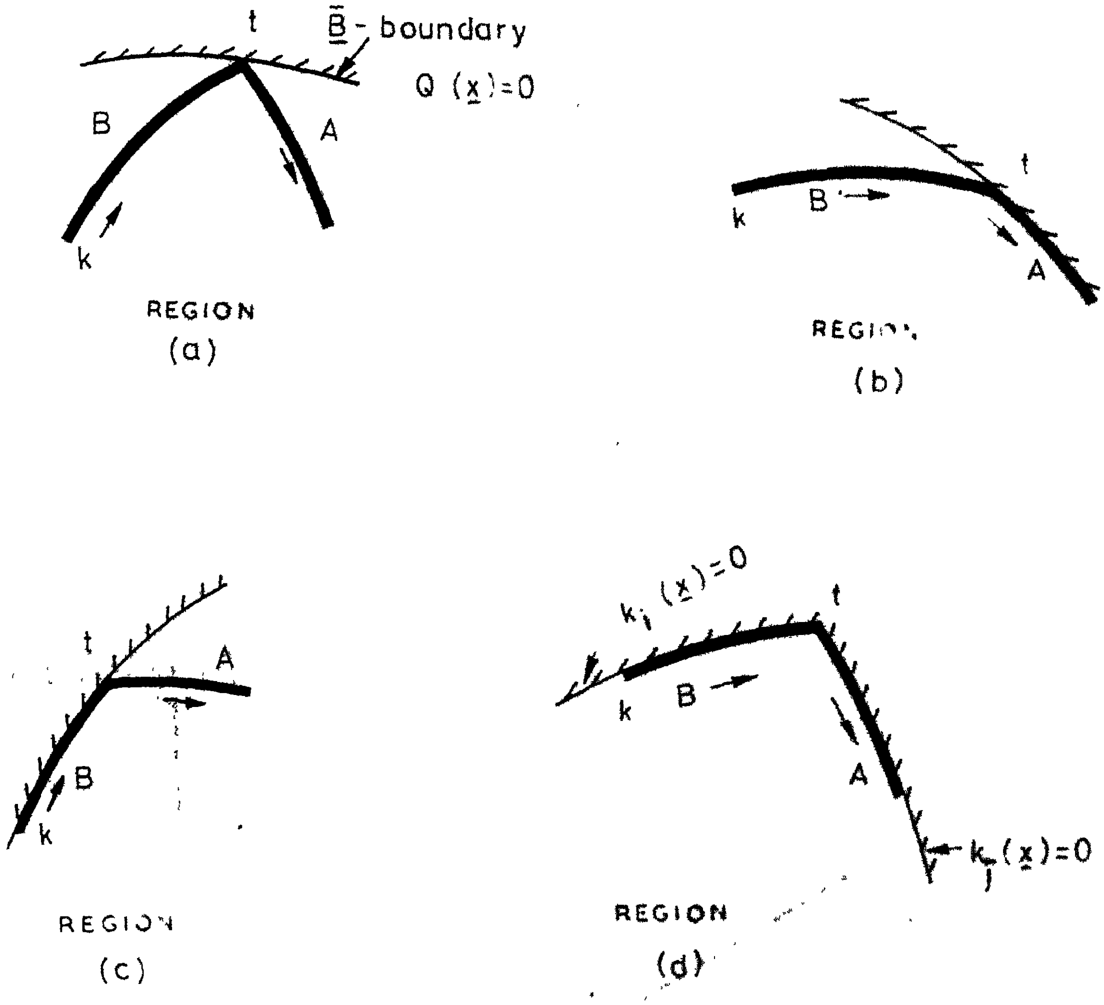
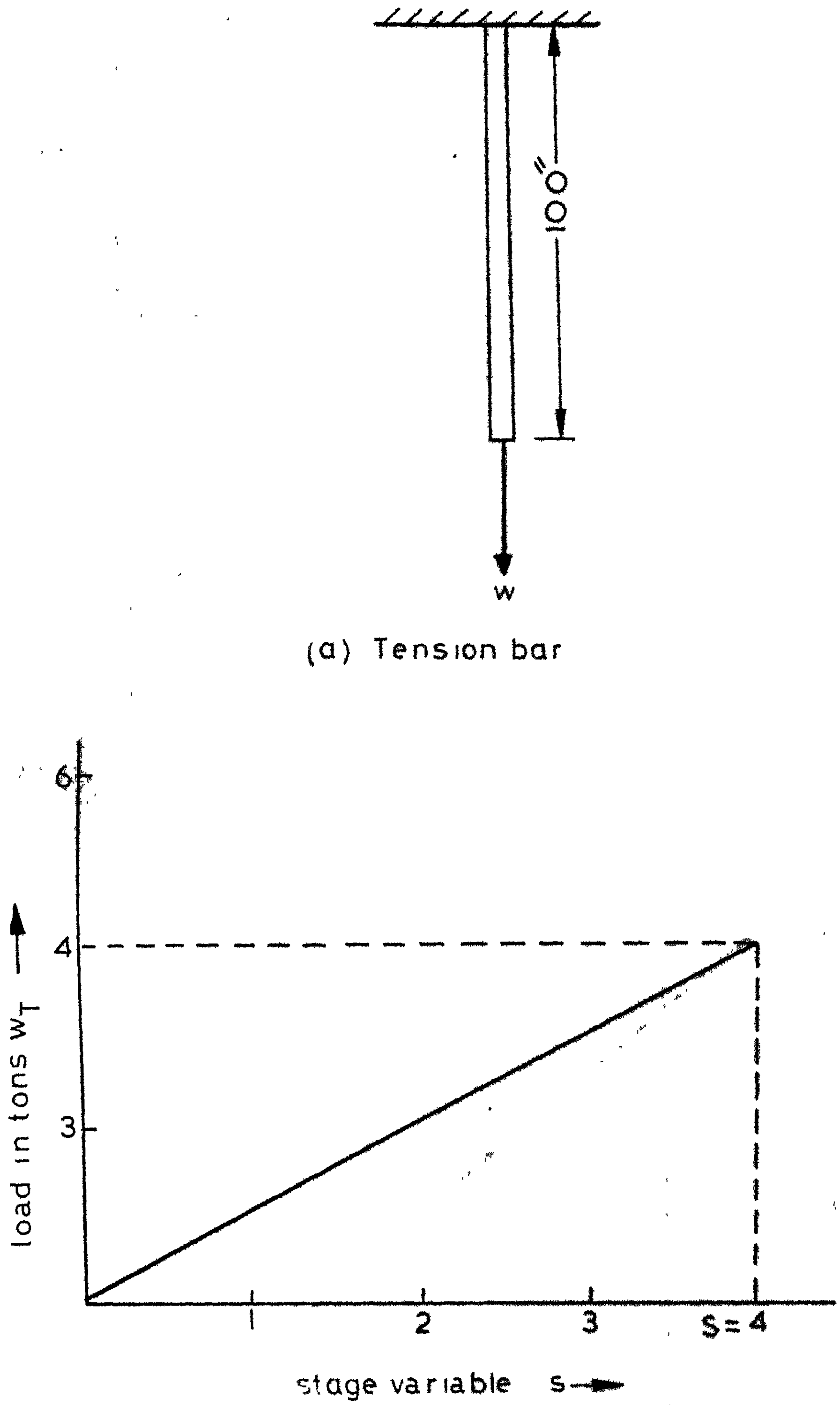
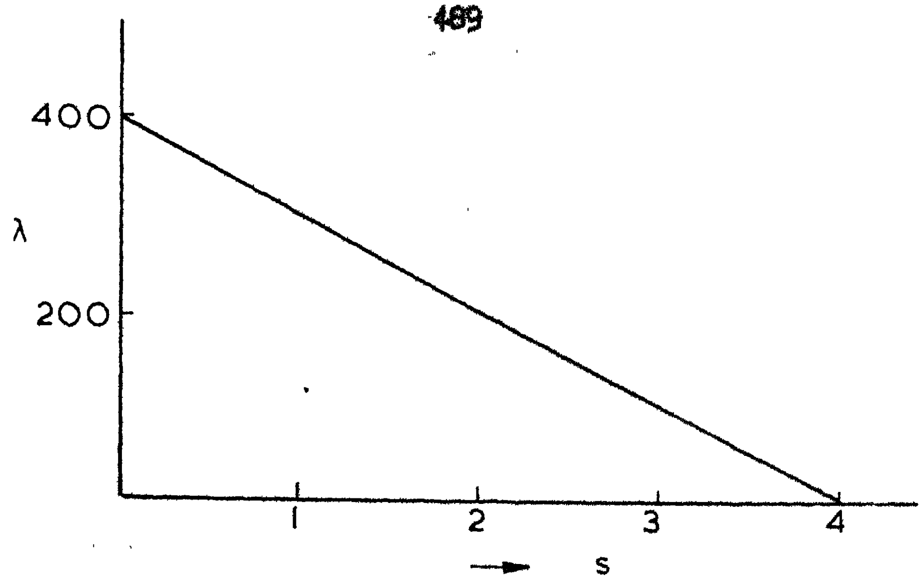


FIGURE 5.7 CORNER CONDITIONS
(Trajectory k (a) intersects boundary and returns, (b) continues to be on the boundary, (c) moves out of boundary, (d) moves from one boundary to another.)

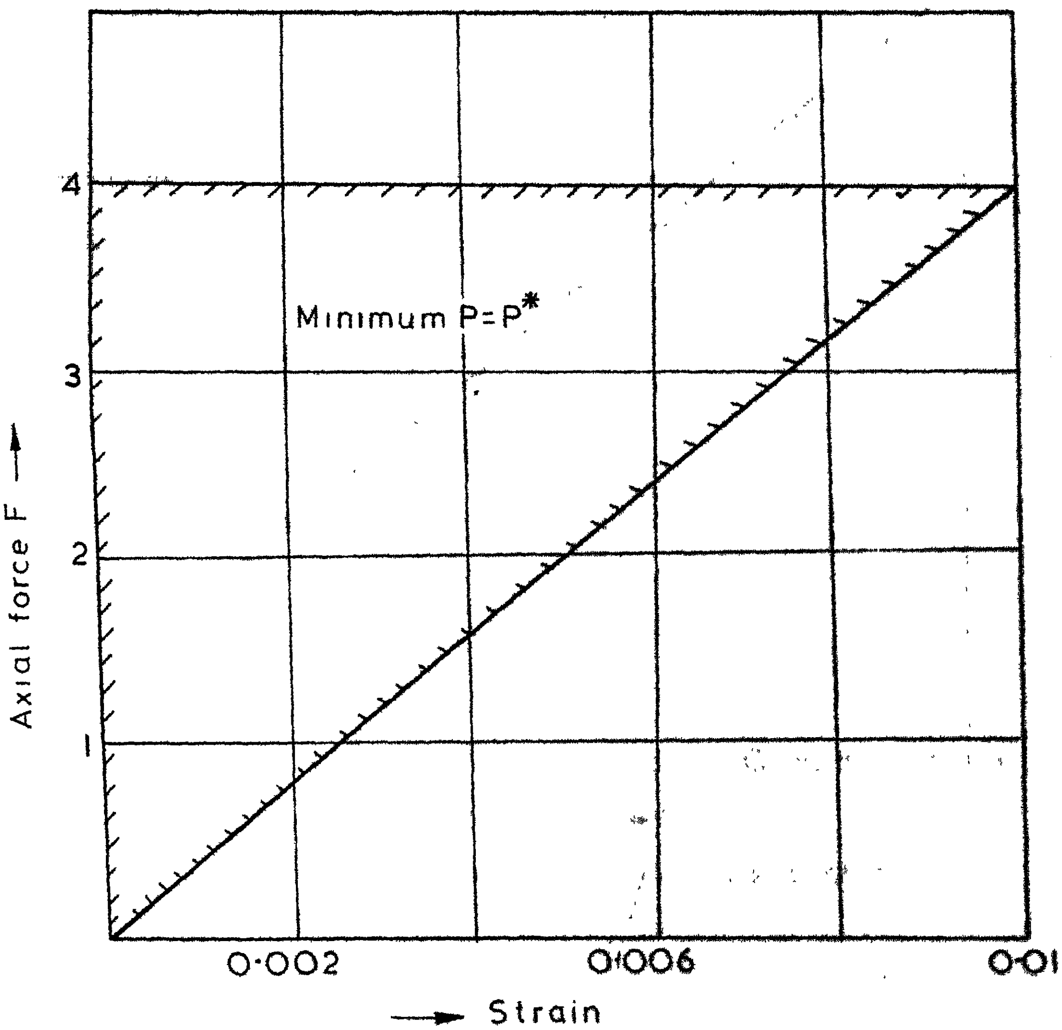


(b) Loading function

FIGURE 5.8 EXAMPLE OF TENSION BAR.

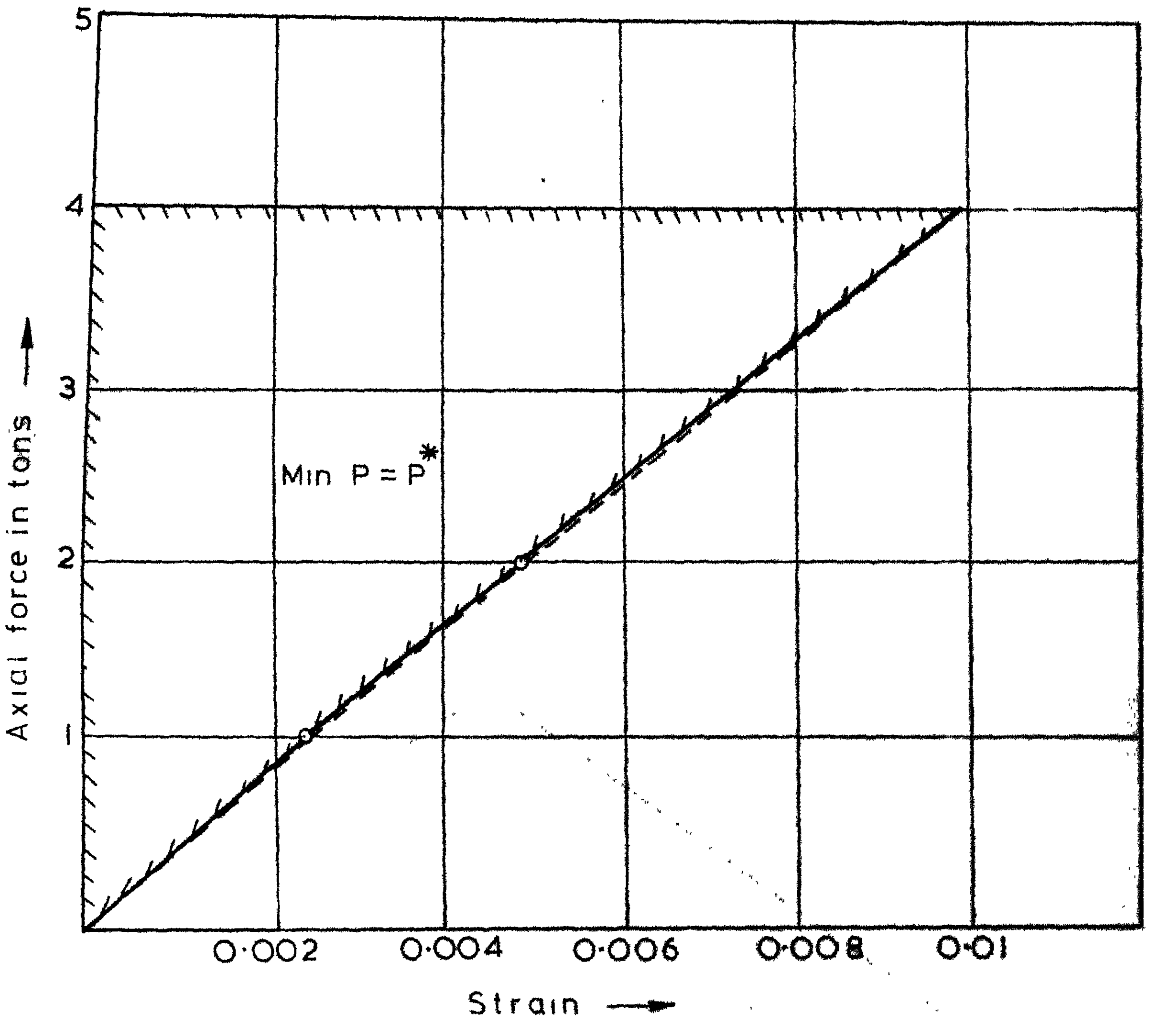


(c) Costate variable λ



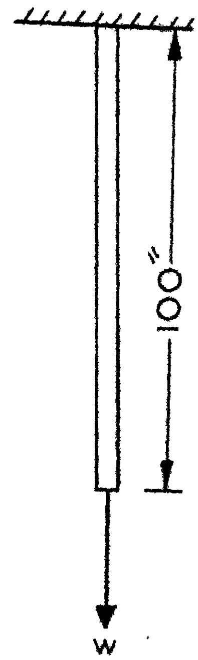
(d) Force deformation relation (task curve) obtained

FIGURE 5.8 (CONTD.)

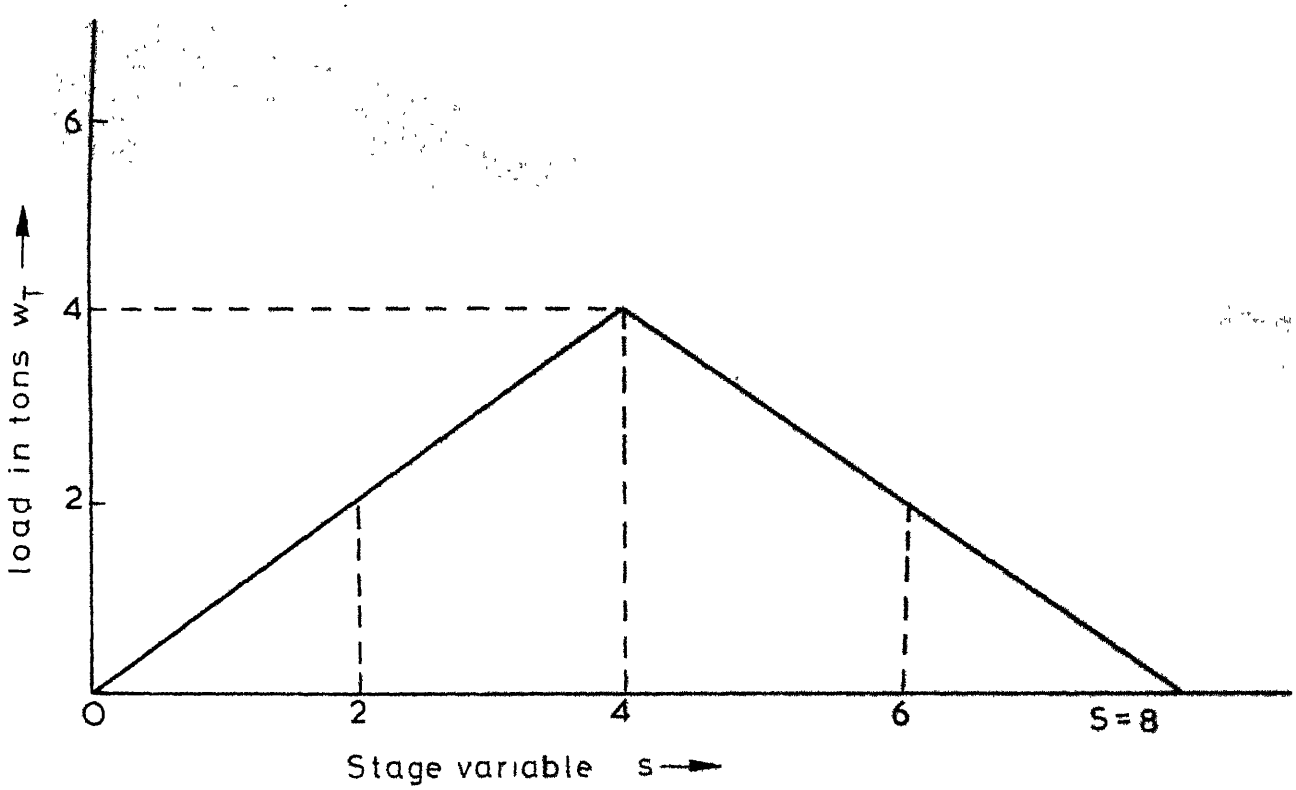


(e) Optimal nonlinear axial force-axial strain relation of tension bar

FIGURE 5.8 (CONTD.)

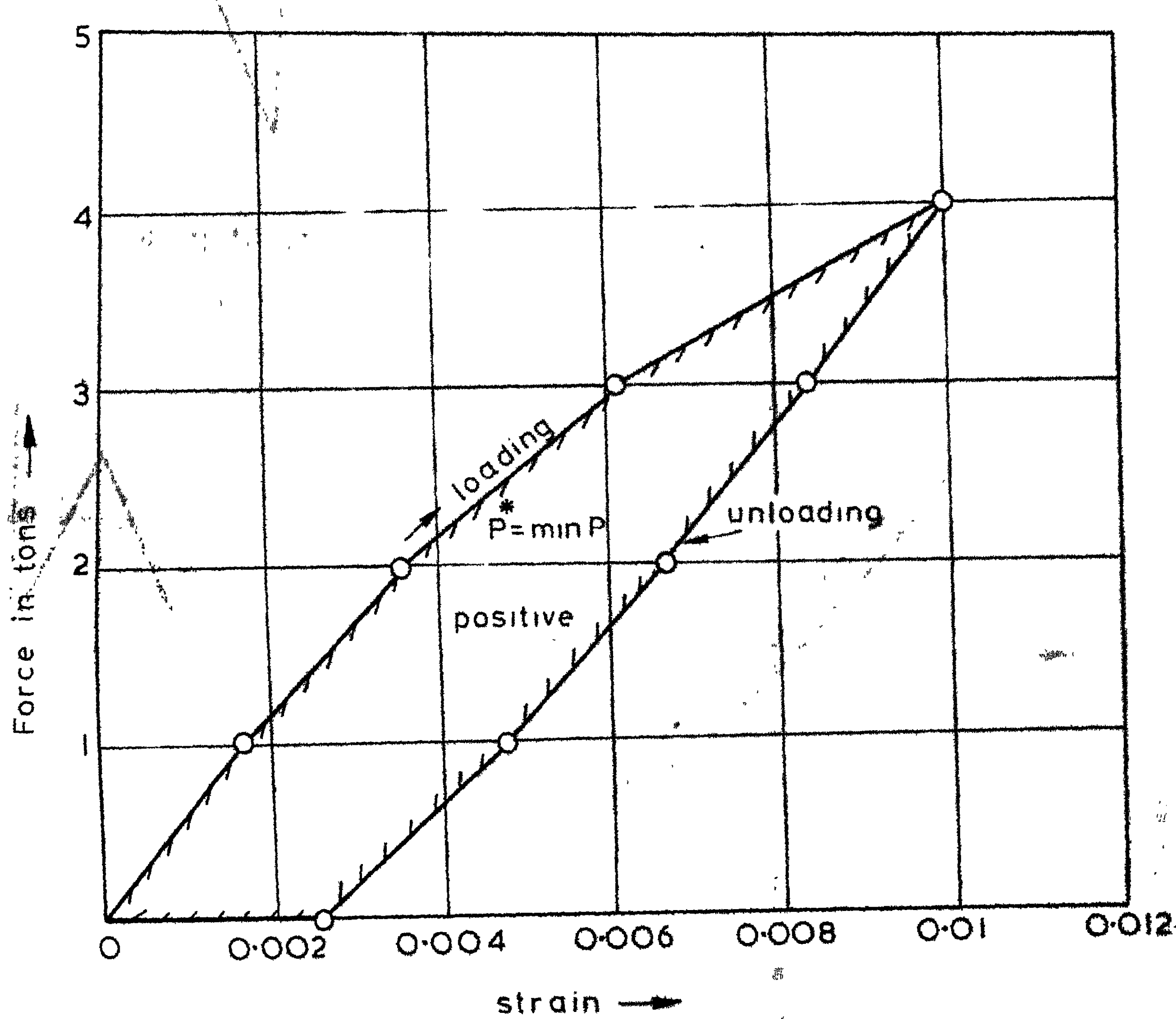


(a) Tension bar



(b) Loading function

FIGURE 5.9 EXAMPLE OF TENSION BAR (CASE 3).



(e) Force deformation relation of tension bar (case 3)

FIGURE 5.9 (CONTD.)

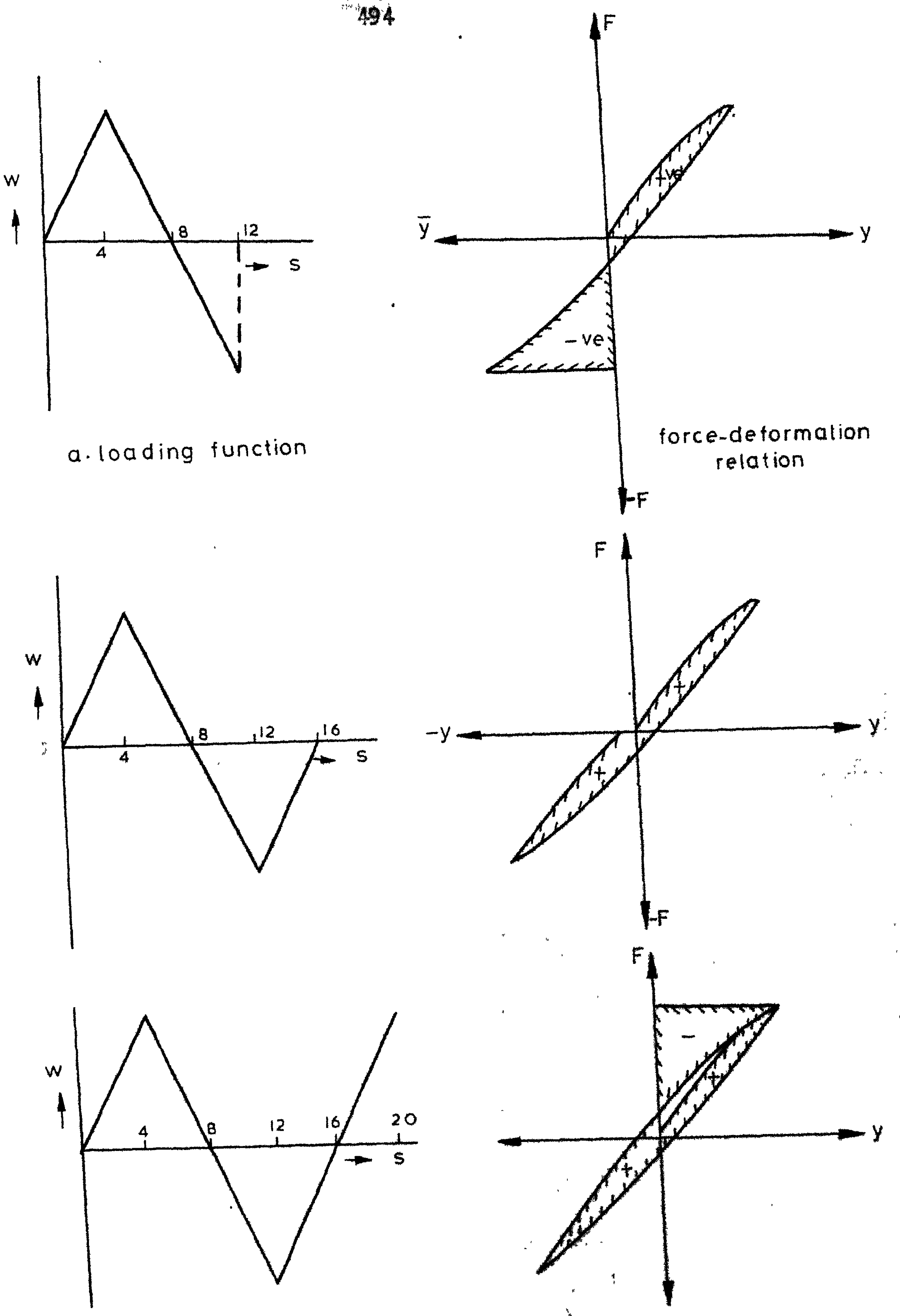
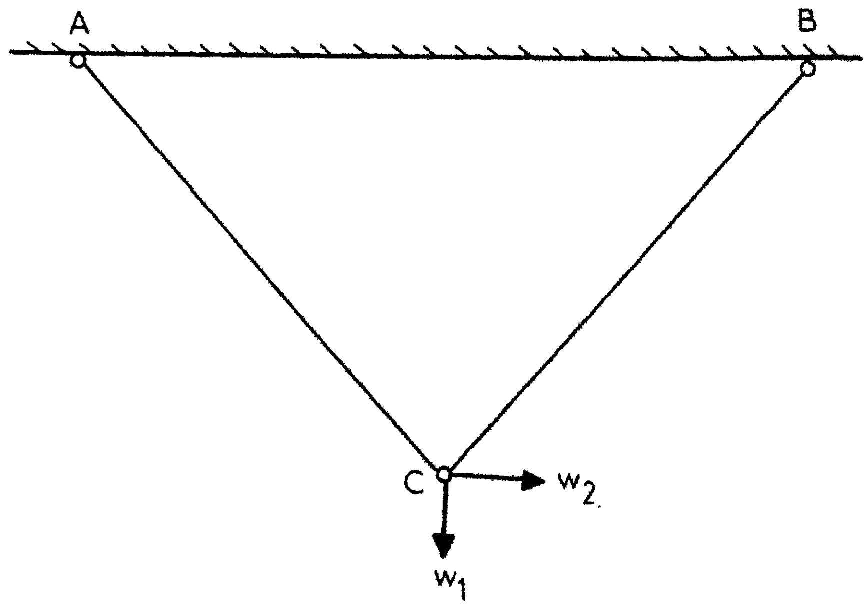
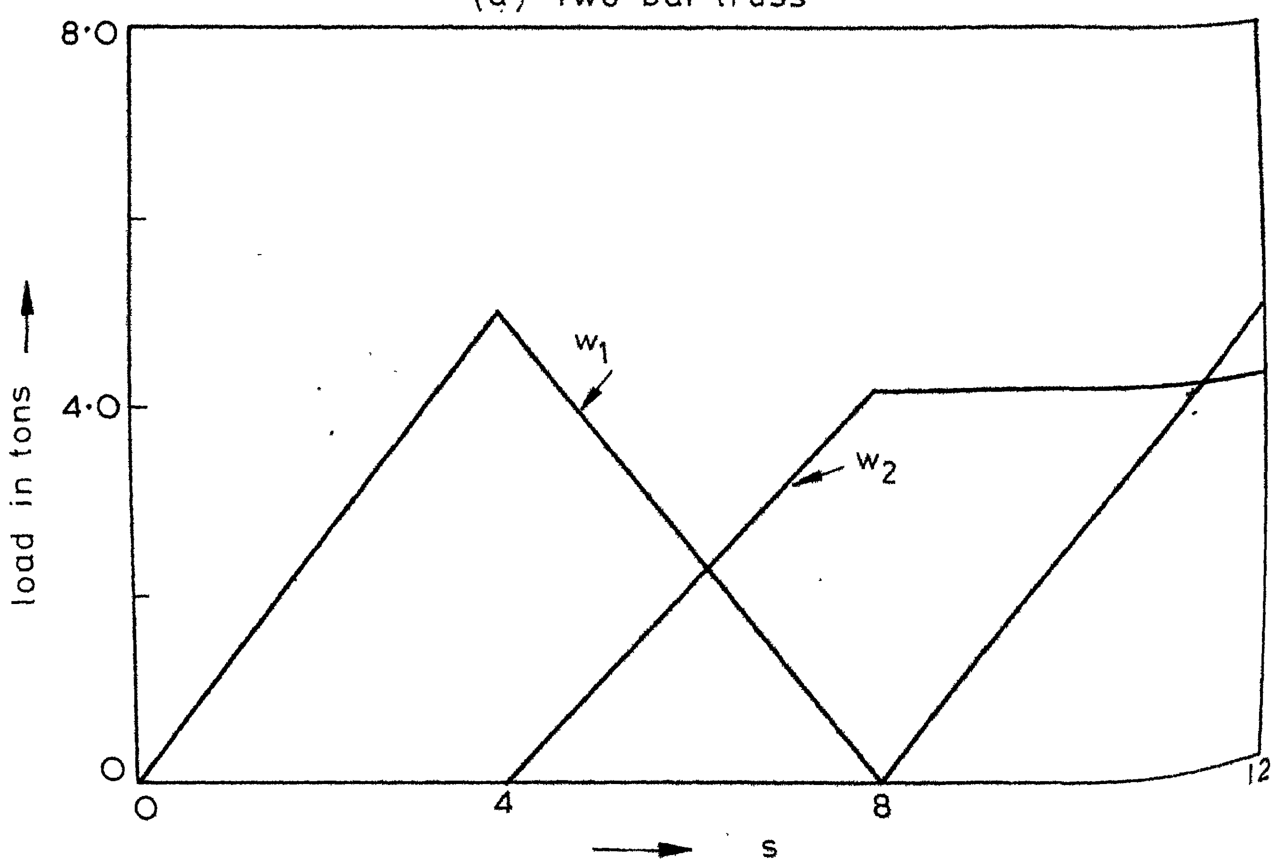


FIGURE 5.10 FORCE-DEFORMATION RELATIONS OF TENSION BAR UNDER DIFFERENT LOADING FUNCTIONS.

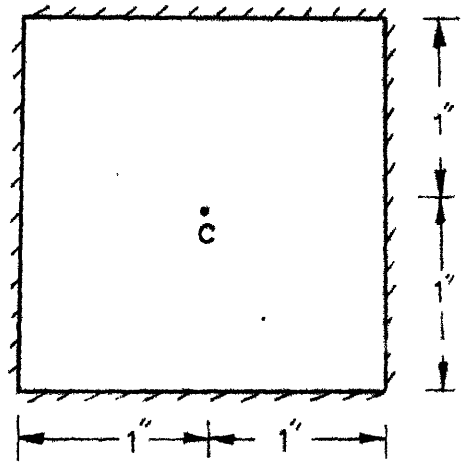


(a) Two bar truss

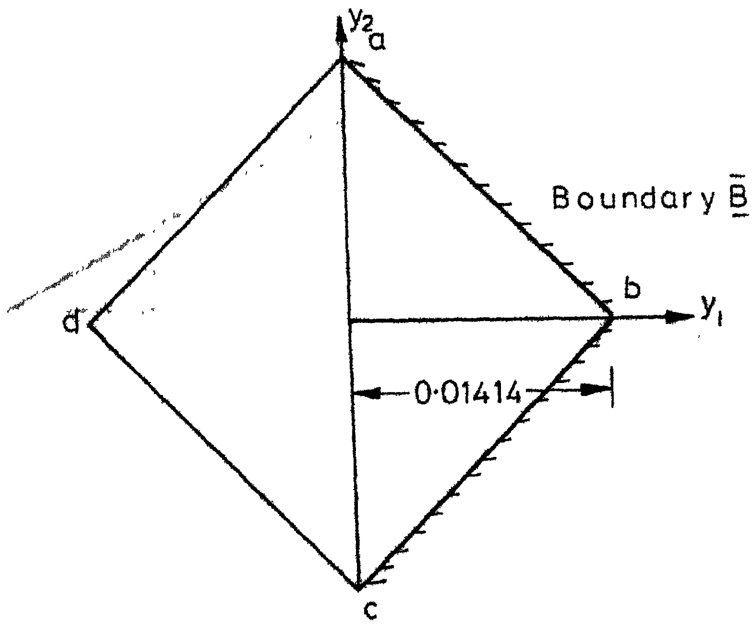


(b) Loading function

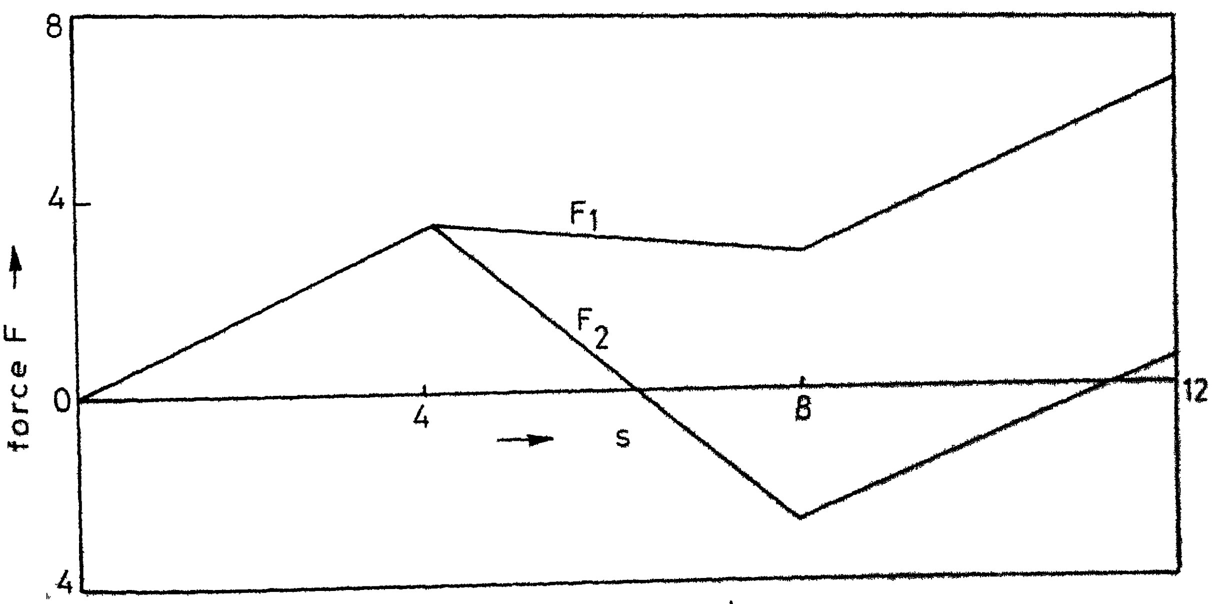
FIGURE 5.11 EXAMPLE OF TWO BAR TRUSS.



(c) Admissible space for c

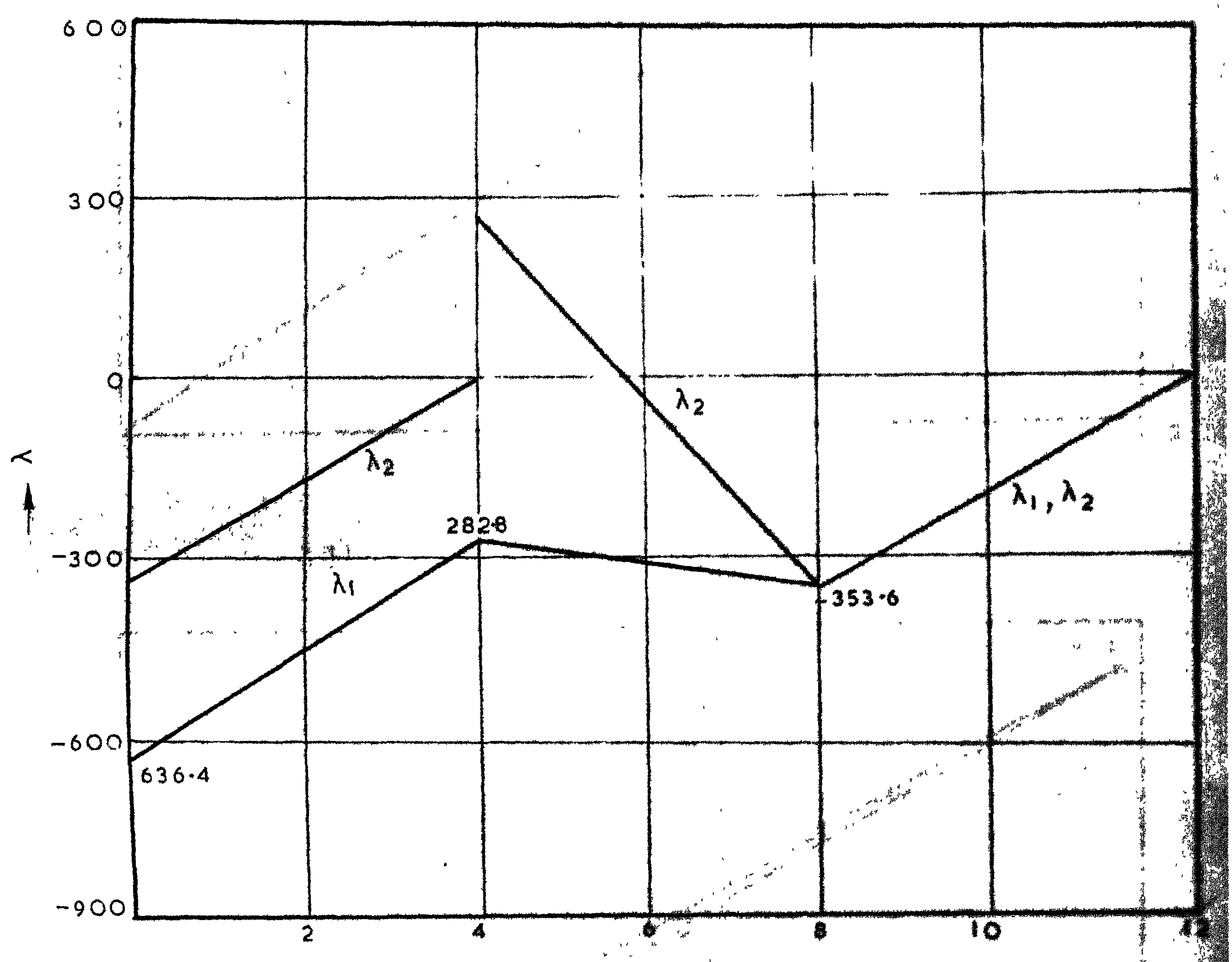


(d) Admissible state space



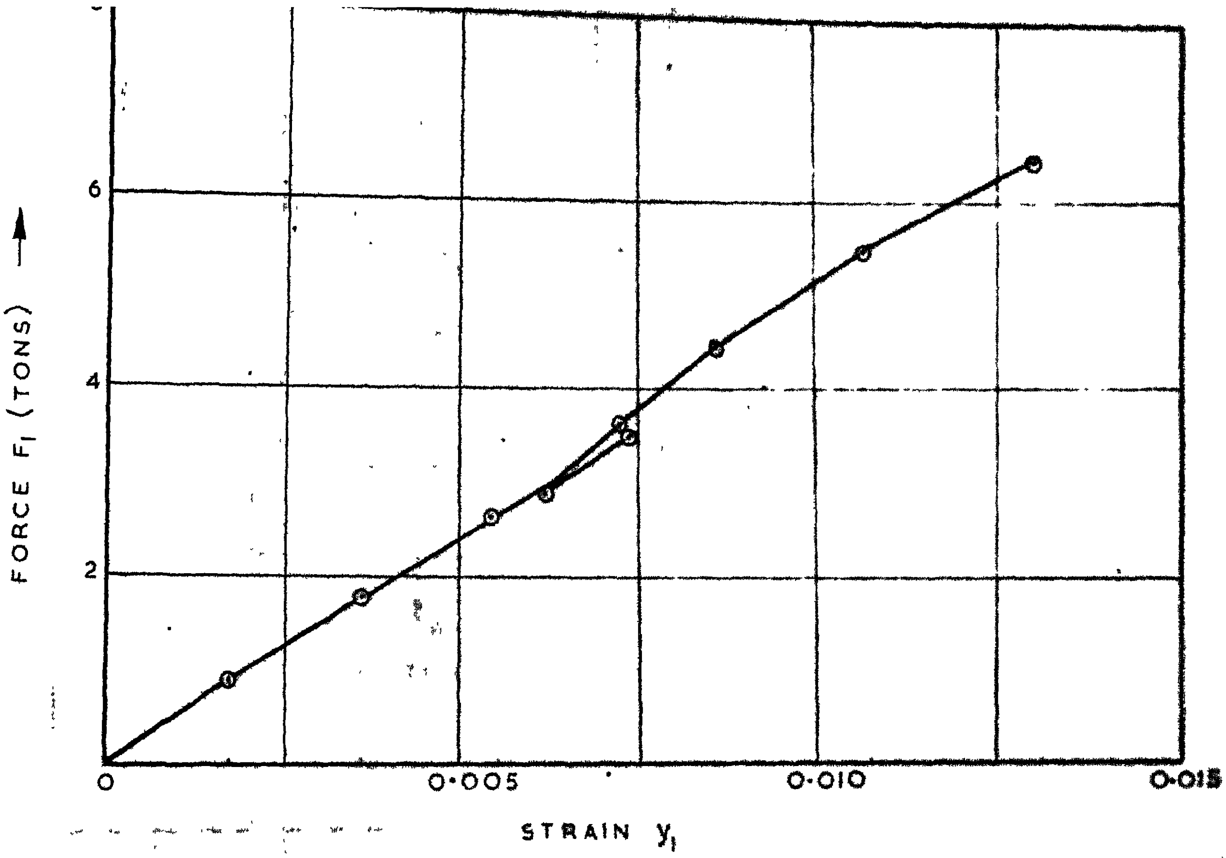
(e) Force in members

FIGURE 5.11 (CONTD.)

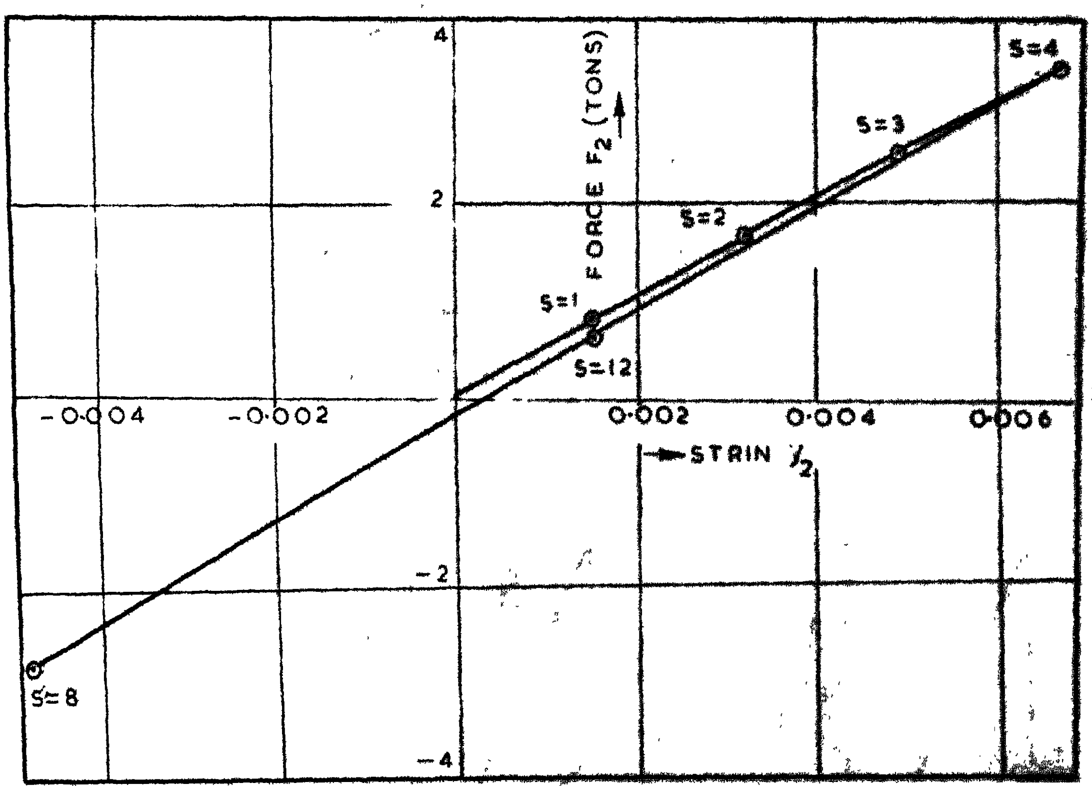


(f) COSTATE VARIABLES λ_1 AND λ_2

FIGURE 5.11 (CONTD.)

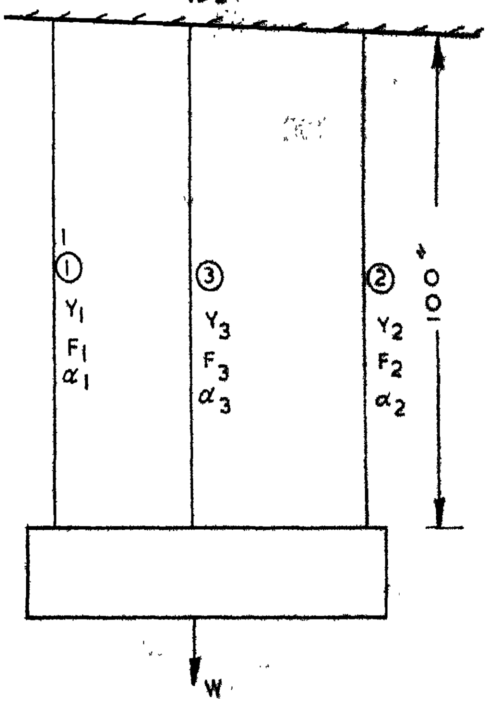


(g) TASK CURVE OF MEMBER 1 AC

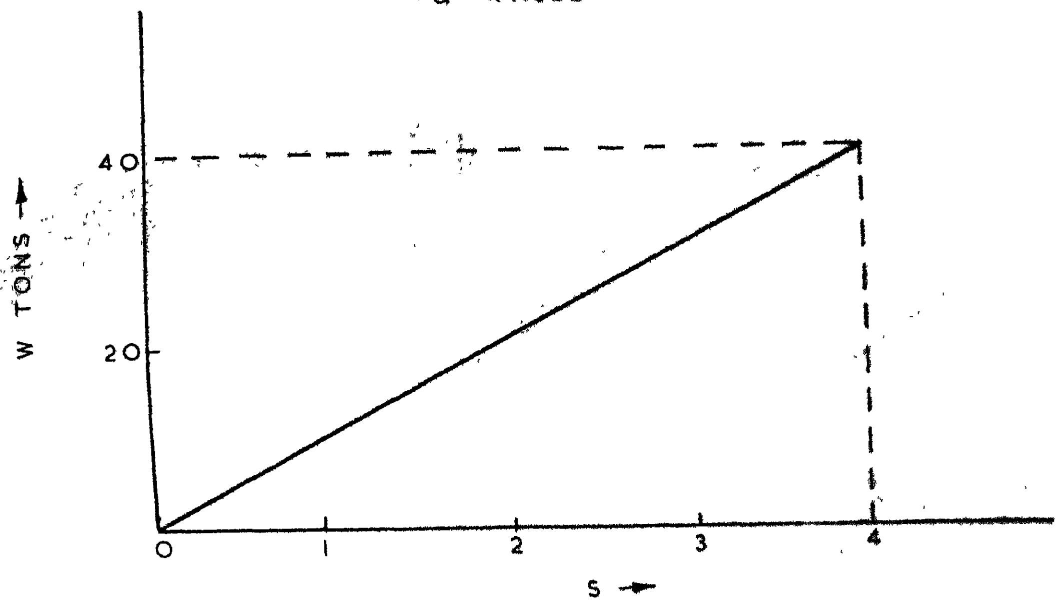


(h) TASK CURVE OF MEMBER 2 BC

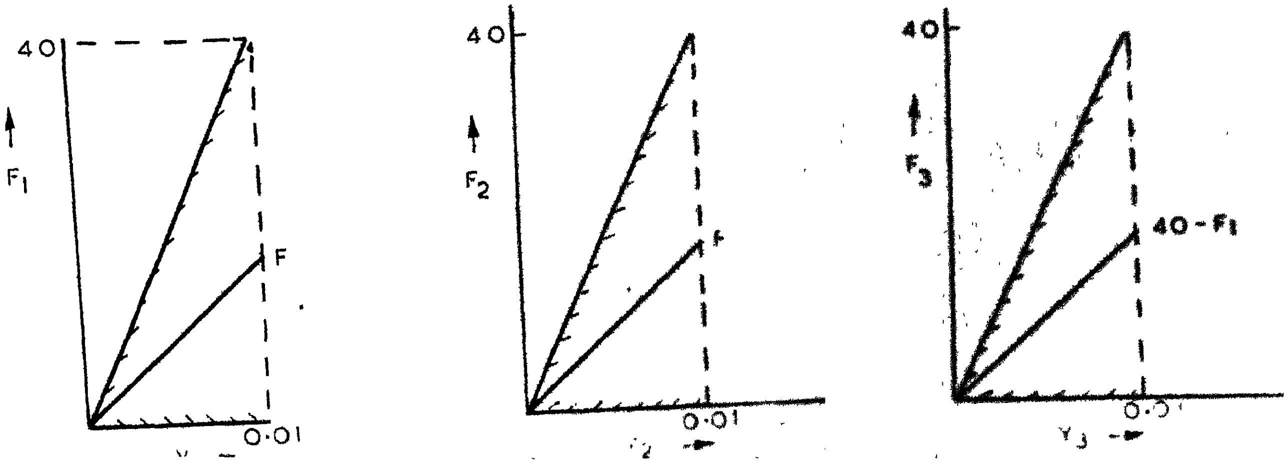
FIGURE 5.11 (CONTD.)

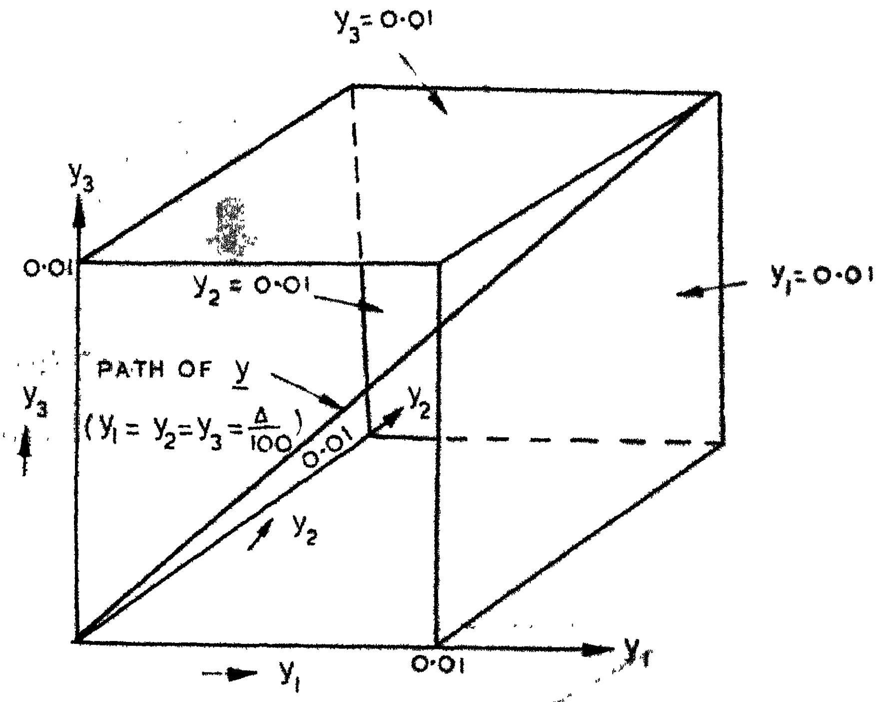


a TRUSS



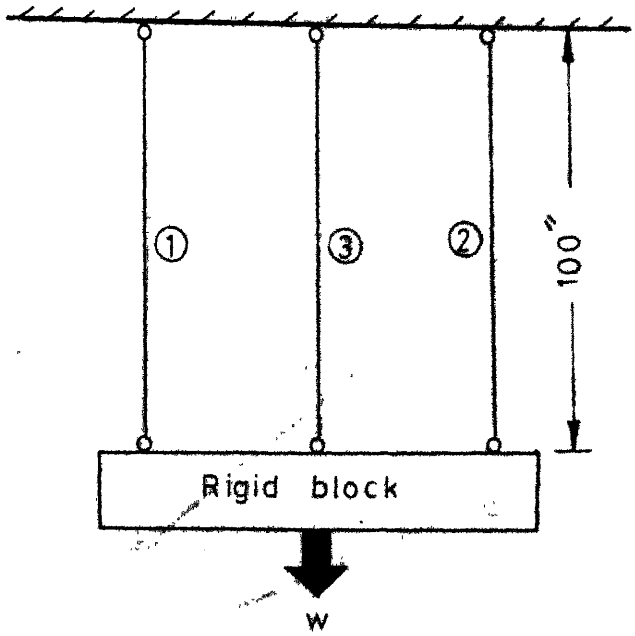
b. LOADING FUNCTION



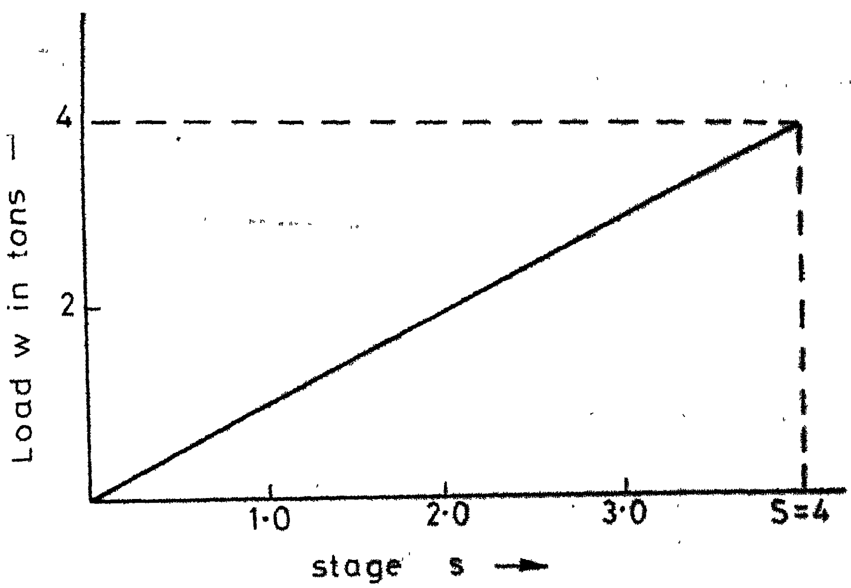


(d) Admissible region in $\bar{Y}$ space and Admissible path of $\underline{y}$.

FIGURE 5.12 (CONTD.)

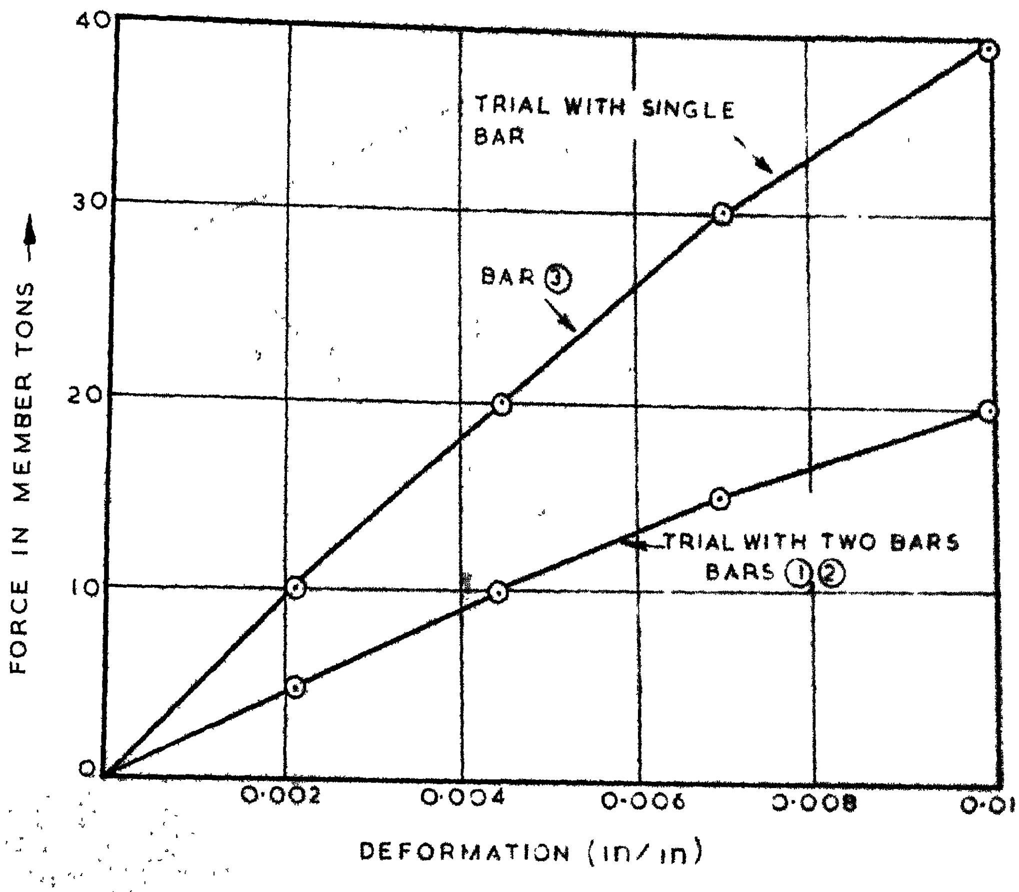


(a) Three bar system

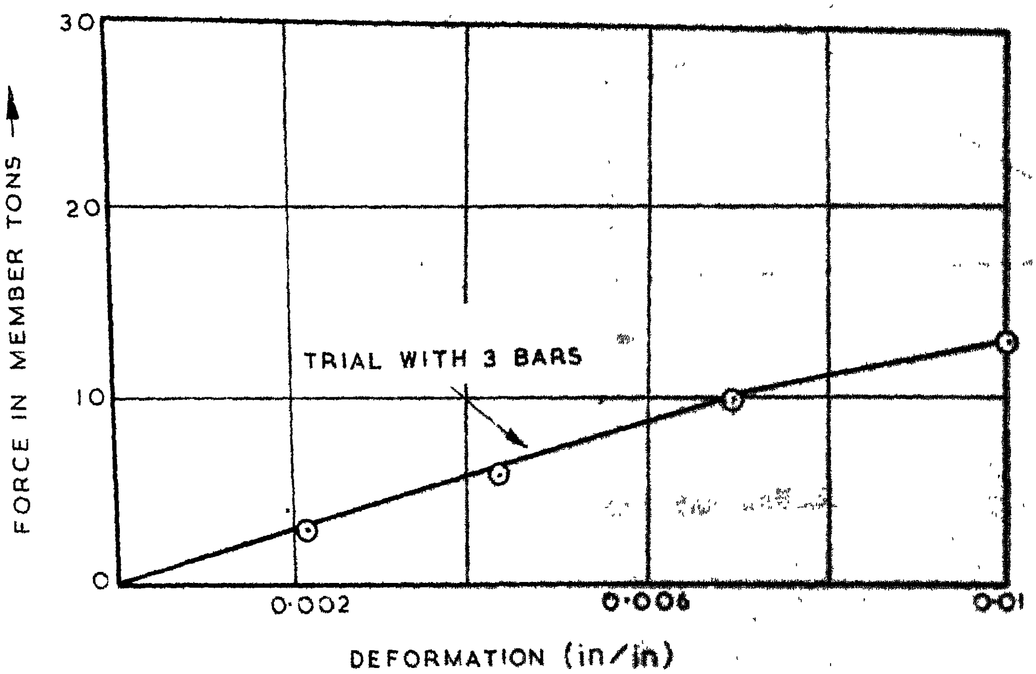


(b) Loading function

FIGURE 5.12 (CONTD.)

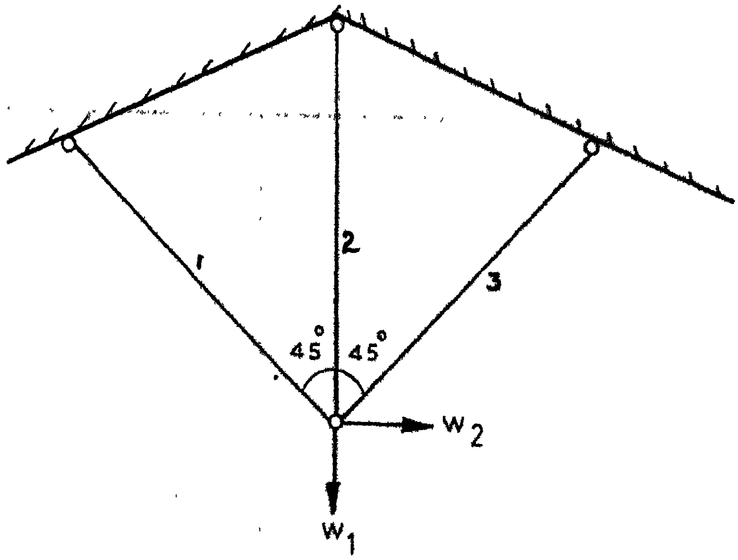


(e) TASK CURVES IN THE CASE OF INELASTIC MATERIAL

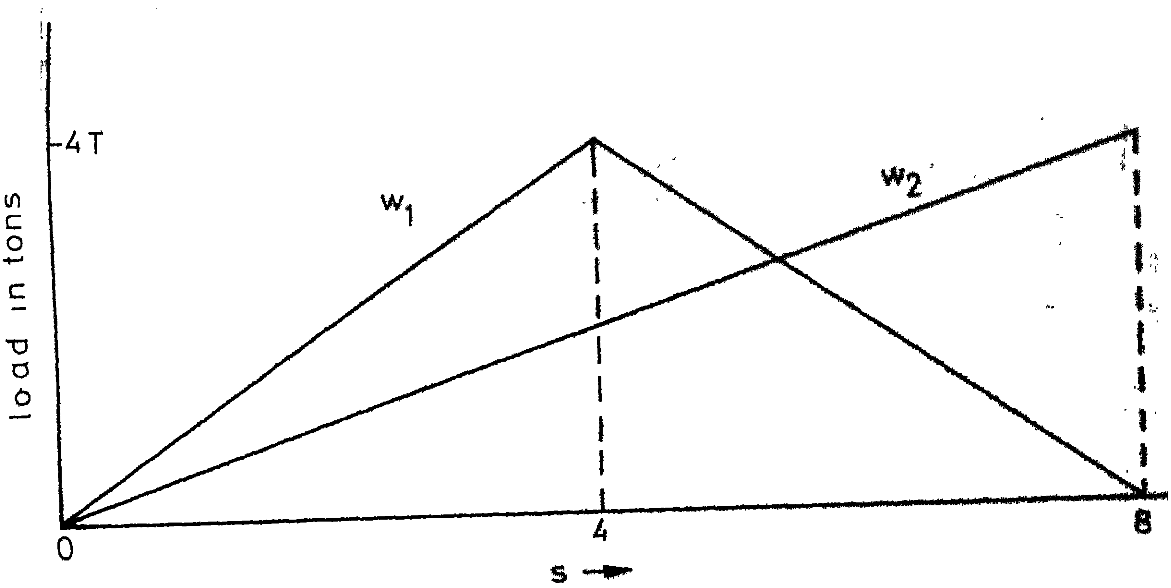


(f) TASK CURVES IN THE CASE OF INELASTIC MATE

FIGURE 5.12 (CONTD.)

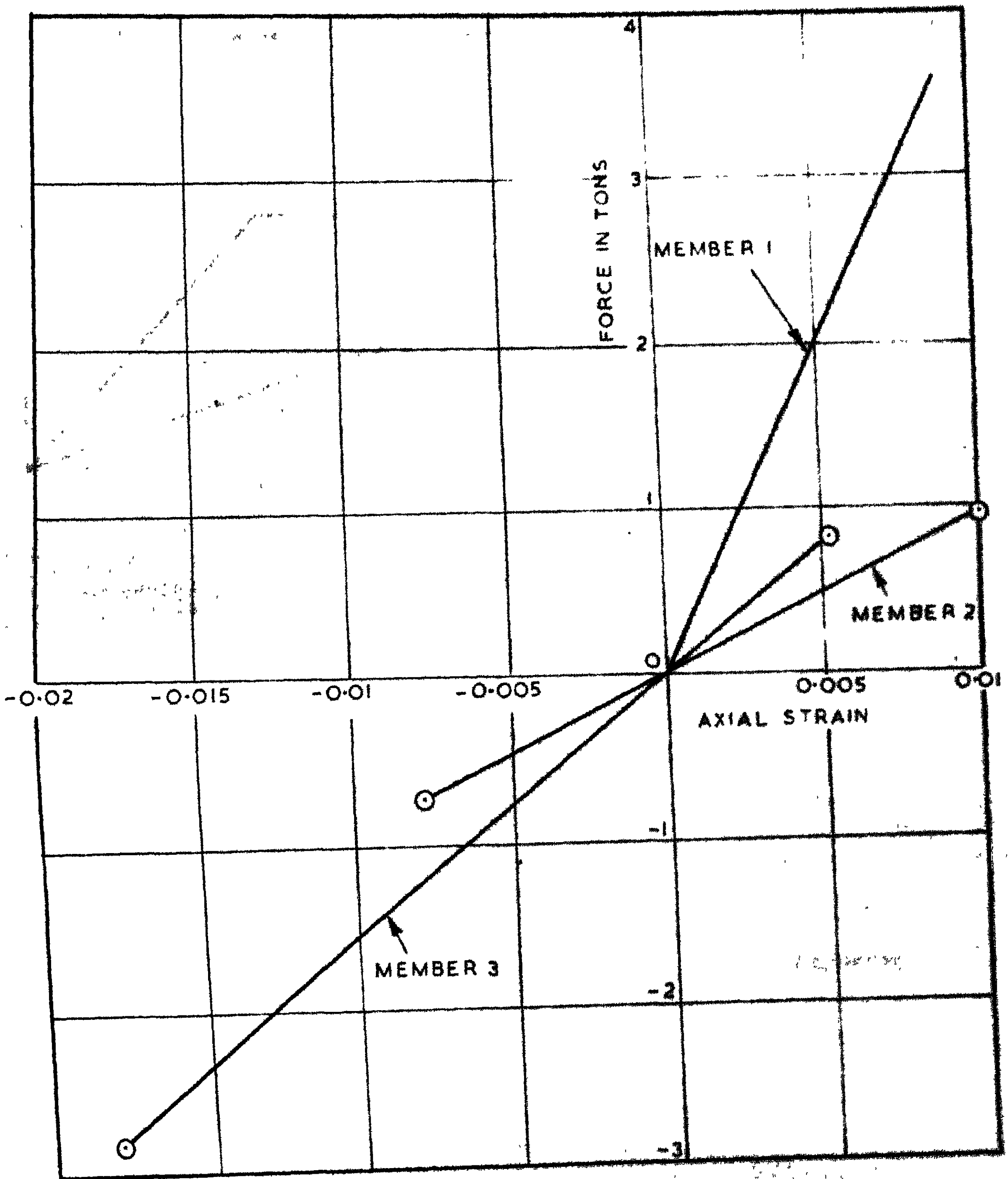


(a) Three bar truss



(b) Loading function

FIGURE 5.13 EXAMPLE OF THREE BAR TRUSS.



(c) TASK CURVE FOR THE TRIAL SHOWN IN TABLE 5·8

Fig. 5·13 (CONTD)

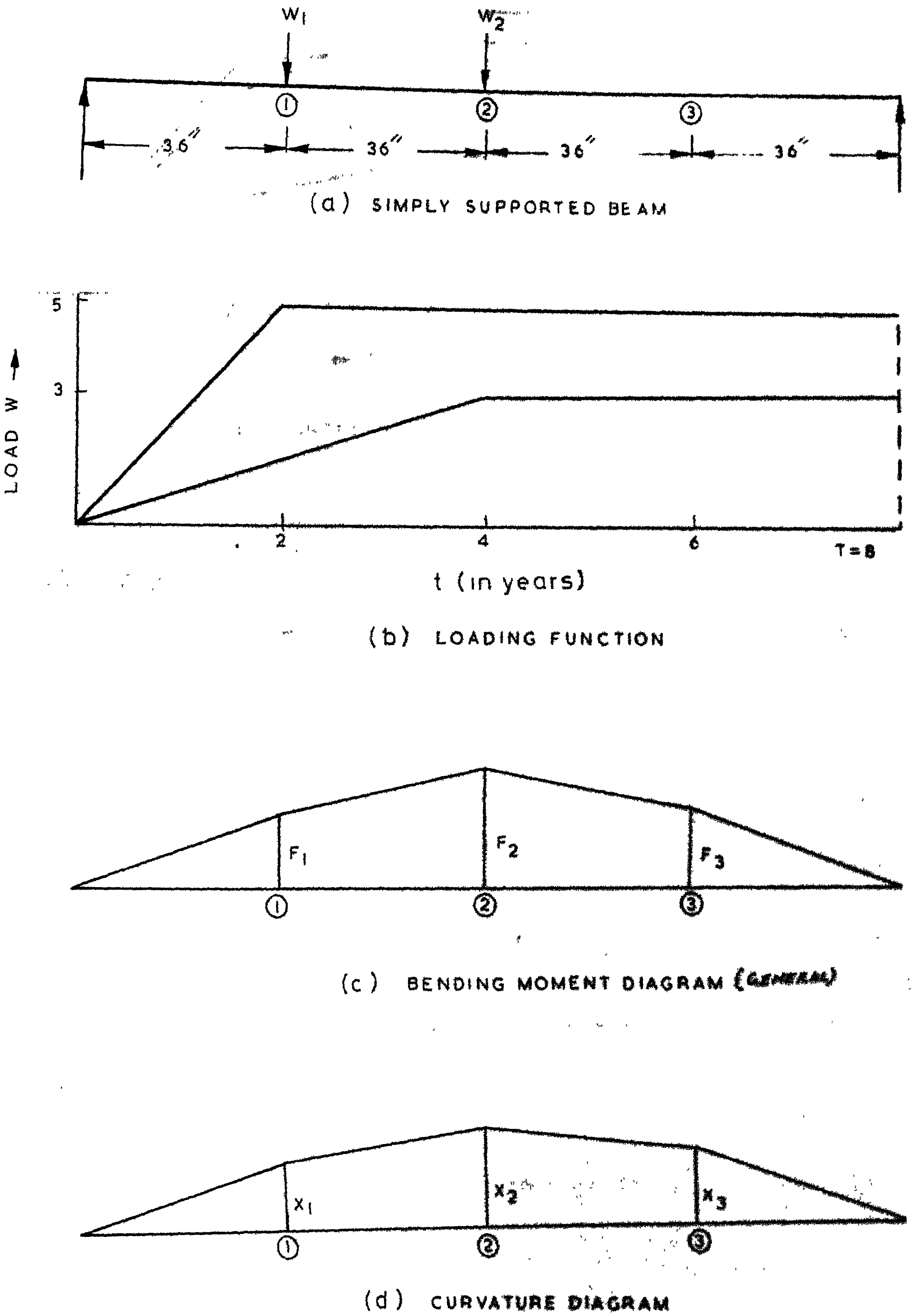
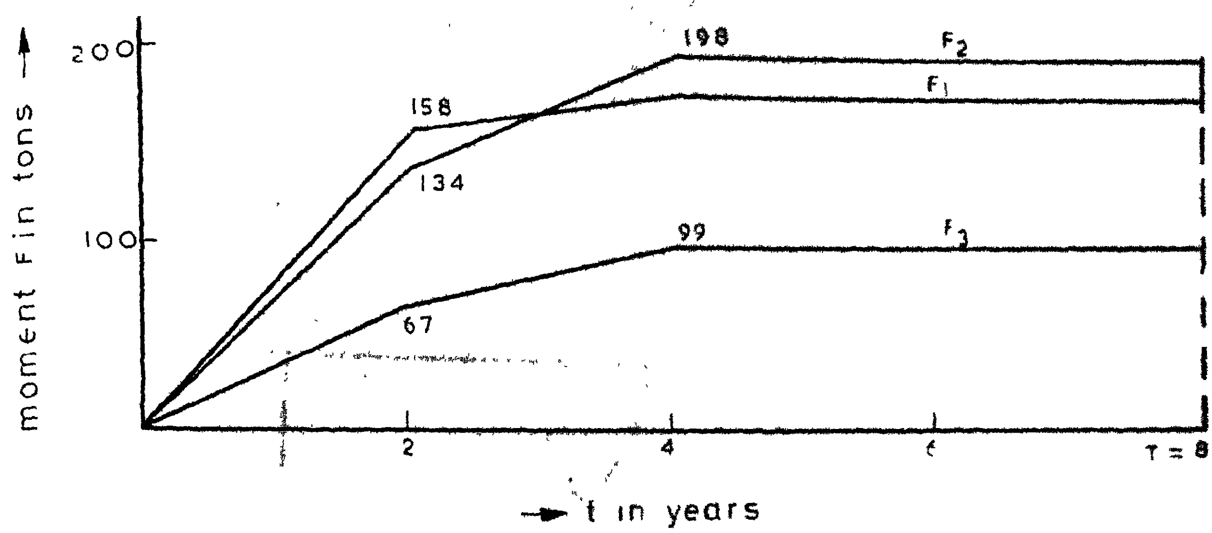
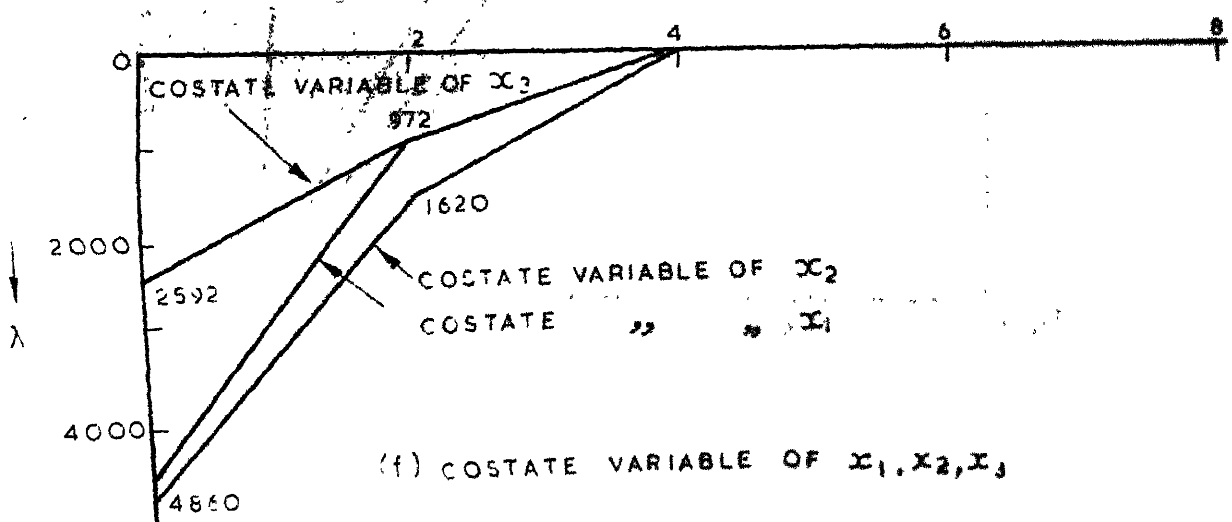


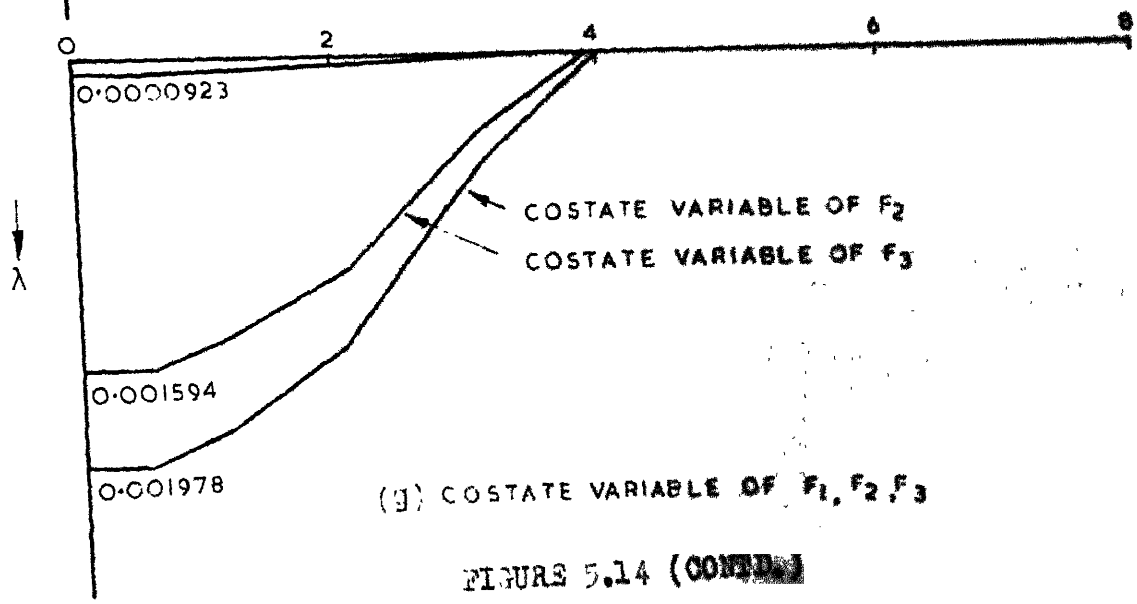
FIGURE 5.14 EXAMPLE OF SIMPLY-SUPPORTED BEAM.



(e) FORCES IN MEMBERS

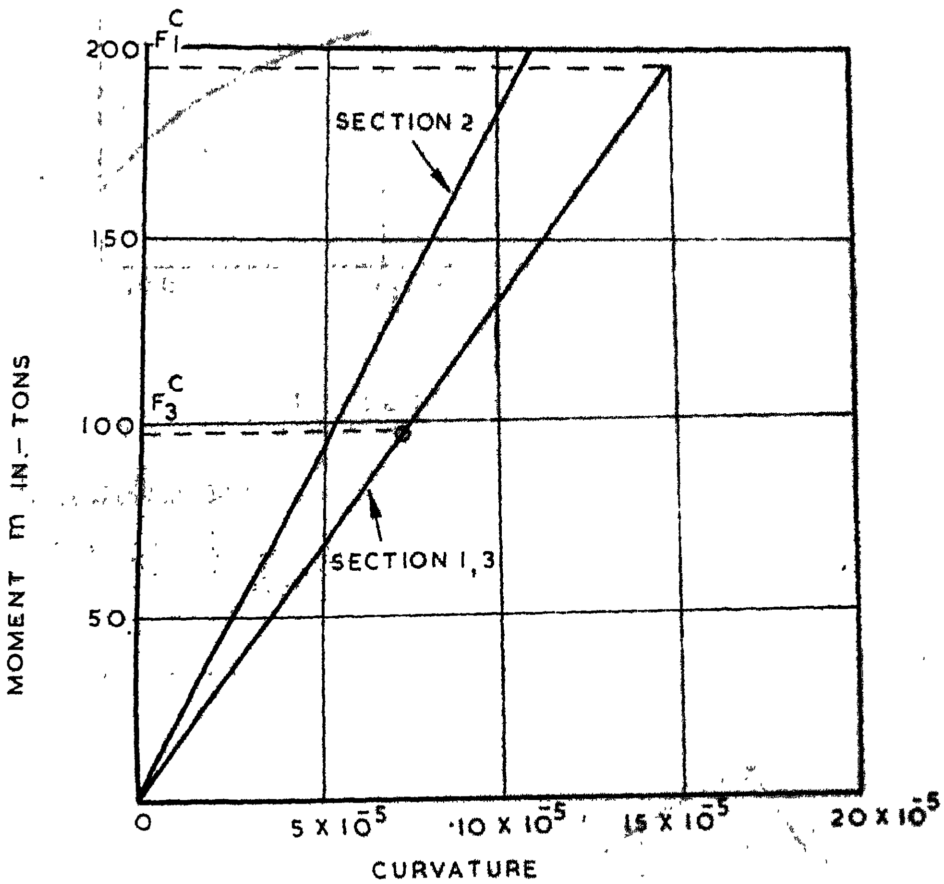


(f) COSTATE VARIABLE OF x_1, x_2, x_3



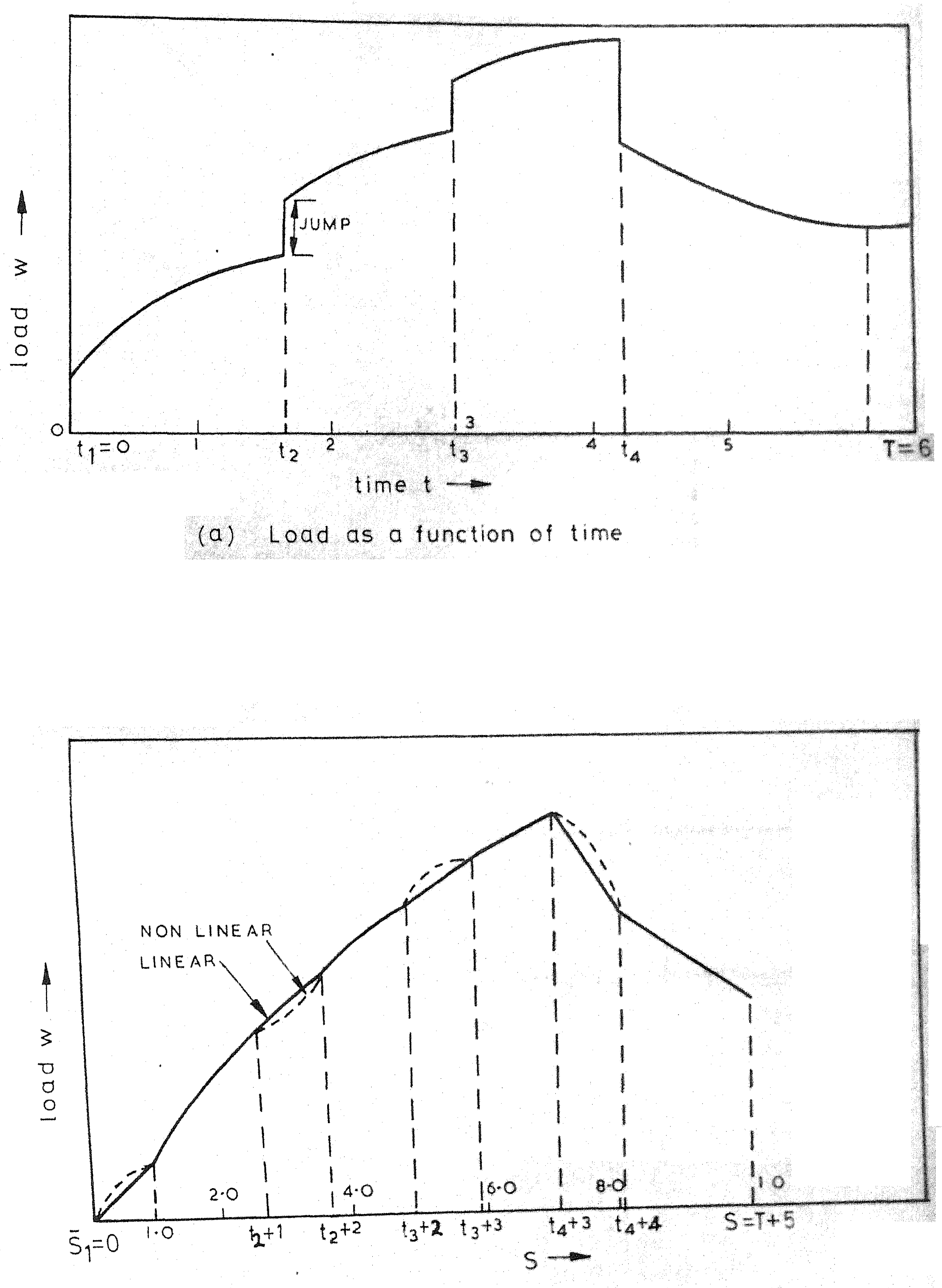
(g) COSTATE VARIABLE OF F_1, F_2, F_3

FIGURE 5.14 (CONTD.)



(h) TASK CURVE

Fig. 5·14 (CONTD)



(a) Load as a function of time

(b) Modified the load function w (t) as a function of s

FIGURE 5.15 MODIFICATION OF LOAD FUNCTION.

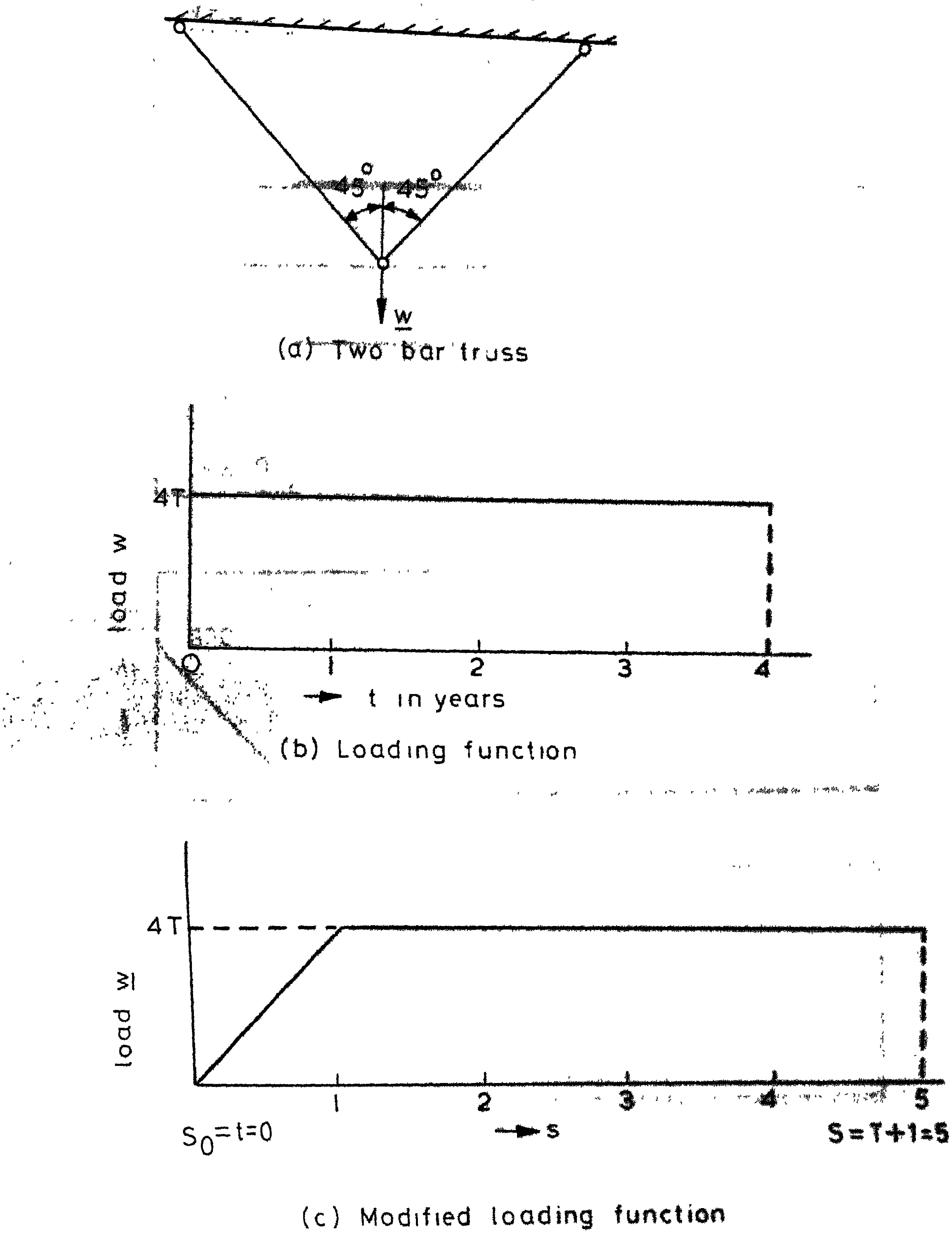
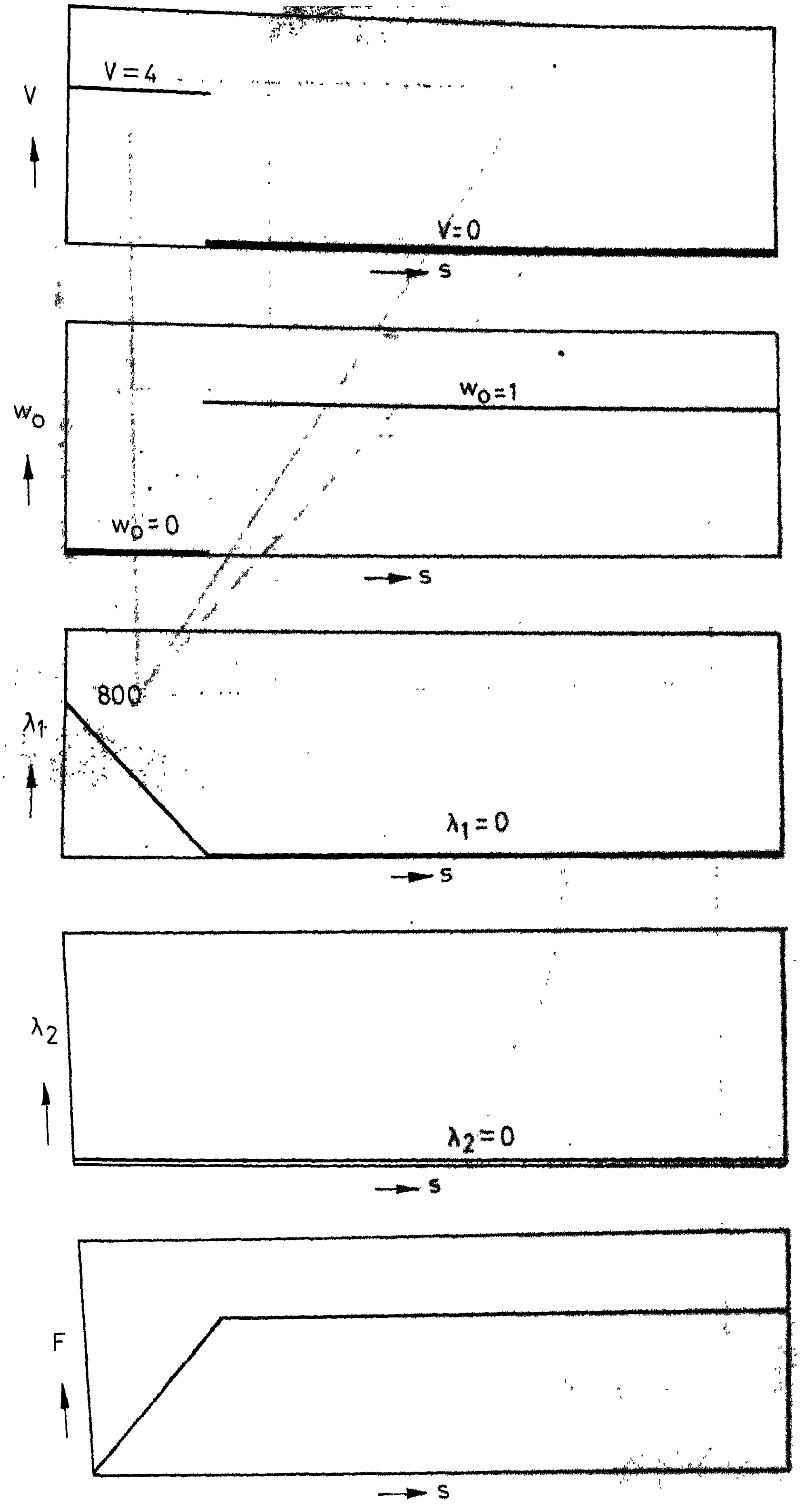
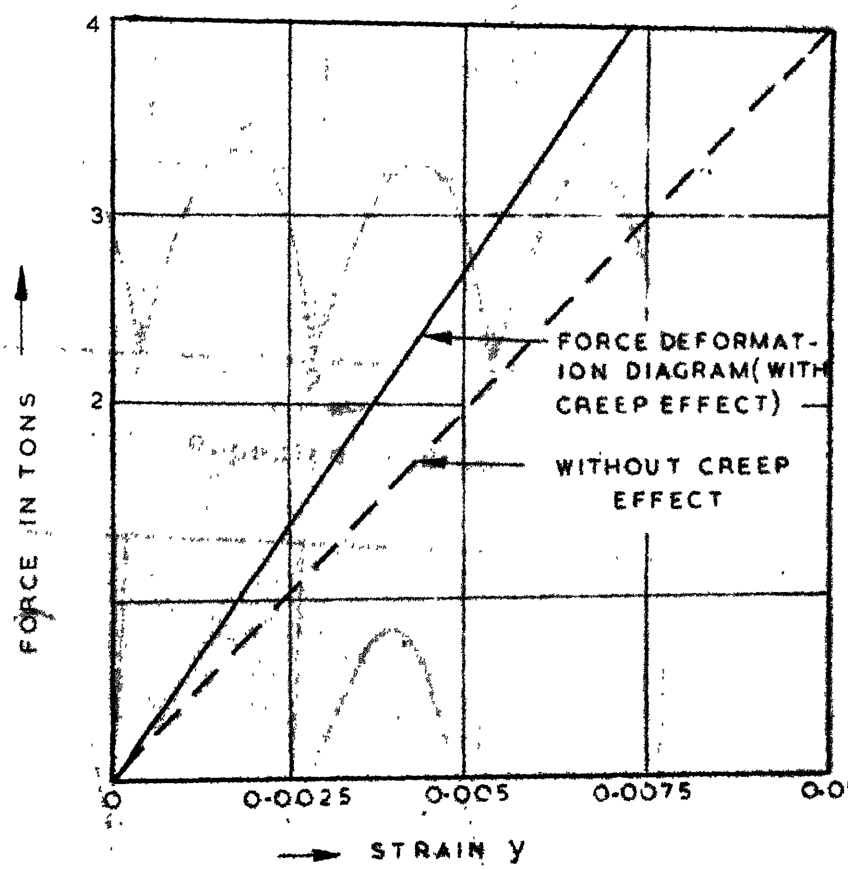


FIGURE 5.16 EXAMPLE OF TWO BAR TRUSS WITH JUMPS IN LOAD.



Variation of V, w_o, λ, λ_2, F with S

FIGURE [illegible].16 (CONTD.)



(d) Task curve

FIGURE 5.16 (CONTD.)

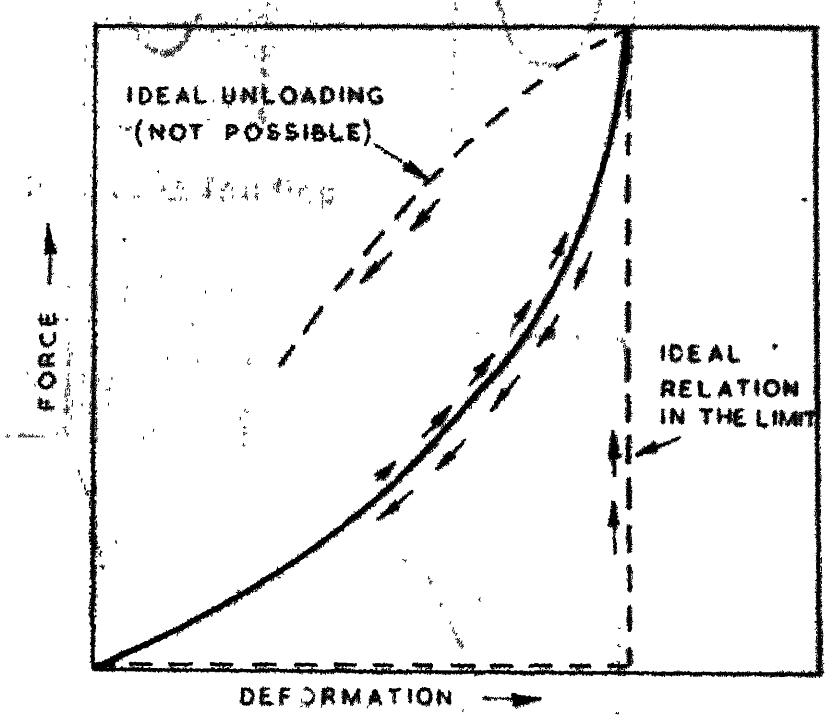
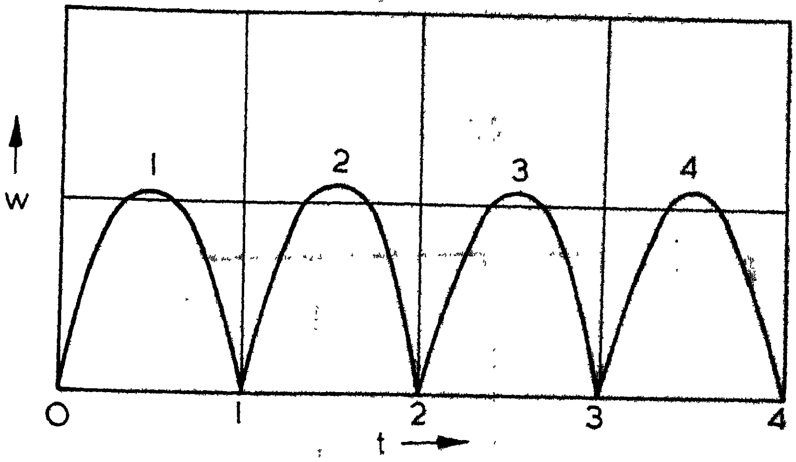
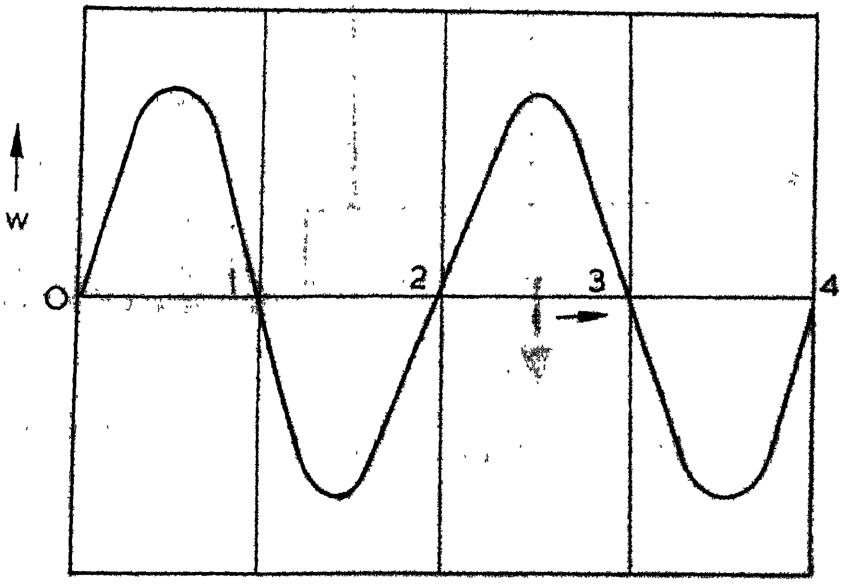


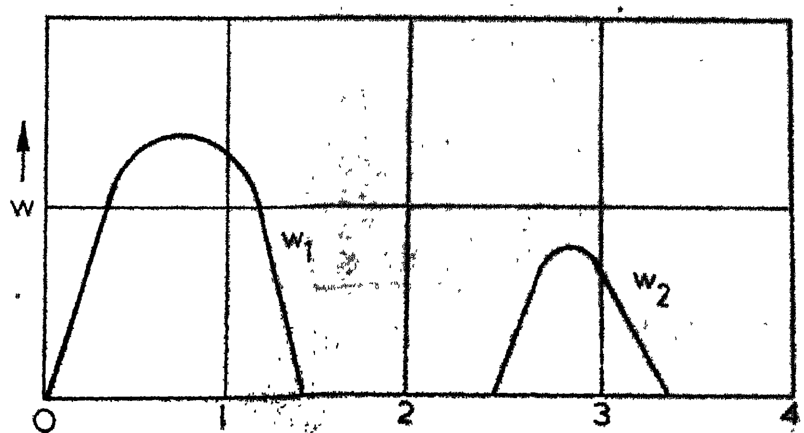
FIGURE 5.17 IDEAL LOCKING MATERIAL



(a) Repeated loading

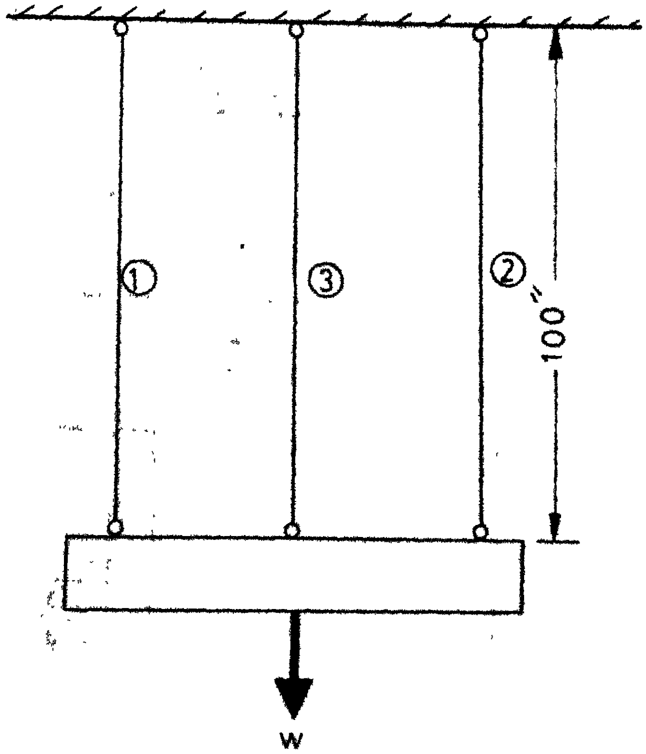


(b) Cyclic loading



(c) Multiple load condition

FIGURE 5.18 COMPLEX LOAD PATTERNS.

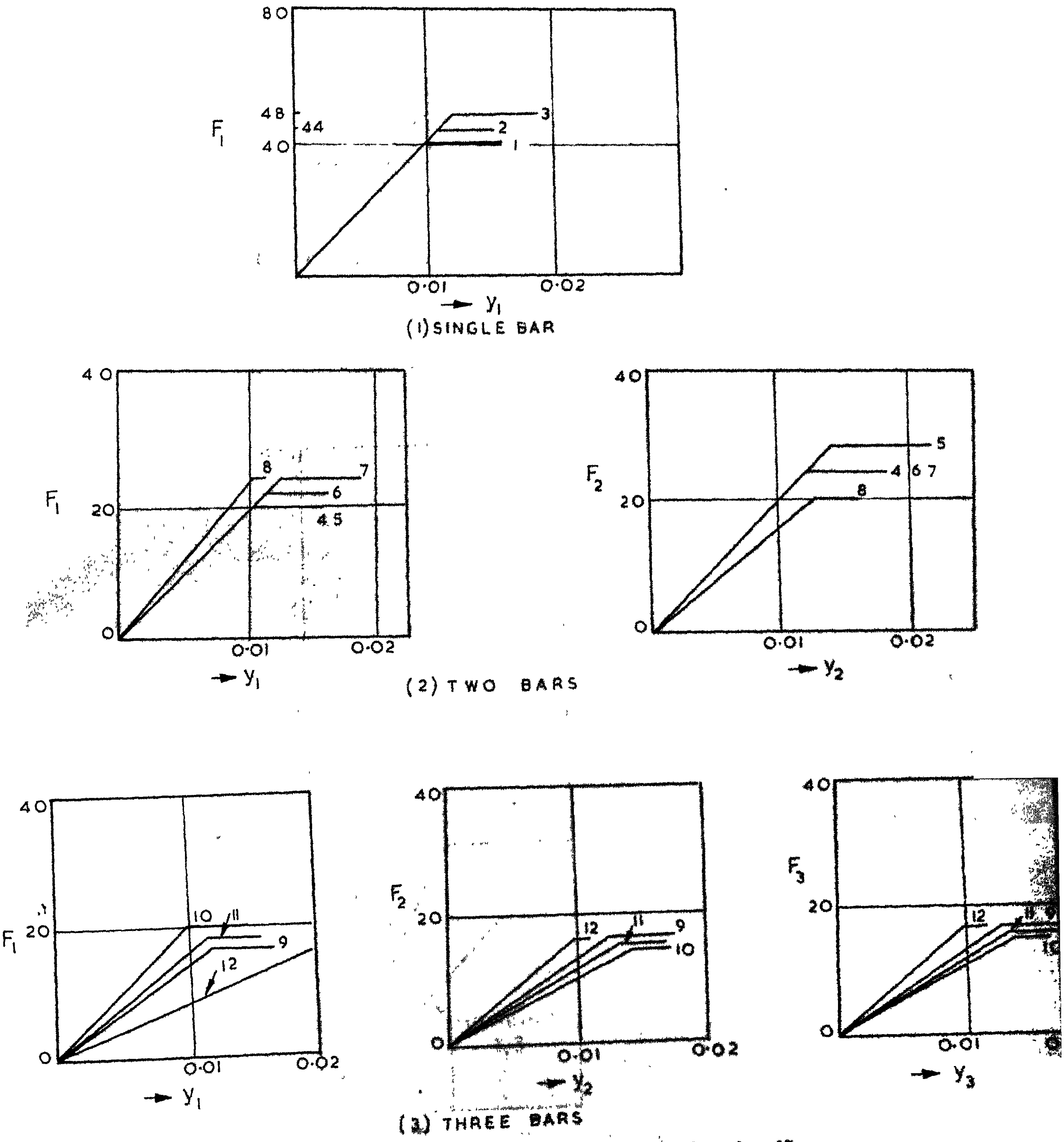


(a) Three bar truss

load	probability
<40 T	0·90
40	0·035
41	0·025
42	0·017
43	0·010
44	0·005
45	0·003
46	0·002
47	0·001
≥ 48	0·002

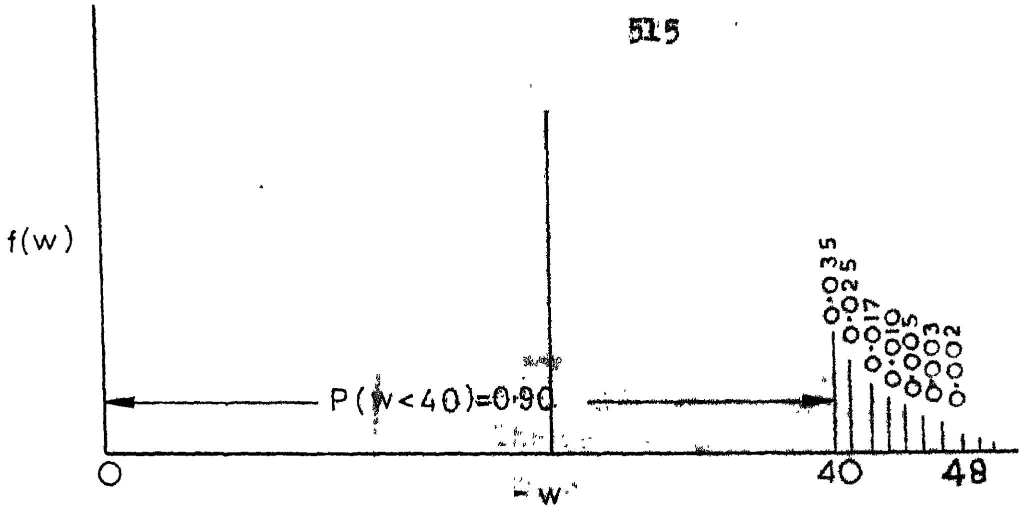
(b) Discrete distribution for w

FIGURE 6.1 THREE BAR SYSTEM.

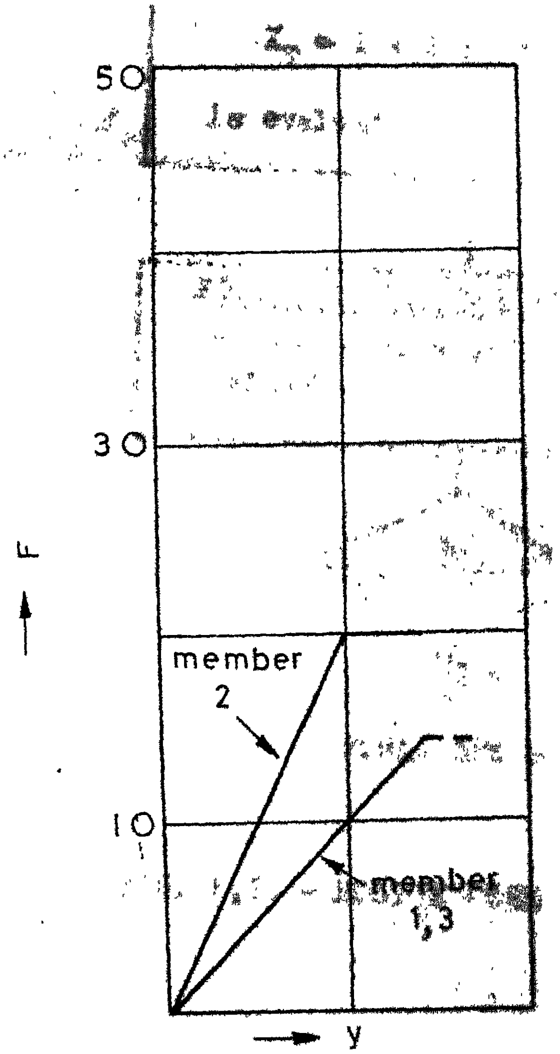


(c) Alternatives chosen for trail.

FIGURE 6.1 (CONTD.)



C. Frequency distribution



(d) Final choice

FIGURE 6.1 (CONTD.)

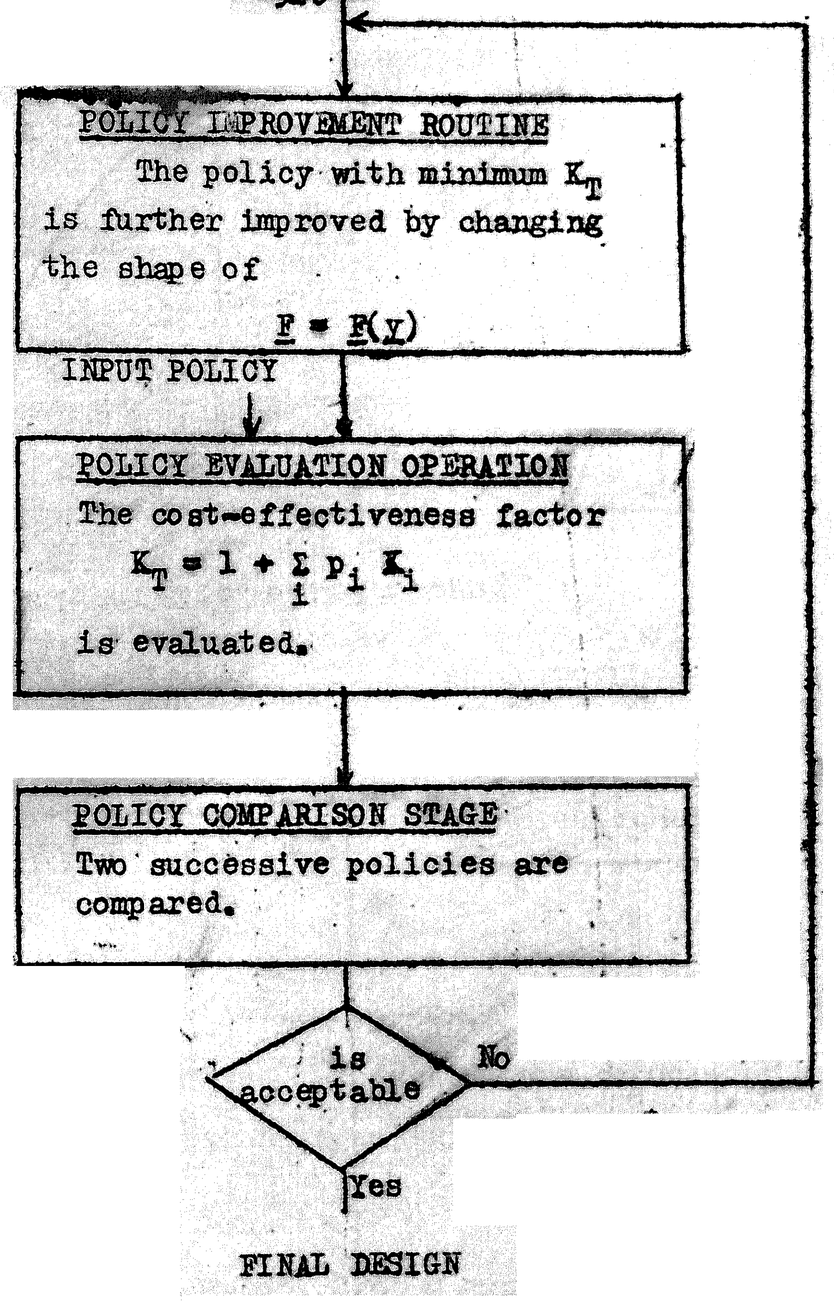


FIG. 6.2. - POLICY ITERATION PROCEDURE

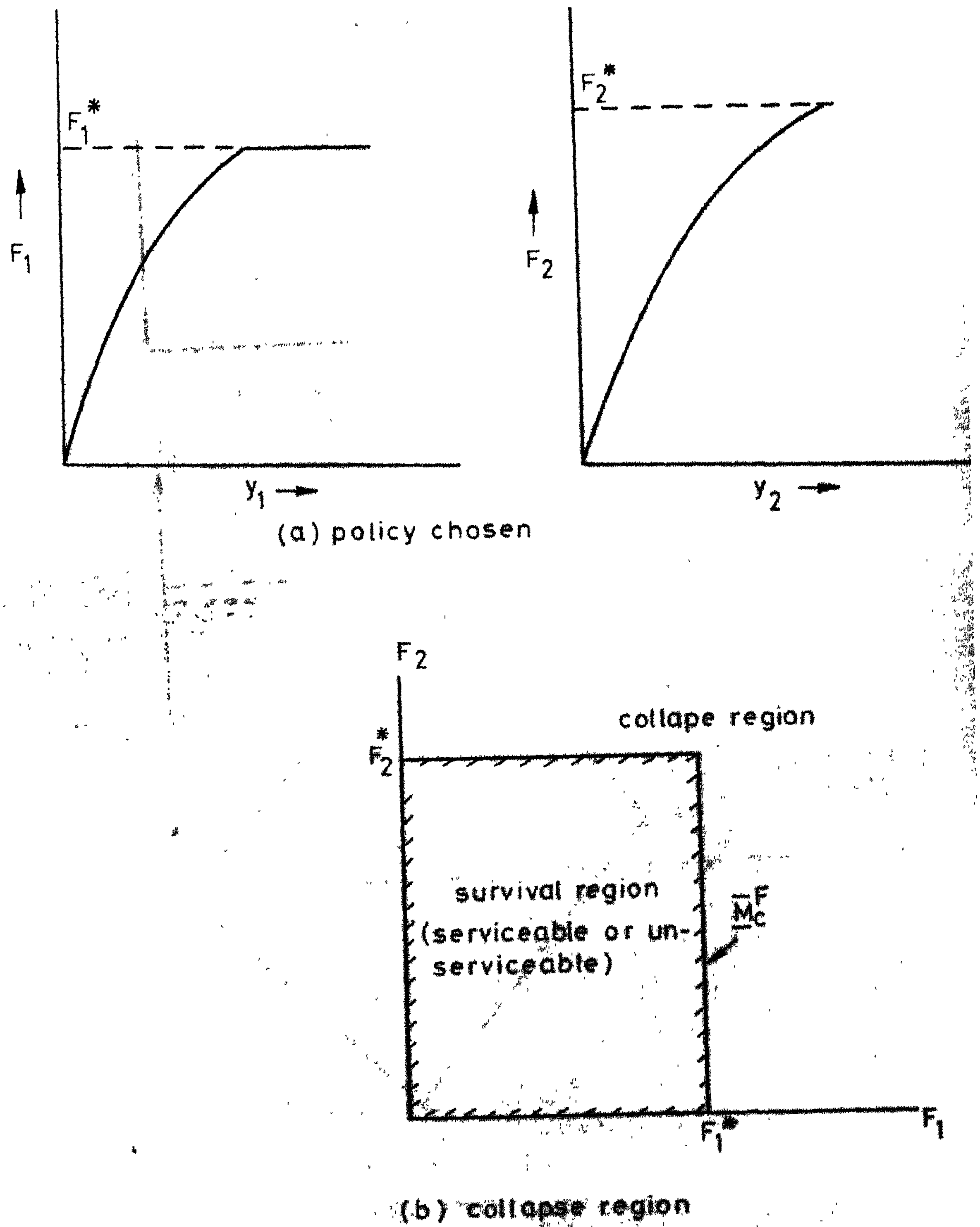


FIGURE 6.3 COLLAPSE REGION ILLUSTRATED FOR A TWO-DIMENSIONAL FORCE-SPACE.

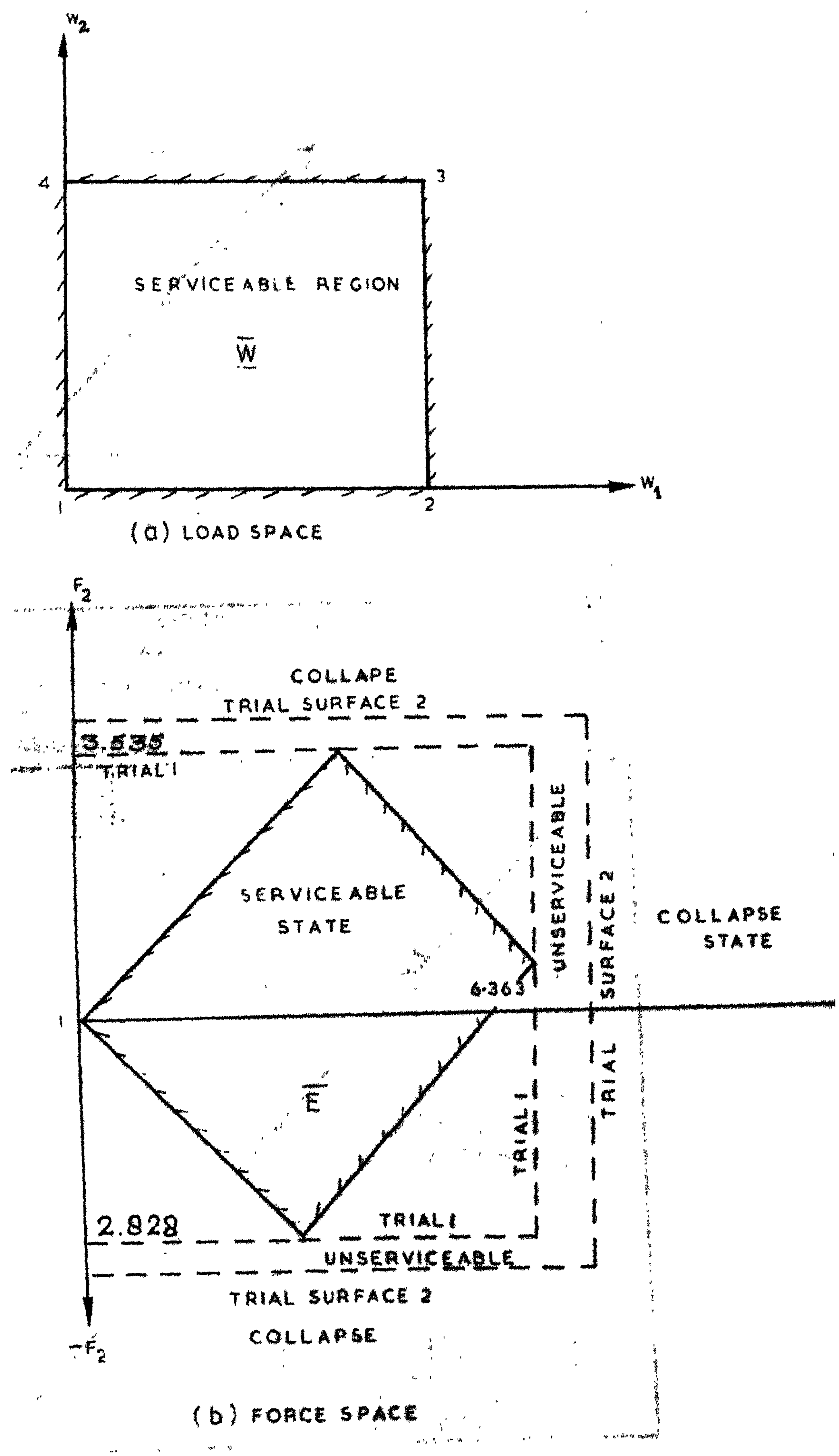
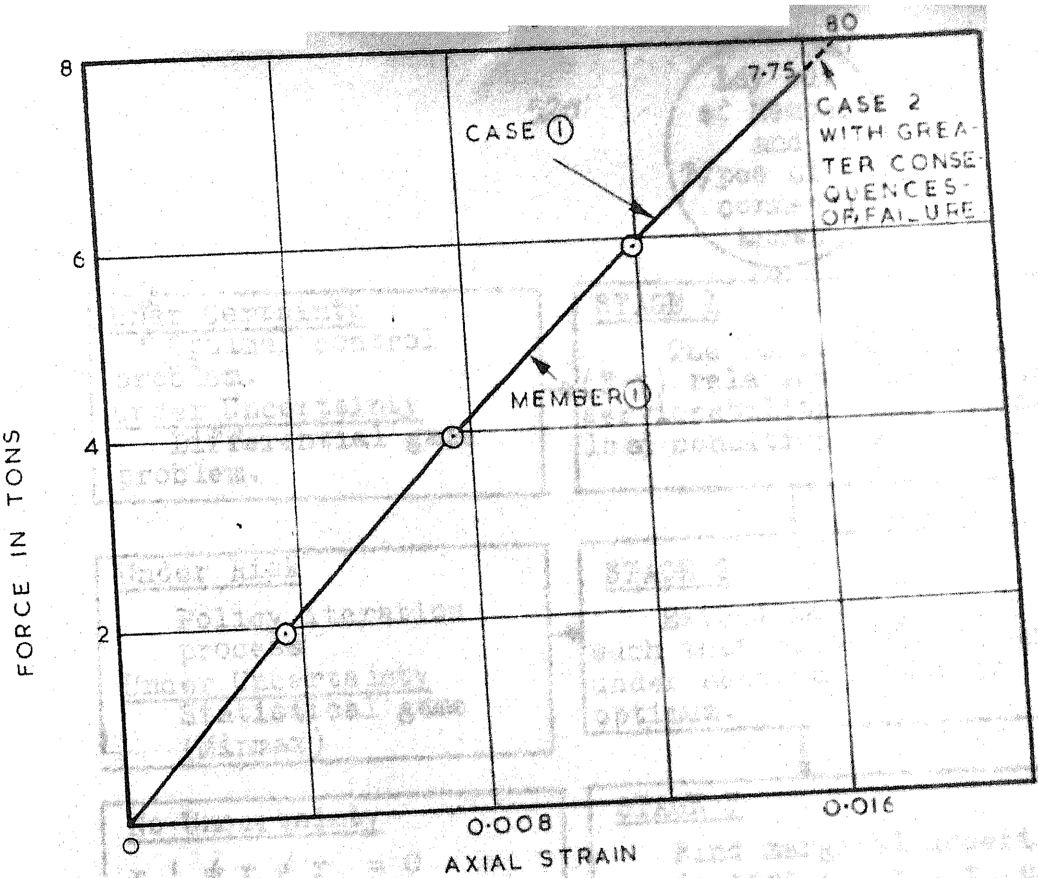
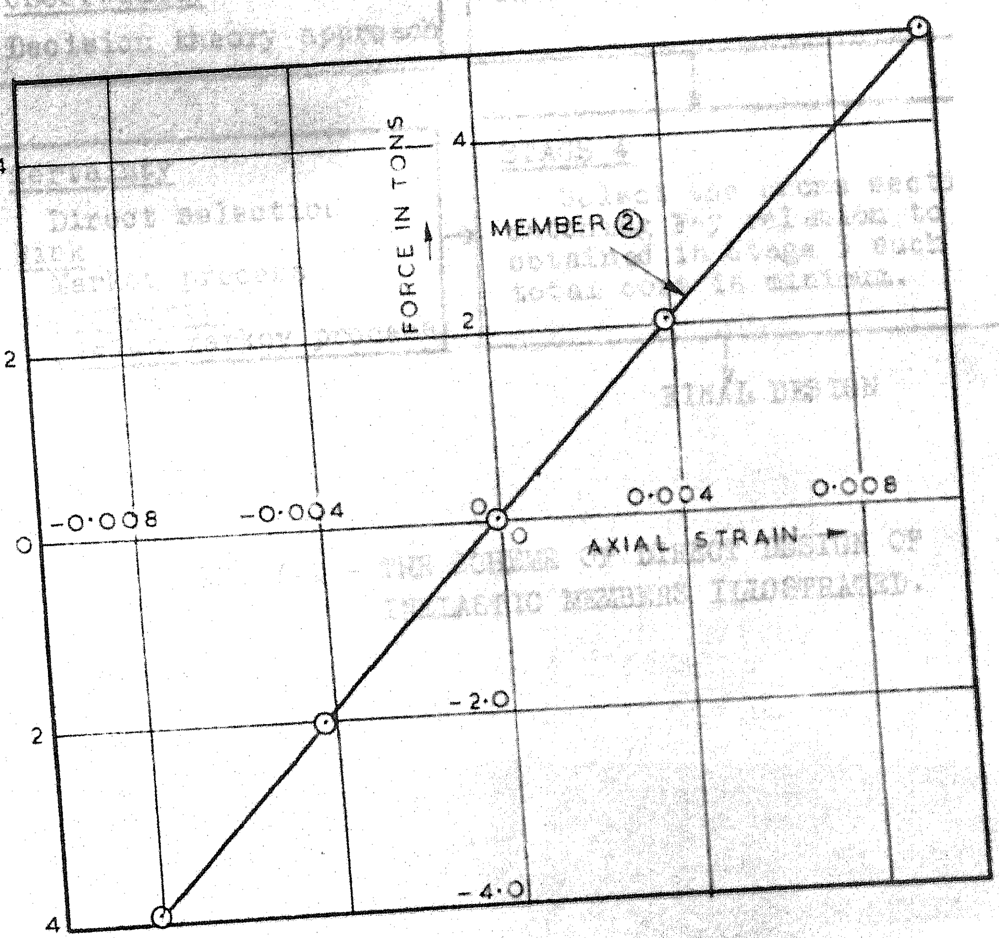


FIGURE 6.4 LOAD AND FORCE SPACES OF TWO BAR TRUSS WITH ILLUSTRATION OF VARIOUS STATES.



(c) TASK CURVE OF MEMBER ONE



(d) TASK CURVE OF MEMBER TWO

FIGURE 6.5 TASK CURVES OF TWO BAR TRUSS.

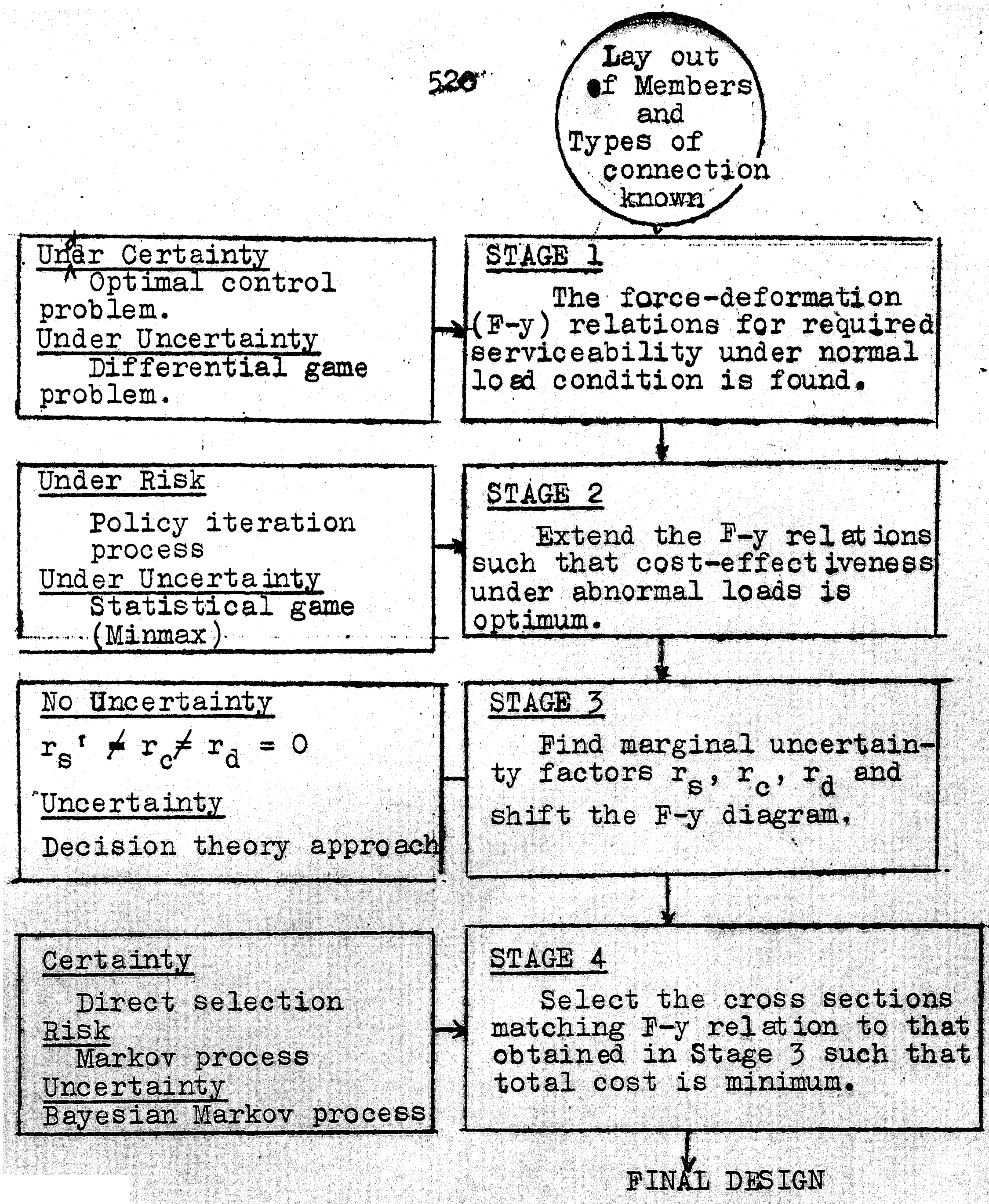


FIG. 7.1 - THE SCHEME OF DIRECT DESIGN OF INELASTIC MEMBERS ILLUSTRATED.

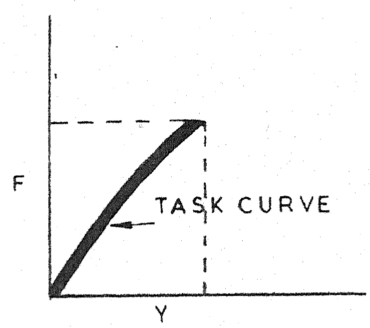
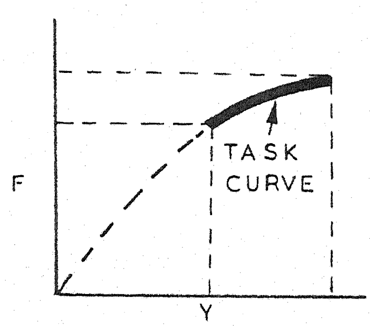
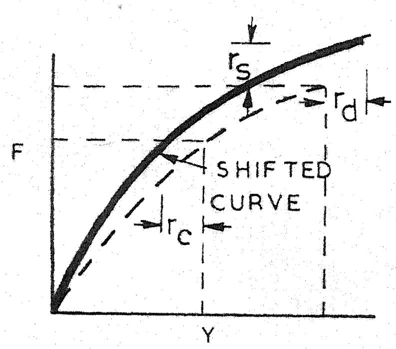
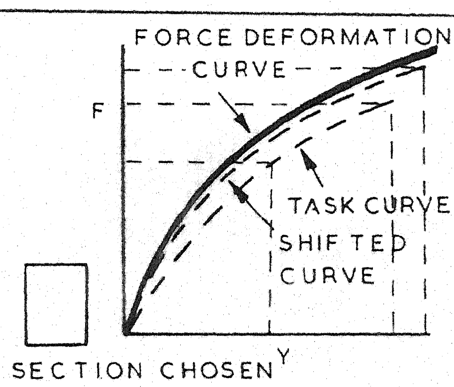
STAGES	OPERATIONS	PURPOSE	
		TYPE OF UNCERTAINTY	VALUES
1		Strategic Uncertainties (Timing, Sequence, direction, position)	Safety and Serviceability for normal load Condition (Not safe for abnormal loads)
2		Statistical Uncertainties probability density function not known)	Safety and ductility (Through cost-effectiveness) (For abnormal loads)
3		Non-measurable Uncertainties	–
4		Uncertainties in Structural behaviour	Minimum Cost

FIGURE 7.2 DECISION OPERATIONS ILLUSTRATED.

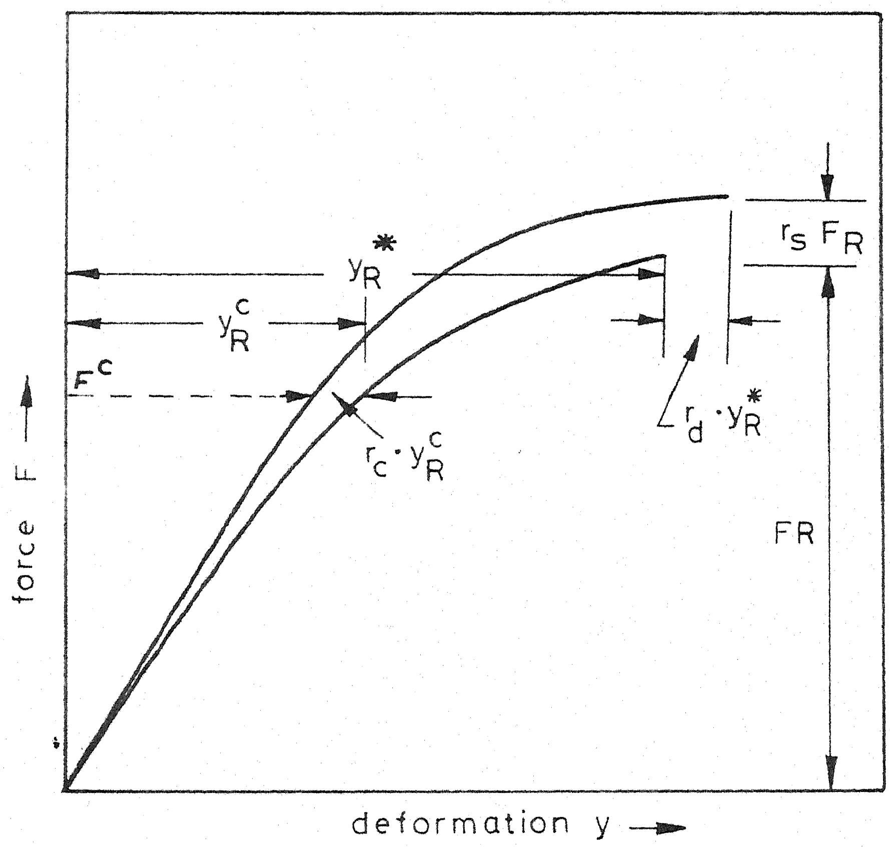
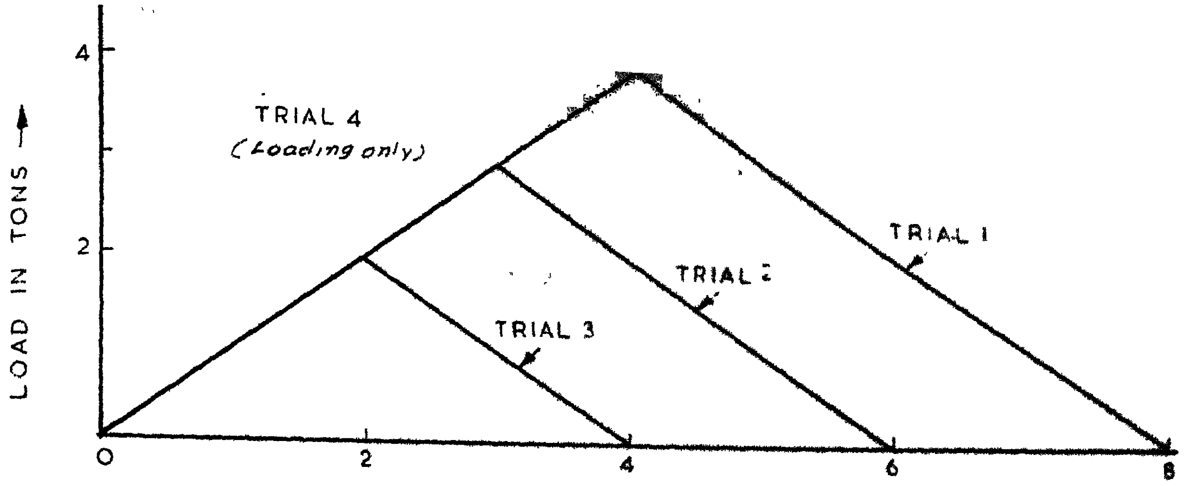
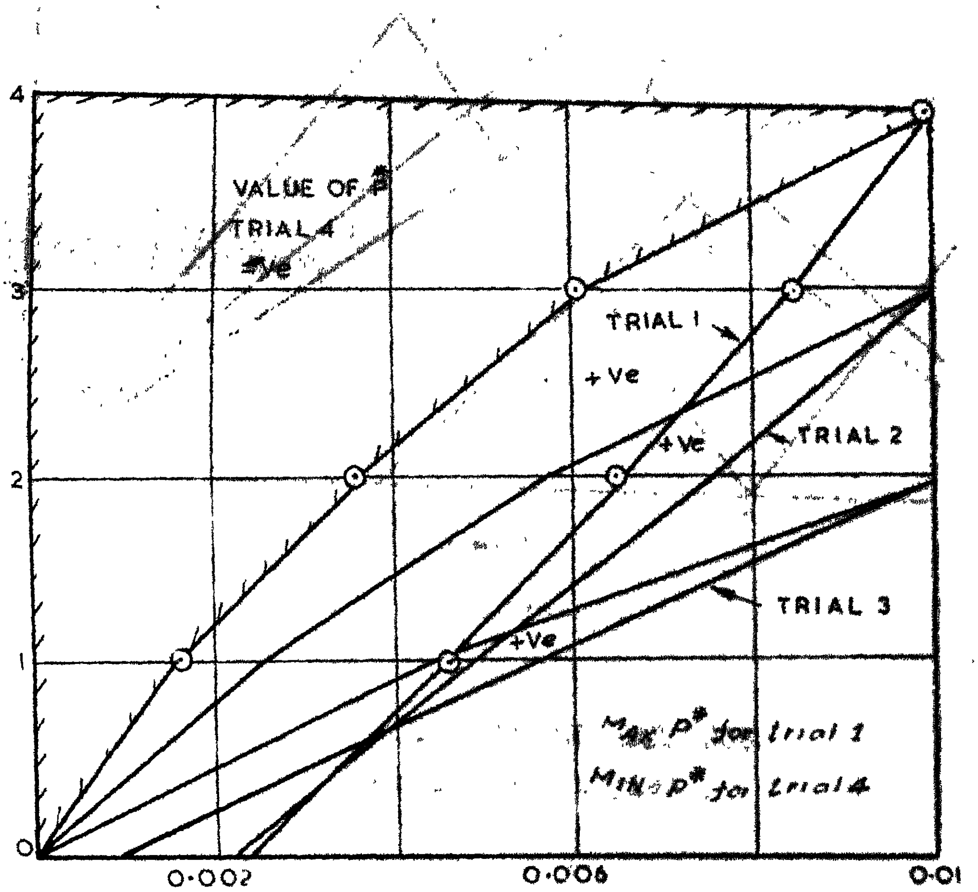


FIGURE 7.3 SHIFTING OF FORCE-DEFORMATION RELATIONS ILLUSTRATED.

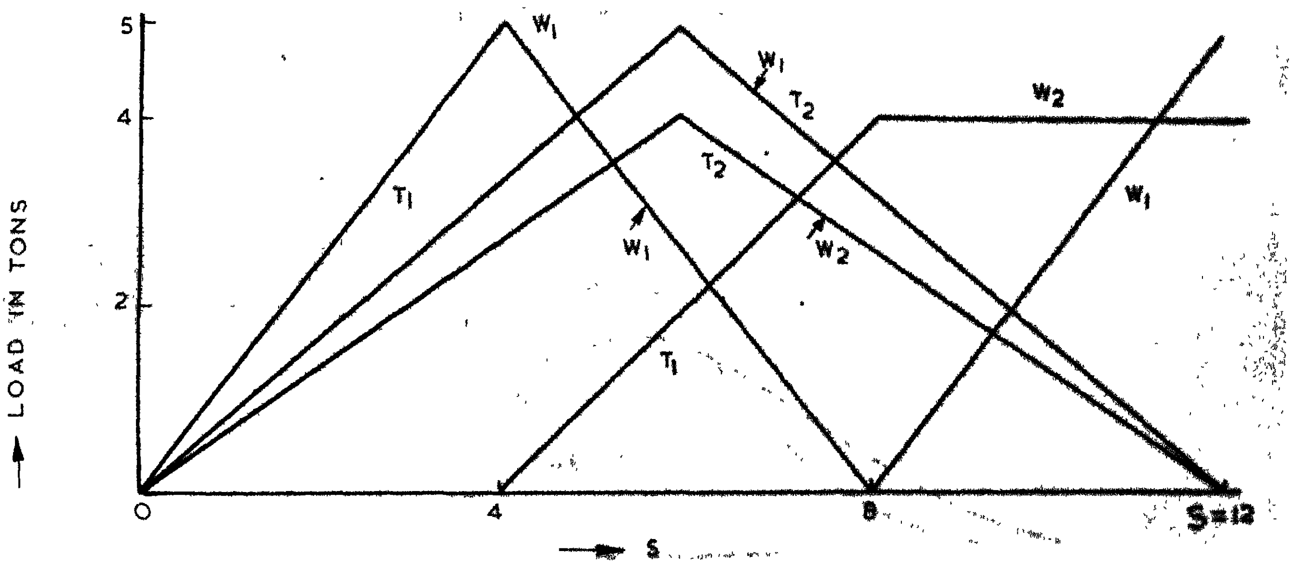


(a) LOADING STRATEGIES OF PLAYER I



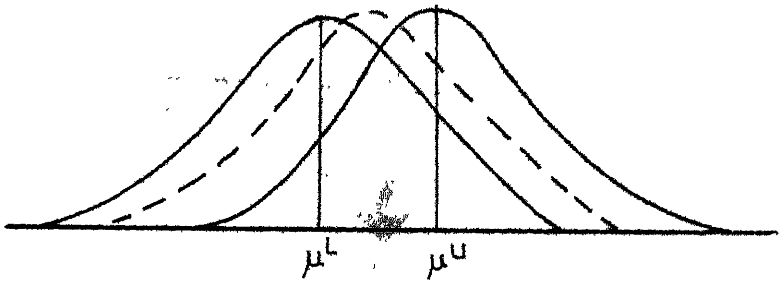
(b) TASK CURVES FOR DIFFERENT STRATEGIES OF LOADING

FIGURE 8. EXAMPLE OF TENSION BAR
(Maximization of P* illustrated).

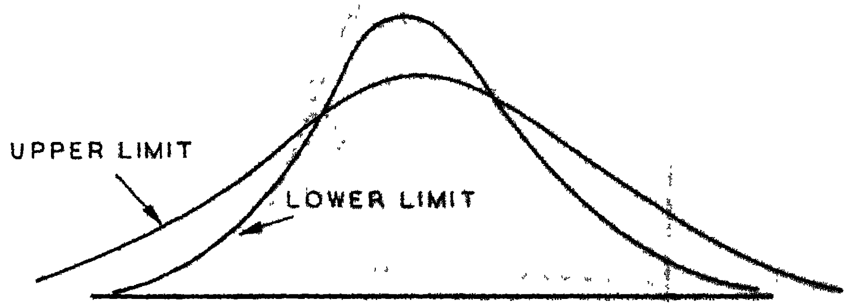


T_1 = TRIAL 1 T_2 = TRIAL 2

FIGURE 8.2 EXAMPLE OF TWO BAR TRUSS
(Loading Strategies).

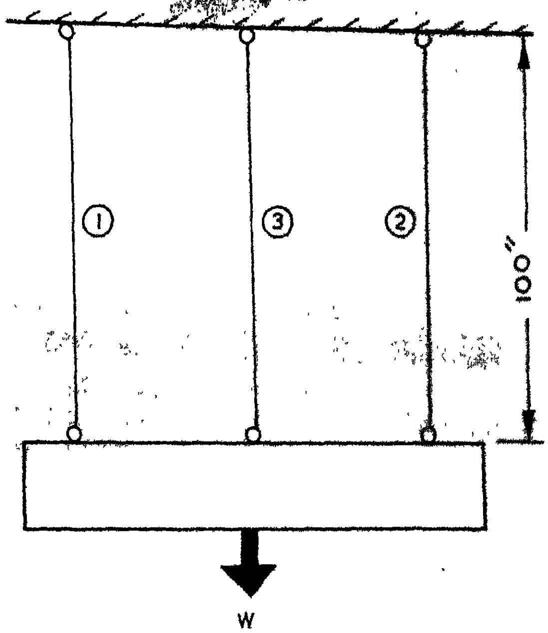


(a) μ-STRATEGIES (MEAN VALUES)

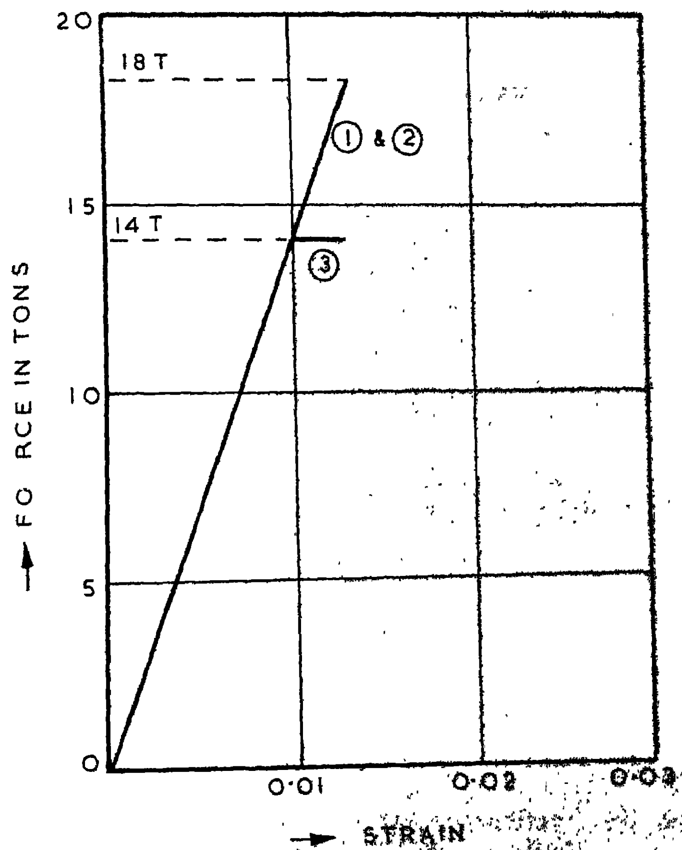


(b) θ-STRATEGIES (STANDARD DEVIATION)

FIGURE 9.1 NATURE'S STRATEGIES IN THE SURVIVAL PLAY OF STRUCTURAL ACTION GAME.



(a) THREE BAR TRUSS



(b) AXIAL FORCE AXAIL STRAIN RELATION

FIGURE 9.2 EXAMPLE OF THREE BAR SYSTEMS.

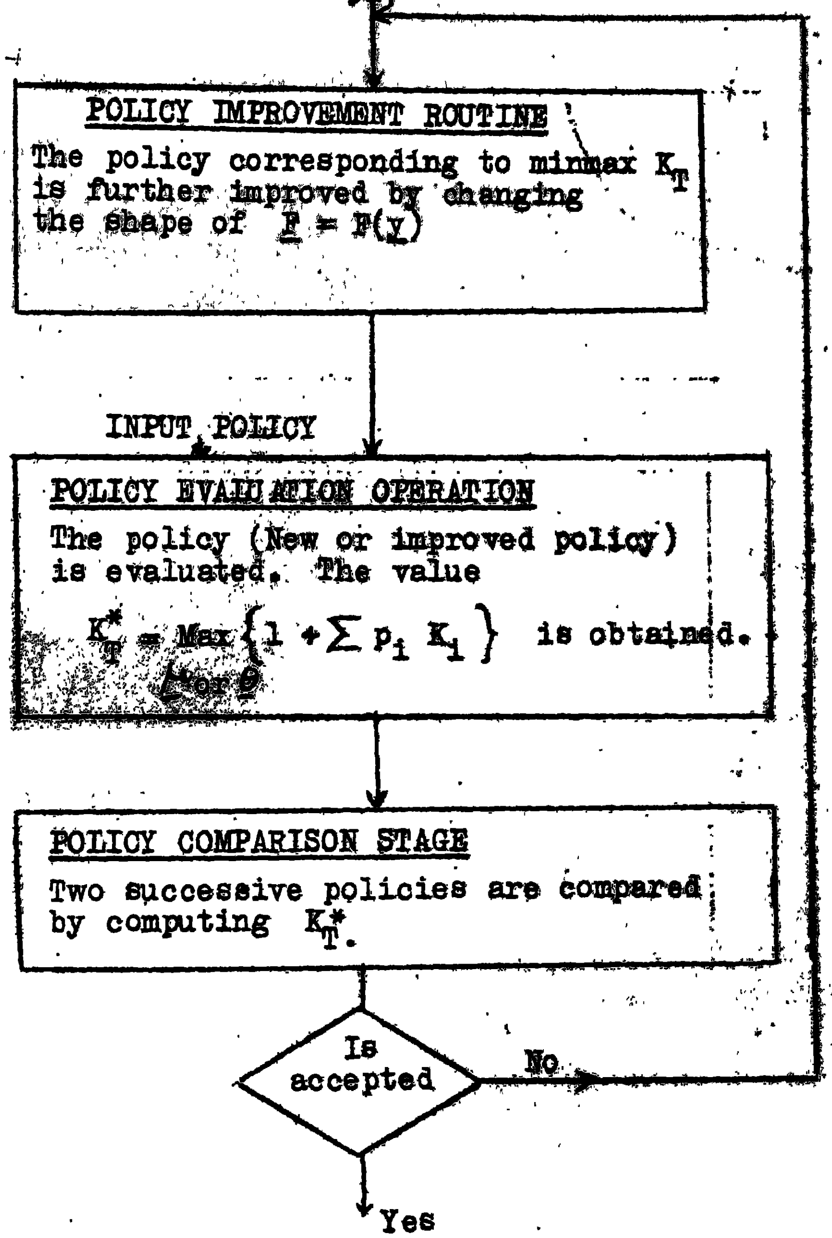
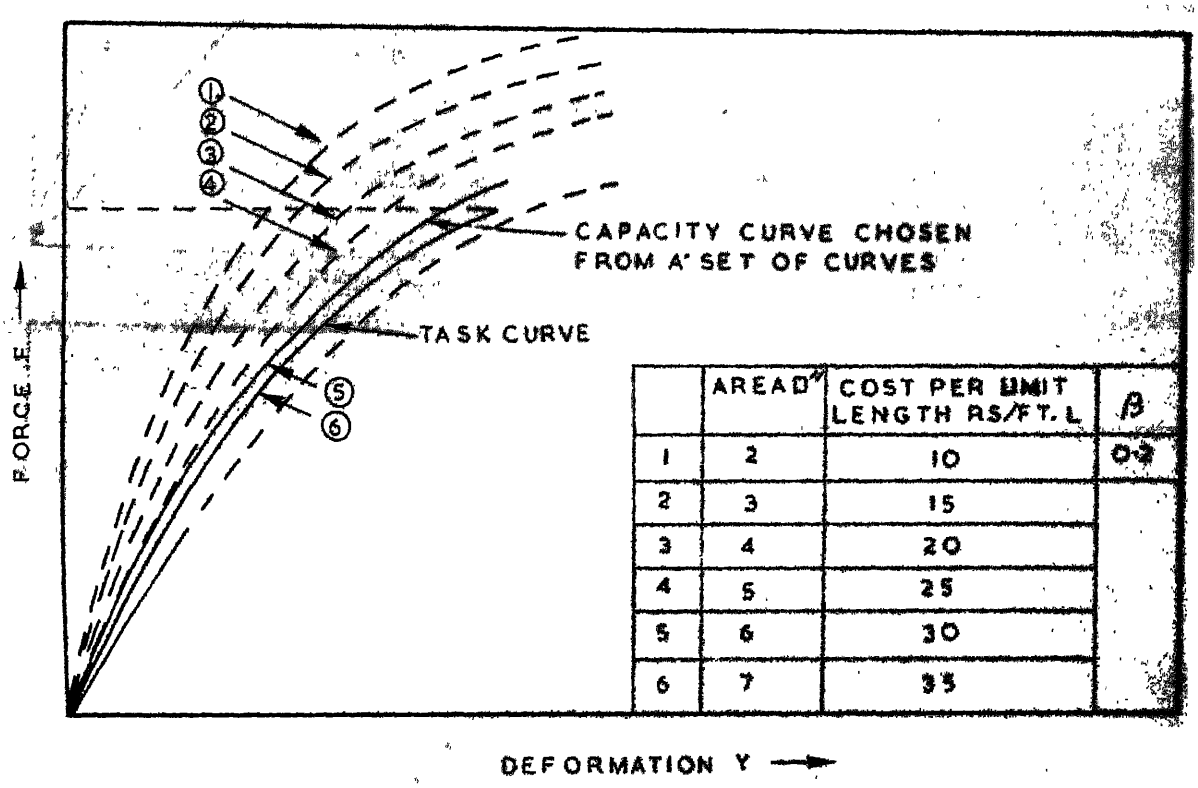


FIG. 9.3. - POLICY ITERATION PROCEDURE FOR SURVIVAL PLAY OF STRUCTURAL ACTION GAME.



	AREA D²	COST PER UNIT LENGTH RS/FT. L	β
1	2	10	0·2
2	3	15	
3	4	20	
4	5	25	
5	6	30	
6	7	35	

FIGURE 10.1 MATCHING OF FORCE-DEFORMATION RELATION AND TASK CURVE.

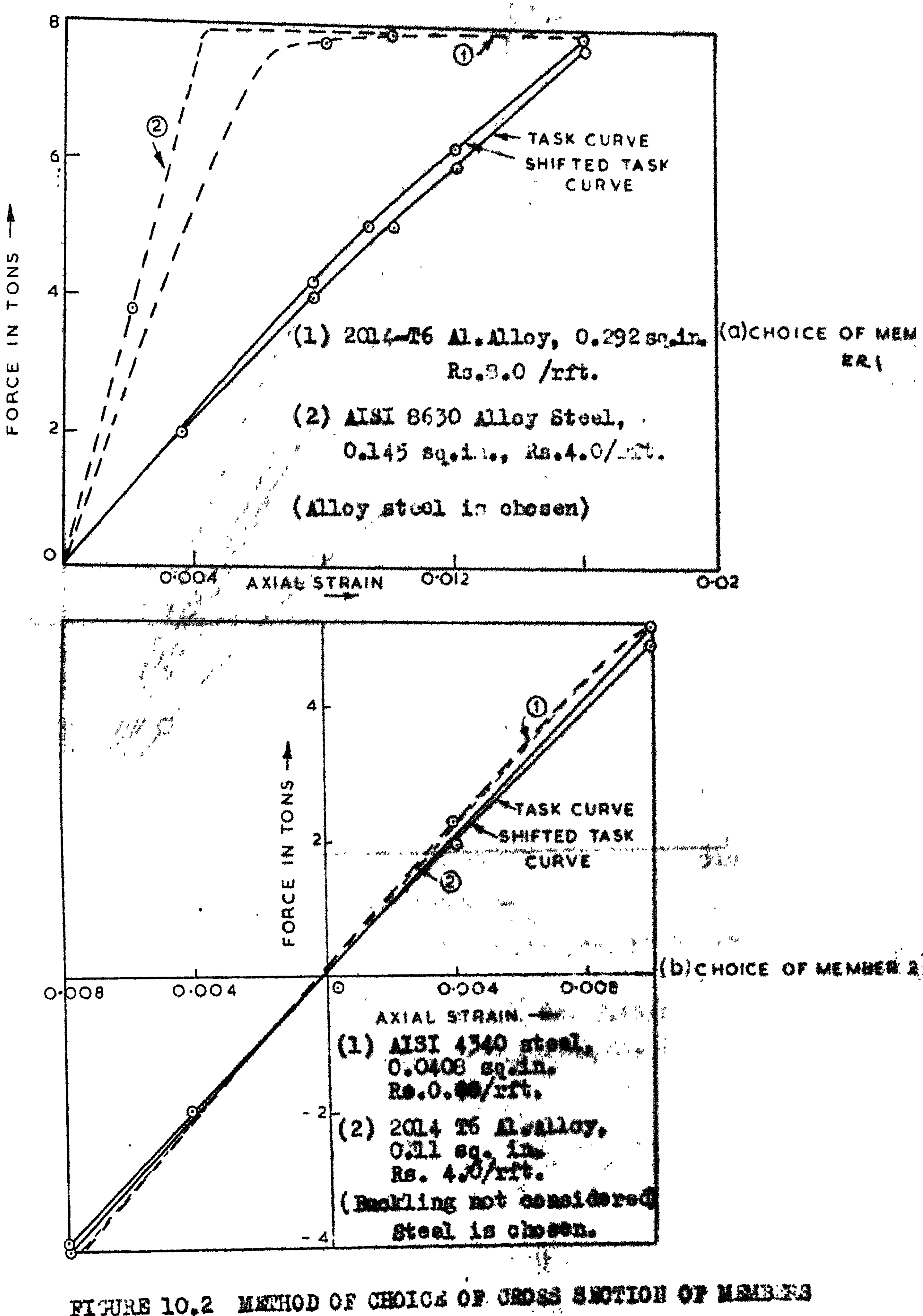


FIGURE 10.2 METHOD OF CHOICE OF CROSS SECTION OF MEMBERS
(Two Bar Truss)

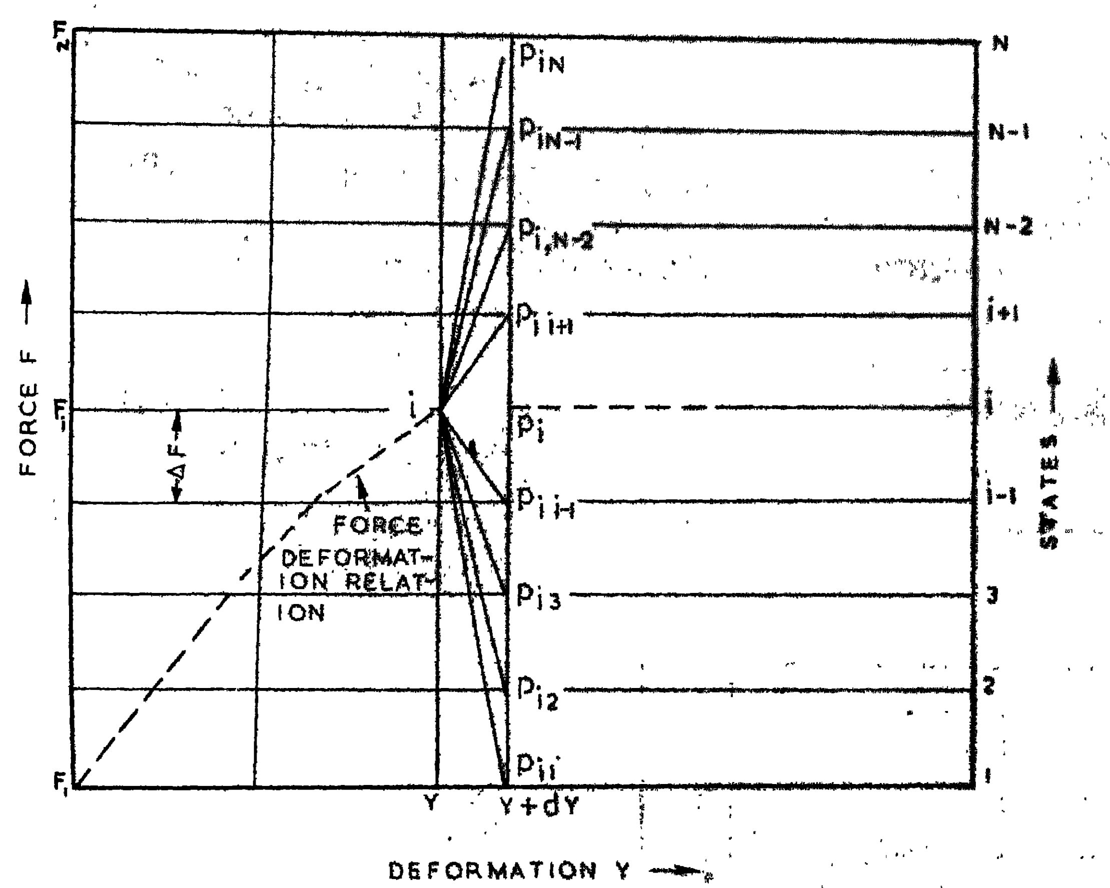


FIGURE 10.4 IDEALIZATION OF THE STOCHASTIC PROCESS OF FORCE-DEFORMATION RELATIONS AS MARKOV PROC[illegible]

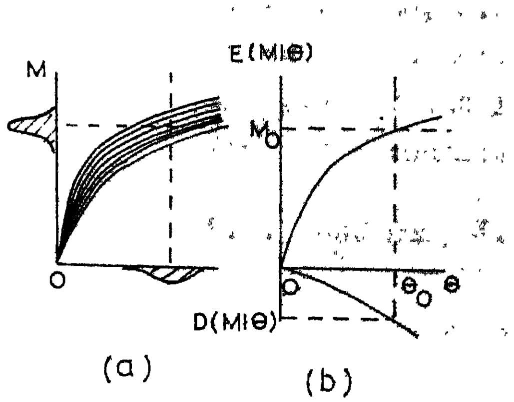


FIGURE 10.5 RANDOM FORCE-DEFORMATION RELATIONS.
(Ref. 113)

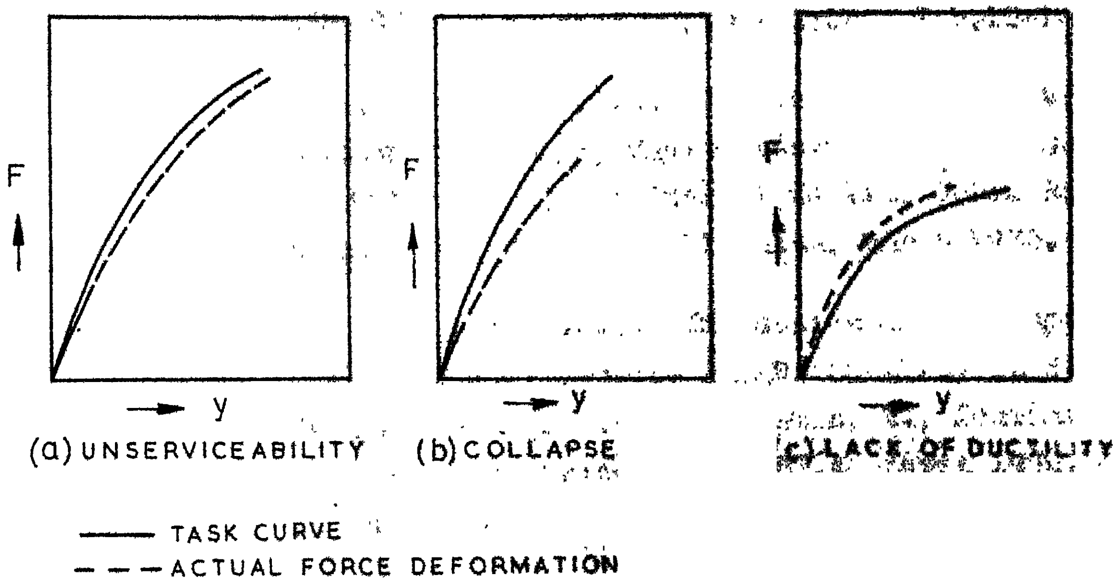


FIGURE 10.6 FAILURE MODES IN COST-EFFECTIVENESS ANALYSIS OF [illegible]

LIST OF PUBLICATIONS

1. Nair, N.G. and Rao, J.K.S., 'On Continuum Mechanics and Structural Concrete', Paper presented at the 13th Congress of the Indian Society of Theoretical and Applied Mechanics held at Durgapur, Dec. 1968.

2. Nair, N.G., and Rao, J.K.S., 'Structural Planning and Design as a Multistage Optimization Process', Paper presented at the 3rd Annual Meeting of the Computer Society of India at Trivandrum, Jan. 1969.

3. Nair, N.G., 'Game-Theoretic Methodology of Structural Planning and Design' in Intensive Course on Optimization in Structural Design, Vol. 2, Ed. by J.K.S. Rao and P.N. Murthy, Indian Institute of Technology, Kanpur, March 1969.

4. Nair, N.G., Rao, C.V.S.K., and Rao, J.K.S., 'Economy in Floor Construction by Rational Choice and Design', Proceedings of the N.B.O. Symposium on Cost Reduction Case Studies, New Delhi, Feb.12-14, 1970.

5. Nair, N.G., and Rao, J.K.S., 'On the Nature of Structural Design Decisions', Paper presented at the Seminar on Modern Trends in Design, Motilal Nehru Regional Engineering College, Allahabad, March 1970.

6. Nair, N.G. and Rao, J.K.S., Discussion on the paper titled 'Safety Factors and Probability in Structural Design' by Ang, A.H.S. and Amin, M., Journal of Structural Division, ASCE, ST.4, April 1970, pp. 853-856.